He II Film Profile and Relative Importance of Height and Surface Pressure*

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The thickness d of the He II film at 1.40°K *vs* height h to 40 cm has been measured by an optical method. Results agree better with an expression of the form $d=kh^{-\frac{1}{3}}$ than with $d=kh^{-\frac{1}{2}}$ or $h=(a/d)^3+(b/d)^2$. Evidence was obtained suggesting that the contour of the film cannot be accounted for by considering it to be an adsorbed film subjected to a varying pressure. The pressure would decrease with height due to gravitational effects in a vertical gas column.

INTRODUCTION

DISCUSSIONS of early measurements of the thickness of the He II film can be found elsewhere.[1,2] More recently Bowers,[3] using a microbalance to weigh the film covering an aluminum foil, found that the thickness d varied with height h in accord with the relation, $d=11.8\times10^{-6}h^{-\frac{1}{2}}$. Measurements were made to heights of about 6.5 cm. The thickness at a given height was found to be independent of temperature from 1.2°K up to 0.006° below the lambda point. Burge and Jackson[4] used an optical method to measure the thickness of the film up to heights of 1.2 cm and found $d=kh^{-1/z}$ with z varying from 3.5 at 1.1°K to 2.5 at 2.1°K. At a temperature of 1.5°K, $k=1.9\times10^{-6}$. Ham and Jackson[5] found that for heights of 0.4 to 1.6 cm, $d=kh^{-1/z}$ with k varying from 3.15×10^{-6} to 2.96×10^{-6} and z from 2.26 ± 0.02 to 2.59 ± 0.45, respectively, as the temperature was varied from 2.05 to 1.32°K.

Frenkel[6] and Schiff[7] have discussed the formation of the film in terms of the Van der Waals attraction between the helium atoms and the substrate, and the gravitational potential of the helium atoms. These considerations led to the expression:

$$d=kh^{-\frac{1}{3}}, \tag{1}$$

in which k depends on the material of the substrate. From a consideration of the zero-point energy of the superfluid atoms in the film and the gravitational potential, Bijl, de Boer, and Michels[8] obtained

$$d=11.2\times10^{-6}h^{-\frac{1}{2}}. \tag{2}$$

Atkins[9] combined the zero-point energy arising from longitudinal Debye waves with the gravitational and Van der Waals energy to obtain

$$h=(a/d)^3+(b/d)^2. \tag{3}$$

For heights of 0.8 to 1.6 cm Ham and Jackson[5] could fit their measurements to this expression by taking $a=1.89\times10^{-6}$ and letting b vary from 2.78 to 2.54×10^{-6} as the temperature varied from 2.05 to 1.32°K. Rice and Widom,[10] and Meyer[11] obtained $d=kh^{-\frac{1}{3}}$ from the following considerations. If the helium gas in contact with the film is considered to be an ideal gas, the pressure p exerted on the film at a height h above the liquid level will be given by

$$p=p_0\exp(-Mgh/RT), \tag{4}$$

in which p_0 is the vapor pressure of bulk liquid at the temperature T, M is the molecular weight, and g the acceleration due to gravity. They postulated that the thickness of the film at a height h would be that appropriate to an adsorbed film at temperature T and saturation p/p_0. Bowers[12] has measured the adsorption isotherm to be

$$-\ln(p/p_0)=K/\nu^3, \tag{5}$$

in which ν is the number of adsorbed layers and K is a constant depending on temperature. Combining Eqs. (4) and (5) gives Eq. (1). In support of this point of view Jackson[13] has reported observation of a film on a solid body which is in contact only with the vapor, and that this film has the same profile as when the body is in contact with the liquid.

EXPERIMENTAL

Figure 1 shows some of the essential parts of the apparatus used. The purpose of the work was to measure the variation in thickness of the He II film on mirror M_3 as a function of height h from the surface of the bulk liquid supplying the film. Windows W_1 and W_2 sealed off an inner chamber which was made of brass except for the stainless steel supporting tube Y. This

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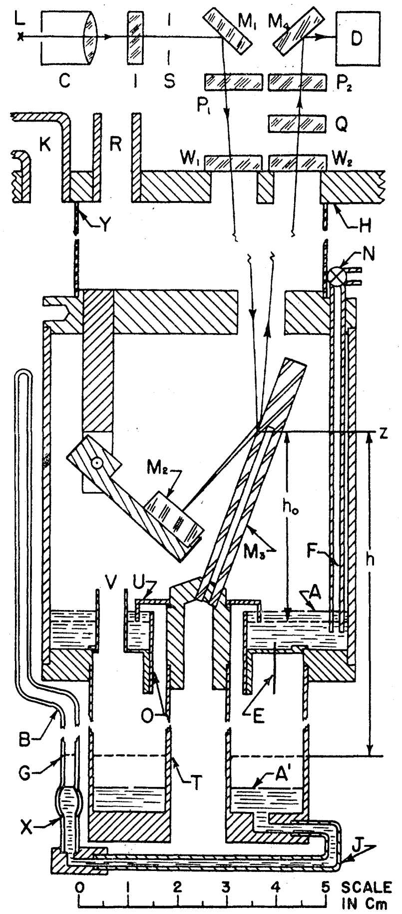

