in which the first approximation does not fit the experimental result, we must seek the origin in the difference between the real antiferromagnet and the simple Heisenberg model.

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Positron Annihilation in Metals

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A correlation function for a positron-electron pair within a metal is obtained by solving a Bethe-Goldstone equation. Thus one may take account of the many body effects, of screening and the exclusion principle, in computing the electron density at the position of the positron. The results indicate that one may, in this fashion, reconcile the Sommerfeld model of a metal with the experimental data on positron annihilation in metals.

I. INTRODUCTION

IN recent years attempts have been made to explain the failure of the Sommerfeld model of a metal to account for the observed positron annihilation rates in metals. It was apparent that no progress could be made until some account was taken of the interactions between valence electrons and positron within the metal. We propose to present an adequate treatment of these interactions. Our work follows most closely that of Ferrell.[1] The basis of the latter's discussion was the Bohm and Pines theory[2] of collective motion in an electron gas. The introduction of the collective or plasmon modes allows one to effect a separation between the long range and short range, or screened, Coulomb interactions in the electron gas. In the present work we will make no attempt to calculate the effect on the positron annihilation rate of the plasmon part of the Hamiltonian. This will be discussed in a later paper. For now we only refer to the earlier work of Ferrell, whose estimate of the plasmon effects indicate they are probably small.

Where we differ from Ferrell is in the treatment of the screened electron-positron Coulomb force. Ferrell reasoned that this weakened interaction should admit of a perturbation treatment. Our calculations indicate this was not a justifiable assumption, at least not for most of the metals of interest. We have attempted to arrive at an electron-positron correlation function by setting down a two-body equation embodying the screened Coulomb positron-electron force and the equally important Pauli exclusion principle. This can be done by formulating the appropriate Bethe-Goldstone equation[3] for an electron-positron pair in a metal. No attempt is made here to include the proper energy-momentum relationship (dispersion law) for the electron and positron, resulting from the short range collisions experienced by these particles. It is assumed that both electron and positron propagate as free particles.

In Sec. II we set out in detail the basic wave equation for our problem. In Secs. III and IV an approximate solution of the equation is discussed and the results for electrons annihilating at zero tabulated. The calculation is extended to include electrons at the surface of the Fermi distribution in V and the resulting annihilation lifetimes computed.

II. BETHE-GOLDSTONE EQUATION FOR ELECTRON-POSITRON PAIRS

We are of course ignoring lattice effects and treating the metal as an electron gas (at zero temperature) with the neutralizing charge of the positive ions smeared throughout space. We assume that the positron on entering the metal is rapidly thermalized[4] and assign zero momentum to this particle. It is convenient to carry out our calculations entirely in momentum space, for it is then possible to state both the Pauli principle and the screening as algebraic restrictions on the intermediate relative momenta. It is easiest to set down the two-body equation we have in mind and explain the notation after.

The equation describing the interaction of a zero momentum positron with a representative electron of momentum $\mathbf{k}_e=2\mathbf{a}$ is

$$(k^2-a^2)\phi(\mathbf{k})=\frac{4\pi}{(2\pi)^3 a_0}\int' \frac{d^3k'}{|\mathbf{k}-\mathbf{k}'|^2}\phi(\mathbf{k}'), \qquad (1)$$

where $\phi(\mathbf{k})$ is the relative-momentum wave function of

[1] R. A. Ferrell, Revs. Modern Phys. **28**, 308 (1956).
[2] D. Bohm and D. Pines, Phys. Rev. **92**, 609 (1953).
[3] H. A. Bethe and J. Goldstone, Proc. Roy. Soc. (London) **A238**, 551 (1957).
[4] G. E. Lee-Whiting, Phys. Rev. **97**, 1557 (1955).

