Determination of (Σ, K) Relative Parity*

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Different relative parity assignments are shown to lead to distinguishable angular distributions in the $K^+ + K^+ + \bar{K}^0$ annihilation mode of unpolarized ($\bar{\Sigma}^-, p$) systems. An impact parameter argument is used to limit consideration to the two cases λ, $l=0, 1$ and λ, $l=0, 1, 2$, where λ denotes the relative angular momentum of the $2K^+$ system and l, that of the $\bar{K}^0$ with respect to the center of mass of the $2K^+$ system.

THE question of the relative parities of the strange particles[1,2] has led recently to a number of interesting theoretical proposals regarding their determination.[3–7] In particular, much attention has been given to the problem of the determination of the parities of specific (Hyperon, K) systems relative to the nucleon.[8–10] We wish to indicate here some considerations relating the observation of the $3K$ annihilation mode of $\bar{\Sigma}^- + p$ to a possible determination of the relative (Σ,K) parity. Specifically, different relative parity assignments lead to distinguishable angular distributions in the products of the annihilation in flight of unpolarized ($\bar{\Sigma}^-$,p) systems. We consider the annihilation mode

$$\bar{\Sigma}^- + p \rightarrow K^+ + K^+ + \bar{K}^0. \tag{1}$$

(Like considerations would hold for $\bar{\Sigma}^+ + p \rightarrow K^0 + K^0 + \bar{K}^0$; this would presumably be more difficult to observe.) Let L, λ denote the relative angular momenta[11] of the ($\bar{\Sigma}^-$,p) and $2K^+$ systems, respectively, and l the relative angular momentum of $\bar{K}^0$ with respect to the center of mass of the $2K^+$ system. From the indistinguishability of the $2K^+$'s which obey Bose statistics, λ is even. Conservation of parity in strong interactions implies

$$(-1)^{L+1}\omega_\Sigma = (-1)^l \omega_K, \tag{2}$$

where ω_Σ, ω_K are the parities of Σ and K, respectively, relative to the nucleon.[12] Then

$$r = \omega_\Sigma \omega_K = (-1)^{L+l+1}, \tag{3}$$

where r is the relative parity of the K with respect to the Σ.

Further, conservation of total angular momentum implies

$$\mathbf{L} + \mathbf{S} = \boldsymbol{\lambda} + \mathbf{l}, \tag{4}$$

where $\mathbf{S}$ is the total spin of the ($\bar{\Sigma}^-$,p) system and may take the values 0 and 1.

For sufficiently low relative energies of the ($\bar{\Sigma}^-$,p) system (i.e., energies $\lesssim 30$ Mev) one may safely restrict oneself to $L=0$, 1 only. An impact parameter estimate of the angular momenta involved in the final state indicates that only the values λ, $l=0$, 1 are to be expected. (The product of the maximum momentum of a final particle and the K-meson radius $\sim \frac{1}{2}\hbar$.) λ must be 0 since it is even. Let $\mathbf{p}$, $\mathbf{k}$ denote unit vectors in the directions of the relative momenta of the ($\bar{\Sigma}^-$,p) and $2K^+$ systems, respectively, and $\mathbf{q}$ the unit vector in the direction of the momentum of the $\bar{K}^0$ relative to the center of mass of the $2K^+$ system. In Table I the possible forms of the amplitudes and angular distributions are tabulated.

The presence of a term $\propto (\mathbf{p}\cdot\mathbf{q})^2$ in the angular distribution would establish $r=-1$; no conclusion could be drawn in its absence. Note that the direction of $\mathbf{k}$ is uncorrelated.

Should experiment show a correlation between $\mathbf{k}$ and the directions $\mathbf{p}$, $\mathbf{q}$, this would indicate that $\lambda=2$, $l=2$ waves are present in the final state, requiring suitable extension of the above analysis. Proceeding as before one finds fifteen possible terms in the angular distribution for $r=1$ and twelve possible terms for $r=-1$. The twelve types of angular correlation common to the

TABLE I. The possible forms of the amplitudes and angular distributions for $r=\pm 1$.

$r=1$		$r=-1$	
Angular momenta	Transition amplitude	Angular momenta	Transition amplitude
$L=0$, $l=1$	$\mathbf{q}\cdot\mathbf{S}$	$L=0$, $l=0$	1
$L=1$, $l=0$	$\mathbf{p}\cdot\mathbf{S}$	$L=1$, $l=1$	$\mathbf{p}\cdot\mathbf{q}$, $\mathbf{S}\cdot\mathbf{p}\times\mathbf{q}$
Angular distribution for unpolarized ($\bar{\Sigma}^-$,p)			
$r=1$		$r=-1$	
$a+b\mathbf{p}\cdot\mathbf{q}$		$a'+b'\mathbf{p}\cdot\mathbf{q}+c'(\mathbf{p}\cdot\mathbf{q})^2$	

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[11] We use natural units, $\hbar=c=1$.
[12] In particular ω_K is the relative parity of the $\bar{K}^0$. This distinction is necessary only if (K^-,$\bar{K}^0$) do not form an isotopic doublet. In this connection see reference 7.