FIG. 1. Apparatus for measurement of film thickness *vs* height.

chamber was connected to a vacuum system for evacuation through tube R. The distance from the head plate H to point Z was 56 cm. With the level G of liquid at its lowest value, h was 40 cm, its maximum value. Except for some optical components shown schematically above the head plate H, the drawing is to the scale shown. A Dewar, not shown, containing liquid helium, surrounded this inner chamber and made a vacuum tight connection to an extension of head plate H. The temperature of the helium in this Dewar was lowered to the desired value by pumping through tube K. A second Dewar containing liquid air surrounded the helium Dewar. Narrow strips in the silvering of both Dewars permitted the viewing of the liquid level G in the glass sight tube B by means of a cathetometer. This tube was connected to the inner chamber by means of Kovar seal X and copper tube J.

Light source L, collimator C, interference filter I, aperture stop S, front silvered mirror M_1, and polaroid P_1 provided a beam of plane polarized light directed down the Dewar. The average wavelength of the light was 5458 A with a half-width of 50 A as determined by the filter. L was an incandescent projection lamp the intensity of which was varied by means of a variac in the power line. The two coated glass windows W_1 and W_2 permitted the light beam to enter and leave the inner chamber. The light beam, after being reflected from mirror M_3 which was covered by a He II film, struck mirror M_2. Mirror M_2 was adjusted to return the light beam to a second reflection from the film covered mirror M_3 and then out of the Dewar through window W_2. After passage through quarter wave plate Q and polaroid P_2, the light was reflected into D by front silvered mirror M_4. D was an RCA-1P21 photomultiplier tube the output of which was fed to a dc high-gain current amplifier connected to an Esterline-Angus chart recorder. The high voltage for the photomultiplier tube was supplied by an Atomic Instrument Company model 312 power supply.

A collimated beam of plane polarized, monochromatic light incident on a polished metal surface is reflected as elliptically polarized light. The orientation and eccentricity of the ellipse are functions of the optical constants of the metal and the wavelength, angle of incidence, and orientation of the plane of polarization of the incident light. A layer of transparent material, the He II film in this case, present on the metal surface M_3, changes the orientation and eccentricity of the reflected light. These changes are interpreted in terms of the variation in thickness of the film on M_3. Burge and Jackson[4] were the first to make use of these changes to measure the thickness of the He II film.

M_3 was a $\frac{3}{4}$ in.$\times 1\frac{3}{8}$ in.$\times \frac{3}{16}$ in. highly polished stainless steel mirror set such that the angle of incidence of the incident light was 70°. Polaroid P_1 was adjusted such that the angle between the plane of polarization of the incident light and the plane of incidence was 38°. M_2 was a front silvered mirror adjusted for approximately normal incidence so as to introduce no changes in the light as a result of the film on it.

Measurements were taken by adjusting Q to give plane polarized light with no film on M_3. Polaroid P_2 was adjusted so as to transmit a minimum of this plane polarized light as denoted by D. With film on M_3 the quarter wave plate Q was left fixed and P_2 was rotated through an accurately measurable angle to the new position for minimum light transmission. The relative thickness of the film was then determined from the change in angle of P_2.[5]

A direct calculation of the film thickness from the characteristics of the elliptically polarized light involves many constants which are not accurately known. Instead of using a film such as one of barium stearate of known thickness for calibration, the variation in the He II film thickness d with height h above the bulk liquid supplying the film was determined by comparing the measured thickness at h with that measured at a fixed height h_0 of 4 cm.

The experimental procedure was as follows. The inner chamber was pumped through a liquid air trap to a pressure of about 10^{-5} mm of Hg by a diffusion pump before a run and sealed off just before adding liquid helium to the surrounding helium Dewar. After adding helium to this Dewar and reaching temperature equilibrium at about 1.40°K, the angular setting of P_2 for minimum light transmission with a dry mirror was found. Valve N, controlled from the head, was then opened to admit liquid to the inner chamber through tube F. With N below the liquid level in the helium Dewar, the helium admitted to the inner chamber should have been quite free of impurities as such would have been frozen out on the walls of the helium Dewar. When the liquid level in the center part of the inner chamber reaches A, liquid will spill over into the lower part through tube O until the lower level is at some chosen value such as A' at which N is closed. At this time the film on M_3 is dependent only on level A since film from the lower liquid level A' is cut off from M_3 by the umbrella U. Vapor pressure equilibrium between the upper and lower parts of the inner chamber was maintained through vent pipe V.