the positron electron pair, a_0 is the Bohr radius, and the prime on the integration sign indicates the restrictions of screening and of the exclusion principle. The latter demands that the electron go outside the Fermi sea before interacting, in an intermediate state, with the positron. Momentum is of course conserved in this interaction and the resulting restriction on the intermediate relative momentum is seen to be

$$|\mathbf{k}'+\mathbf{a}|>k_F, \tag{2a}$$

or

$$\mathbf{k}'=\mathbf{a} \text{ (corresponding to forward scattering).} \tag{2b}$$

Following Bohm and Pines, the screening due to collective effects of the electron-electron interactions can be accounted for by demanding that the relative momentum transferred in the electron-positron collision satisfy

$$|\mathbf{k}-\mathbf{k}'|>k_c. \tag{3}$$

This leads to an interaction in configuration space with a screening length $\sim 1/k_c$. We have decided to use a value for the cutoff momentum k_c, appreciably higher than the Bohm and Pines result[5]

$$k_c=0.353k_F(r_s)^{\frac{1}{2}}, \tag{4}$$

where k_F is the Fermi momentum of the valence electrons and r_s is the usual metal parameter describing the density of valence electrons. The higher value we use,

$$k_c=0.470r_s^{\frac{1}{2}}k_F, \tag{5}$$

is a reflection of more recent work[6] indicating plasma oscillations exist in an electron gas for higher momenta than supposed by Bohm and Pines. (1) is now brought into a more manageable form by concentrating our attention on the distorted part of the wave function $\phi(\mathbf{k})$. We write

$$\phi(\mathbf{k})=\delta(\mathbf{k}-\mathbf{a})+fu(\mathbf{k}). \tag{6}$$

The δ-function represents an initial plane wave of relative momentum $\mathbf{a}=\frac{1}{2}\mathbf{k}_e$, and the parameter multiplying the distortion $u(\mathbf{k})$ is $f=[4\pi/(2\pi)^3](1/a_0)$. Inserting (6) into (1) we obtain

$$u(\mathbf{k})=\frac{1}{k^2-a^2}\frac{1}{|\mathbf{k}-\mathbf{a}|^2}+\frac{f}{k^2-a^2}\int_{\substack{|\mathbf{k}'+\mathbf{a}|>k_F\\|\mathbf{k}'-\mathbf{a}|>k_c\\|\mathbf{k}-\mathbf{k}'|>k_c}} d^3\mathbf{k}'\,\frac{1}{|\mathbf{k}-\mathbf{k}'|^2}u(\mathbf{k}'). \tag{7}$$

[5] D. Pines, *Solid State Physics*, edited by F. Seitz and D. Turnbull (Academic Press, Inc., New York, 1955), Vol. I, Eq. (6–8).

[6] R. A. Ferrell, Phys. Rev. **107**, 450 (1957), Eq. (40); Sawada, Brueckner, Fukuda, and Brout, Phys. Rev. **108**, 507 (1957), Eq. (17).

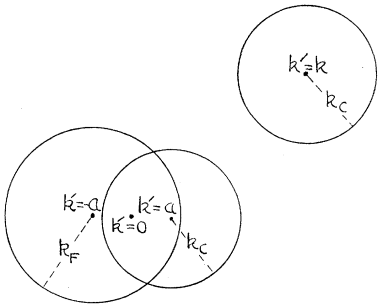

FIG. 1. Excluded regions in relative momentum space.

We have not paid any attention to a definition of the energy denominator in (7) since our restrictions (2a) and (3) do not permit it to vanish. This is simply a statement of the suppression of all but forward scattering by the exclusion principle.

We note because of screening (7) is valid only for $|\mathbf{k}-\mathbf{a}|>k_c$. When $|\mathbf{k}-\mathbf{a}|<k_c$ the equation for $u(\mathbf{k})$ is homogeneous and we then take $u(\mathbf{k})=0$. This leads to the further restriction $|\mathbf{k}'-\mathbf{a}|>k_c\cdots$(2c) in (7). The excluded regions of $\mathbf{k}'$-space are indicated in Fig. 1. Centered about the origin in k'-space there are two overlapping spheres of radii k_F and k_c, excluded, respectively, by the Pauli principle and screening. In addition one must delete the region interior to the sphere of radius k_c centered at $\mathbf{k}$.