distributions for $r=1$ and $r=-1$ are:

$$1,\ \mathbf{p}\cdot\mathbf{q},\ (\mathbf{k}\cdot\mathbf{q})^2,\ (\mathbf{k}\cdot\mathbf{p})^2,\ (\mathbf{p}\cdot\mathbf{q})^2,\ \mathbf{q}\cdot\mathbf{k}\mathbf{k}\cdot\mathbf{p},\ (\mathbf{k}\cdot\mathbf{p}\times\mathbf{q})^2,\ (\mathbf{p}\cdot\mathbf{q})(\mathbf{k}\cdot\mathbf{q})^2,\ (\mathbf{p}\cdot\mathbf{q})(\mathbf{k}\cdot\mathbf{q})(\mathbf{k}\cdot\mathbf{p}),\ (\mathbf{p}\cdot\mathbf{k})^2(\mathbf{k}\cdot\mathbf{q})^2,\ (\mathbf{k}\cdot\mathbf{q})^4,\ (\mathbf{p}\cdot\mathbf{k})(\mathbf{k}\cdot\mathbf{q})^3. \quad (5)$$

The three terms present for $r=1$ and absent for $r=-1$ are of the form

$$(\mathbf{p}\cdot\mathbf{q})^2(\mathbf{k}\cdot\mathbf{q})^2,\ (\mathbf{k}\cdot\mathbf{p}\times\mathbf{q})^2(\mathbf{k}\cdot\mathbf{q})^2,\ (\mathbf{p}\cdot\mathbf{q})(\mathbf{p}\cdot\mathbf{k})(\mathbf{k}\cdot\mathbf{q})^3. \quad (6)$$

These terms would all arise from the $L=1$, $l=2$, $\lambda=2$ contingency for $r=1$. For $r=-1$, $L=1$ implies $l=1$ so that terms in which $\mathbf{p}$ occurs twice and $\mathbf{q}$ four times cannot appear in the angular distribution.

It is possible to discern the presence of these terms in the observed angular correlation via the following analysis. On averaging the angular distribution weighted with $[3(\mathbf{p}\cdot\mathbf{k})^2-1]$ over the directions of $\mathbf{p}$, the resulting angular correlation between $\mathbf{q}$ and $\mathbf{k}$ has the form

$$\alpha+\beta(\mathbf{q}\cdot\mathbf{k})^2+\gamma(\mathbf{q}\cdot\mathbf{k})^4. \quad (7)$$

Absence of the terms (6) implies $\gamma=0$. Deviation from a straight line plot in $(\mathbf{q}\cdot\mathbf{k})^2$ would serve to indicate a nonvanishing γ and further imply $r=1$. We wish to thank members of the Physics Department for their interest and helpful discussions, and in particular, Dr. M. Goldhaber, Dr. G. C. Wick, and Dr. C. N. Yang for their valued criticism.

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Electromagnetic Properties of π and K Mesons*

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A formalism is proposed which can give a smaller mass for the charged than for the neutral K meson, but a larger mass for the charged than for the neutral π meson. The theoretical prediction agrees with the experimental mass difference $M(K^0)-M(K^+)\approx 9.4$ electron masses if the rms radius of the charge distribution of the K meson is equal to 0.48×10^{-13} cm.

I. INTRODUCTION

According to the principle of charge independence, the charged meson (meaning π meson) and the neutral meson should have the same mass before the electromagnetic interaction is switched on. The present explanation of the mass difference $M(\pi^\pm)-M(\pi^0)$ is therefore based on the electromagnetic self-mass of the $\pi^\pm$, the electromagnetic self-mass of the π^0 being zero.

In the lowest-order perturbation theory, two Feynman diagrams give rise to the electromagnetic self-mass of the meson.[1] The first diagram is the familiar one corresponding to the virtual emission and reabsorption of a photon. In the second bubble diagram, the virtual photon is emitted and absorbed by the meson at the same point. The second diagram owes its existence to the requirements of gauge invariance of the interaction of mesons with the electromagnetic field.

Since both of these contributions are divergent, the integrals have been evaluated with an invariant cutoff function. Then the charged meson is found to be heavier than the neutral meson in agreement with experiments for the π mesons.[1] It has been found recently, however, that the neutral K meson is heavier than the charged K meson.[2] This poses a serious challenge to an explanation of the meson mass differences on electromagnetic grounds.

There is, however, an important distinction between the two contributions. In the first case, the nucleon-antinucleon pairs surrounding the meson can provide a natural cutoff; thus, as in the case of the electromagnetic self-mass of the nucleons, the use of the invariant cutoff functions that depend on a four-vector has a questionable meson-theoretical basis.[3] In the second case, since we disregard terms above the second order in the electromagnetic coupling constant e, the meson-nucleon interaction does not modify the interaction that occurs at one point and hence does not provide a natural cutoff. Hence we employ as usual an invariant cutoff function as a formal device to make a divergent integral finite in the second case.

In a previous article,[3] the Wick-Sorensen[4] (WS) method was used to obtain the electromagnetic self-mass of a physical nucleon. This paper presents an

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