Angular settings of P_2 for minimum light intensity into D were recorded *vs* time as shown in Fig. 2 for a height of 17.7 cm. Points prior to time t_0 show the gradual decrease in film thickness on the mirror at the place of light incidence Z due to a change in the upper level from A to a position h_0 cm below Z. This level dropped due to film flow through tubes V and O and bulk flow through the 0.006-in. i.d. nickel tube E. This caused the lower level to rise up from A'. At t_0 the upper level broke contact with the cylindrical umbrella U, and at that instant the film at Z on M_3 must have been supplied by bulk liquid h cm below, instead of h_0 cm below. Points after t_0 show that the film thickness at Z does not change measurably with the relatively small changes in h occurring after time t_0. $\theta(h_0)$ is the angular setting of P_2 corresponding to a minimum in transmitted intensity just before t_0 and $\theta(h)$ is the setting just after t_0 as determined by drawing straight lines through the measured points to t_0. The time t_0 is taken as that at which the recorder deflects abruptly to a lower value.

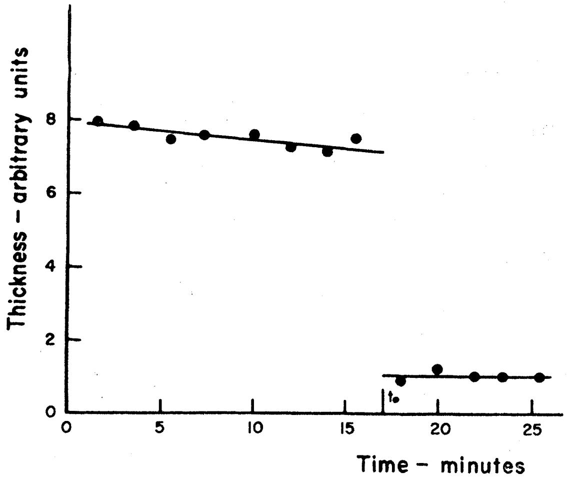

FIG. 2. A typical thickness determination.

More liquid was then let into the inner chamber to bring the upper level back to A and the lower level to a new A' and values of $\theta(h_0)$ and $\theta(h)$ were determined for the new h. By viewing level G in the glass sight tube B at time t_0 with a cathetometer, h was obtained.

The bath temperature was controlled by pumping through tube K, and determined by measuring the helium vapor pressure with an oil manometer. Precautions were taken to maintain as nearly as possible isothermal conditions within the lower part of the inner chamber containing the mirror and tube T. Except for the small solid angle subtended by the opening to admit the light beam, these structures were surrounded by opaque walls immersed in liquid helium. The brass plate forming the top of the mirror chamber had holes drilled radially into it to permit liquid to flow in to help maintain it at the temperature of the helium in the Dewar. The tube surrounding the mirror chamber was of $\frac{1}{16}$-in. brass and that surrounding T was of $\frac{1}{32}$-in. brass. M_3 was mounted by a Wood's metal joint at the top of a brass tube T, 36 cm long and with a wall thickness of $\frac{1}{32}$ in. The mirror had a $\frac{1}{8}$-in. diameter hole drilled into its center as shown. Thus, during an experiment bath helium filled T and the hole in M_3 to aid further in maintaining the surfaces on which the film was being studied at the bath temperature. Minimum light intensity compatible with the sensitivity of the apparatus was used. Evidence that the light was not seriously affecting the film was obtained by increasing this minimum intensity by a factor of 5 without detecting any change in the film thickness. Further work (DHL) has indicated that limiting the periphery over which film could be supplied to the mirror from the lower bath by a factor of 1:20 does not affect the film thickness.

Special mounts for glass windows W_1 and W_2 had to be developed which would apply minimum stress to them. Otherwise, changes in stress on these windows due to variations in stress on the head plate H introduced an intolerable amount of birefringence in the glass. The difference method of measurement employed here eliminated remaining small effects.

As in previous work, small amounts of impurity strongly affected the film thickness. Reproducible results, presumably free from the effects of impurities, were obtained by pumping out the inner chamber to 10^{-5} mm Hg before a run. Admitting helium to the inner chamber as liquid from the bath rather than by condensing in gas further reduced impurities added to the system.

RESULTS

The data in Fig. 2 show that the thickness of the film, as measured at Z on mirror M_3 of Fig. 1, decreased abruptly as the level of bulk liquid supplying the film dropped from a distance h_0 to a distance h below point Z.