It is difficult to specify a simple analytic procedure for the solution of (7). It is clear that the relative angular momentum is no longer a constant of the motion. Hence one obtains a coupled set of integral equations for the various partial waves. However, one may begin by examining a limiting situation, the annihilation of the positron with a zero momentum electron. The total momentum of the interacting pair vanishes and the resulting equation for the distorted part of the wave function is spherically symmetric. We consider this case in the following section.

III.

Taking $\mathbf{a}=0$ as indicated in the previous paragraph, we find for (7)

$$u(k)=\frac{1}{k^4}+\frac{f}{k^2}\int_{\substack{k'>k_F\\|\mathbf{k}-\mathbf{k}'|>k_c}} d^3k'\,\frac{u(k')}{|\mathbf{k}-\mathbf{k}'|^2}. \tag{8}$$

It is to be noted since $k_F>k_c$ in any of the metals we consider, the condition $k'>k_c$ is superfluous. A perturbation treatment of (8) would then simply constitute an iteration solution beginning with the inhomogeneous term $1/k^4$. One can quickly reproduce the results of Ferrell[1] at this point by assuming

$$u(k)=1/k^4. \tag{9}$$

Then the configuration space correlation function is[7]

$$\Psi(\mathbf{r})=\int\phi(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{r}}d^3\mathbf{k}$$
$$=1+\frac{4\pi f}{r}\int_{K_F}^{\infty}\frac{dk}{k^3}\sin kr. \tag{10}$$

The second term in (10) represents a concentration of electrons about the position of the positron. If now one assumes that the annihilation rate does not vary too greatly with electron momentum (a statement which is quite accurate in perturbation theory), one can compute the enhancement of the Sommerfeld rate predicted by perturbation theory. With our normalization this is

$$|\Psi(0)|^2=(1+2/\pi a_0k_F)^2=(1+2r_s/6.02)^2. \tag{11}$$

It is easy to establish that the enhancement factor (11) is not sufficiently large to account for the observed lifetimes. We shall see this when tabulating our own results and comparing these with available experimental evidence. More important perhaps, is the failure of (11) to explain the observed variation of lifetimes with the density of valence electrons in metals. The Sommerfeld rate varies like r_s^{-3}, while experimentally the lifetimes are roughly constant over a large range of metals. It is our hope that both the absolute lifetime and the variation with metals can be accounted for by a more detailed solution of (8). If an electronic, digital, computer were available it is likely that an iteration of (8) would produce the desired results. In view of our lack of such computing facilities we sought approximate analytical techniques for obtaining an adequate solution.

To illustrate our approach let us consider (8) in the limit of no screening, but with the exclusion principle still taken into account. Of course one would now expect the annihilation rates to be much too large since the positron would be able to affect an exaggerated number of electrons in its neighborhood.

(8) now reduces to

$$u(k)=\frac{1}{k^4}+\frac{2\pi f}{k^3}\int_{K_F}^{\infty}(dk')k'u(k')\ln\left|\frac{k+k'}{k-k'}\right|. \tag{12}$$

It is evident that a solution for $k>k_F$ can be obtained as a power series in $(k_F/k)^2$ in this instance, since (12) is invariant to the transformation $k\rightarrow -k$. We assume then

$$u(k)=\frac{1}{k^4}\sum_0^{\infty}\alpha_n\left(\frac{k_F}{k}\right)^{2n}. \tag{13}$$

Inserting (13) into (12) we find, making use of the identity[8] $(n=0, 1, \cdots)$

$$\sum_{m=0}^{\infty}\left[\frac{1}{2n+2m+3}-\frac{1}{2n-2m+1}\right]\frac{1}{2m+1}$$
$$=\frac{\pi}{2(n+1)}\tan(n+1)\pi=0, \tag{14}$$

the odd powers in $1/k$ cancel out and there remains the following set of equations for the dimensionless coefficients α_n:

$$\alpha_0=1+\tau\sum_0^{\infty}\frac{\alpha_n}{2n+1},$$
$$\alpha_m=\frac{\tau}{2m+1}\sum_0^{\infty}\frac{\alpha_n}{2n-(2m-1)}, \tag{15}$$

where

$$\tau=2/\pi a_0k_F.$$

The latter set of linear equations represent a restatement of the integral Eq. (12), and can be solved by some iterative procedure to any degree of accuracy required.

An interesting relation can be established between the first coefficient in the series (13) and the amplitude $\Psi(0)$. By considering $\lim_{k\rightarrow\infty}k^4u(k)$ in (12) or by direct calculation one obtains

$$\alpha_0=\Psi(0). \tag{16}$$

We will not display the numerical solutions of (15). It is sufficient to state that the values of the enhancement factors obtained are much too large and moreover the variation with valence electron density is still too marked. The Pauli principle appreciably alters the pure Coulomb interaction at zero electron momentum, but by itself it cannot explain the observed data.

We now show that it is possible to write down a set of equations similar to (15) for the integral equation (8), if we are willing to treat the screening restriction approximately. (8) can be written

$$u(k)=\frac{1}{k^4}+\frac{f}{k^2}\int_{k_F}^{\infty}d^3k'\frac{u(k')}{|\mathbf{k}-\mathbf{k}'|^2}$$
$$-\frac{f}{k^2}\int_{|\mathbf{k}-\mathbf{k}'|<k_c}d^3k'\frac{1}{|\mathbf{k}-\mathbf{k}'|^2}u(k'). \tag{8'}$$

The last term in (8′) represents a reduction in the distortion $u(k)$ due to screening. We will assume this term can be approximated by evaluating $u(k')$ at the point $\mathbf{k}'=\mathbf{k}$ and hence removing $u(k')$ from the integrand. This approximation is prompted by the singularity in the integrand at $\mathbf{k}'=\mathbf{k}$. If $k\gtrsim k_F+k_c$, the region defined by (3) is a sphere and there results for

[7] Reference 1, Eqs. (51) and (52).

[8] L. B. W. Jolley, *Summation of Series* (Chapman and Hall, Ltd., London, 1925), Eq. (421).

the last term in (8′)

$$(f/k^2)u(k)(4\pi k_c). \tag{18}$$

However when $k \lesssim k_F + k_c$ (3) is something less than a sphere and we obtain a different approximation for the screening term. In fact for $k = k_F$ it is easy to show that the relevant term is very nearly

$$(f/k^2)u(k)(2\pi k_c). \tag{19}$$

Since $u(k)$ clearly drops off rapidly with increasing k we will want to weight the interior regions of integration more heavily than the exterior and assume then

$$\frac{f}{k^2}\int_{|\mathbf{k}-\mathbf{k}'|<k_c} d^3k' \frac{1}{|\mathbf{k}-\mathbf{k}'|^2}u(k') \approx \frac{f}{k^2}u(k)x(2\pi k_c), \tag{20}$$

where x is a number between one and two but probably closer to one. We can adjust x at will to minimize the error in our treatment of the screening. It is natural to choose this parameter so as to make the error in $\Psi(0)$ as small as possible.

We must then solve an altered integral equation

$$u_0(k)\frac{(1+2\pi f k_c x)}{k^2} = \frac{1}{k^4} + \frac{2\pi f}{k^3}\int_{k_F}^{\infty} k'dk' \ln\left|\frac{k+k'}{k-k'}\right| u_0(k'). \tag{21}$$

We can regard the solution of (21) as an improved starting point for the iteration of (8). Our treatment of the screening is in any case not without ambiguities, so that we are justified in handling it in the above fashion.