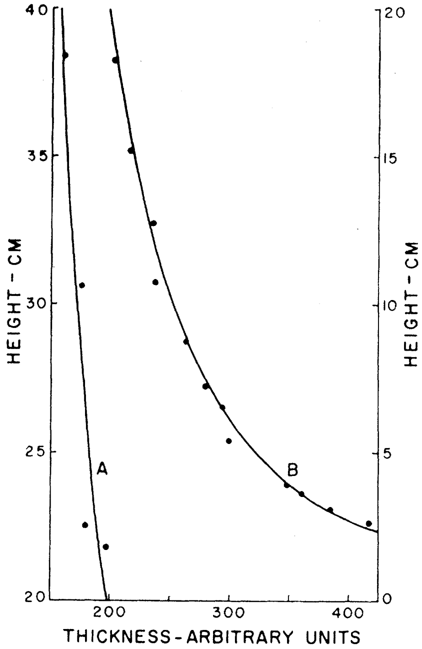

FIG. 3. Profile of He II film at 1.4°K.

This change in thickness occurred in a time less than the time constant of the amplifier and recorder, which was about 0.3 second. This abrupt drop in level of bulk liquid supplying the film accompanied the separation of liquid from the cylindrical umbrella *U*. Vent pipe *V* maintained a constant vapor pressure above the film on M_3 at *Z* where measurements were taken. Since there was an abrupt change in film thickness accompanying the abrupt change in distance to the liquid supplying the film, while there was no change in the vapor pressure to which the film on the mirror was subjected, it would seem that the thickness of the film is dependent principally on its height above the bulk liquid supplying it. Although the pressure in a vertical gas column varies with h as given in Eq. (4), it would seem that either the pressure is not a significant factor in determining the thickness of the film, or the superfluid properties of the film on the surfaces extending throughout and surrounding the gas column prevent the pressure from varying with height as given in Eq. (4).

Thus, it may not be possible to account for the contour of the film by considering it to be an adsorbed film subjected to a pressure varying with height as is implied in combining Eqs. (4) and (5).

The thickness d of the film at height h_0 was determined from the difference between $\theta(h_0)$ and the angular setting of P_2 for minimum transmission with a dry mirror. The thickness at other heights was then found from the difference between $\theta(h)$ and $\theta(h_0)$. Figure 3 shows the thickness of the film as a function of height to 40 cm. Curve *A* is referred to the left scale and curve *B* to the right scale. The points are those taken during a typical run at 1.4°K. The solid curve is a plot of $d=kh^{-1/z}$ where $k=560$ and z is 2.9. These values were obtained from the intercept at $h=1$, and the slope of curve *A* of Fig. 4 which was taken as the best fit to the experimental points plotted as $\log d$ *vs* $\log h$. Errors in determining the setting of P_2 for minimum transmission with a dry mirror could change the value of k by ±30 and z by ±0.2.

Figure 4 also shows that the measured points can be represented well by a single straight line on a log-log plot up to heights of 40 cm. Curve *B* is for $k=560$ and $z=3$ and curve *C* is for $k=560$ and $z=2$. It is seen that the data are represented better by $d=kh^{-\frac{1}{3}}$ than by $d=kh^{-\frac{1}{2}}$ in the range of heights from 2.5 to 40 cm.

Equation (3) can be fitted to the measured values represented by curve *A* of Fig. 4 by taking $a=533$ and $b=208$. Since a is much larger than b and since Eq. (1) fits the data quite well as shown by curves *A* and *B*, it appears that Eq. (1) is adequate in the range of heights from 2.5 to 40 cm. Others[13] have observed results for much smaller heights which followed Eq. (3) with $a=189$ and $b=254$. Curve *D* is a line normalized to the data at a height of one cm and having a slope given by a plot of Eq. (3) using $a=189$ and $b=254$. The dashed portion represents an extrapolation beyond the measured values.

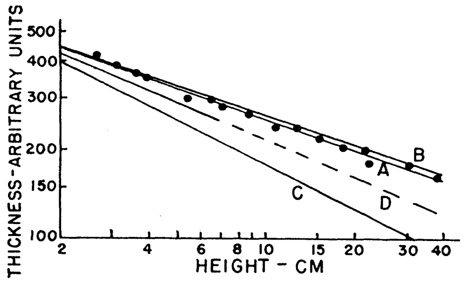

FIG. 4. Power law dependence of film profile.

While the plot of thickness *vs* height measurements occasionally showed deviations from a smooth curve, these could not be correlated with the type of irregularities described previously.[14] The difference might be accounted for by the fact that thicknesses reported here were measured for the static film, whereas, the thicknesses previously reported were determined from measured properties of the moving film.

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