Once again (21) admits of a solution

$$u_0(k) = \frac{1}{k^4}\sum_{n=0}^{\infty}\alpha_n\left(\frac{k_F}{k}\right)^{2n}, \tag{22}$$

with the coefficients in (22) satisfying the equations

$$\begin{aligned} \alpha_0 &= 1+\tau\sum_{n=0}^{\infty}\frac{\alpha_n}{2n+1} \\ \alpha_m+\alpha_{m-1}(\tfrac{1}{2}x\tau\beta) &= \frac{\tau}{2m+1}\sum_{0}^{\infty}\frac{\alpha_n}{2n-2m+1}. \end{aligned} \tag{23}$$

The relation $\alpha_0 = \Psi(0)$ is still valid. We can now fix our attention on a particular metal and solve the set (23) by iteration. This is done in the following section.

IV.

To cover an interesting range of metals we have carried out computations for the following values of the parameter τ,

$$0.6667,\ 1.0000,\ 1.2500. \tag{24}$$

TABLE I. Wave function and enhancement factors for electrons annihilating at zero momentum.

τ	0.6667	1.0000	1.2500
x	1	1	1.315[a]
s	3	3	4
α_0	2.27	4.06	4.98
g_1	−1.27	−4.17	−8.63
α_2	0.40	2.73	10.03
g_3	−0.14	−1.60	−10.52
α_4	−0.03	0.87	10.00
α_5	−0.015	−0.46	−9.47
α_6	−0.0007	0.22	8.41
α_7	...	...	−7.38
α_8	...	...	6.20
α_9	...	...	−5.21
α_{10}	...	...	4.42
α_{11}	...	...	−3.69
α_0 pert.	1.67	2	2.25
$\lvert\Psi(0)\rvert^2$	5.15	16.48	24.75
$T\times10^{+10}$ sec^{-1}	1.31	1.39	1.80

[a] Corresponds to $\frac{1}{2}\tau\beta x = 0.7500$.

These correspond, respectively, to metals with the density parameters (r_s)

$$2.010,\ 3.014,\ 3.768, \tag{25}$$

and screening parameters ($\beta = k_c/k_F$),

$$0.6663,\ 0.8159,\ 0.9124. \tag{26}$$

The first of the values (24) corresponds roughly to aluminum, the last to sodium.

It is not necessary to discuss the actual calculations in great detail. We began by solving a truncated set of equations for $\alpha_m m \lesssim s$. The latter coefficients were used to estimate the coefficients α_m, for $m > s$. One could then reconsider the equations for the first s coefficients, including coupling from the previously ignored coefficients. This procedure was iterated until it was felt the error in the first s coefficients was of the order of a few percent. For the cases corresponding to $\tau = 0.6667$, 1.0000 the coupling between successive coefficients decreased quite quickly with increasing m. However, for the value $\tau = 1.25$ the coupling decreased only moderately with increasing m and it was necessary to use a larger basic set of equations (larger value of s) and to proceed out further in the sequence $[\alpha_m]$. It is to be noted that the parameter x mentioned previously was taken to be larger for the case $\tau = 1.25$.

Using the values of α_0 noted above in Table I we can compute the enhancement factor $|\Psi(0)|^2$. Assuming all electrons annihilate at essentially zero momentum we would obtain mean lives T given (in seconds) by

$$T^{-1} = \left[\frac{1}{4}\left(\frac{k_F^3}{3\pi^2}\right)(8\pi a_0^3)(1.25\times10^{-10})^{-1}\right]|\Psi(0)|^2. \tag{27}$$

The bracketed factor in (27) is the spin-averaged Sommerfeld annihilation rate obtained by comparing the density of valence electrons in the metal to that in positronium. The enhancement factors and corresponding lifetimes are also contained in Table I.

We have included in Table I the perturbation theory values for α_0. From these the inadequacy of perturbation theory in lowest order can be inferred. However it does seem likely that an iteration of the lowest order would achieve some degree of success for the metals with denser valence electron gases.

It is possible to estimate the error in our approximate solution of (8) by using the solution of (21) as a first iterate in (8). This is an impossible task in the light of our computing facilities, but we did compute the error in the lifetimes due to the inhomogeneous term in the integral equation which one obtains for the difference

$$u_1(k)=u(k)-u_0(k). \tag{28}$$

The change in $\Psi(0)$ is seen to be

$$\Delta_1\Psi(0)=4\pi f\int_{k_F}^{\infty}k^2dk\,u_1(k)$$
$$=4\pi f\int_{k_F}^{\infty}k^2dk\left[\frac{2\pi fxk_c}{k^2}u_0(k)\right.$$
$$\left.-\frac{f}{k^2}\int_{|\mathbf{k}-\mathbf{k}'|<k_c}d^3k^1\frac{u_0(k')}{(\mathbf{k}-\mathbf{k}')^2}\right]. \tag{29}$$

Because of the complexity encountered in evaluating (29) we arbitrarily cut off the calculation after the fourth term in the expansion (22). Moreover (29) was evaluated only for the two extreme values of the electron density considered. We should point out that the first order errors in the wave function $u(k)$ were considerably larger than those in the amplitude $\Psi(0)$. This is reminiscent of the situation encountered in a Ritz variational calculation. As a representative calculation we can quote the relative error in $u(k)$ at $k=k_F$. For the case $r=0.6667$ one finds this error to be some 5% while the corresponding error in $\Psi(0)$ is completely negligible. For the case $r=1.25$ one has little confidence in the values $u_0(k_F)$ and $u_1(k_F)$; but for larger $k\sim k_F+k_c$, the ratio u_1/u_0 is of the order of 0.25. Nevertheless for the latter case (29) constitutes only a 3% reduction of $\Psi(0)$.

V. VARIATION OF THE ENHANCEMENT FACTOR WITH ELECTRON MOMENTUM

As we have indicated the task of computing $\Psi(0)$ for nonzero electron momentum is perhaps an order of magnitude harder than that for zero electron momentum. However, we will proceed under the assumption that $\Psi(0)$ varies only slowly across the Fermi sea. A straightforward procedure is to expand $u(\mathbf{k})$ in a series of Legendre polynomials in the fashion

$$u(\mathbf{k})=\sum_{l=0}^{\infty}u_l(\mathbf{k})Pl(\cos\theta), \tag{30}$$

where θ is the angle between the vectors $\mathbf{a}$, $\mathbf{k}$. Of course we are interested only in the s-wave in (30), but unfortunately we cannot separate off this partial wave from the rest. In fact we obtain for the case $l=0$, the following integral equation (ignoring for the moment the restriction $|\mathbf{k}-\mathbf{k}'|<k_c$).

$$u^0(k)=\frac{1}{2ka}\ln\left|\frac{k+a}{k-a}\right|+\frac{f}{k^2-a^2}\int_{\substack{|\mathbf{k}'-\mathbf{a}|<k_c\\|\mathbf{k}'+\mathbf{a}|>k_F}}d^3k^1\frac{1}{2kk'}$$
$$\times\ln\left|\frac{k+k'}{k-k'}\right|\sum_{l=0}^{\infty}u^l(k')Pl(\cos\theta'). \tag{31}$$

If k_c, k_F were equal, we would have an equation similar in structure to the Bethe-Goldstone equation for two identical particles (both subject to the exclusion principle). In such a case it is easy to show the coupling of the p-wave to the s-wave in (31) is eliminated. The reason is simply that the region of angular integration in (31) is then symmetrical about $\theta=\pi/2$. In fact only the partial waves with even l remain. Moreover one expects the coupling of successive waves to decrease with increasing l. This suggests that even in the event $k_c\neq k_f$ we assume

$$u(\mathbf{k})=u^0(k). \tag{32}$$

The latter assumption is best for metals with lower valence electron densities when β approaches unity. We have estimated the error in neglecting the p-wave by using a Born approximation for this wave and inserting it into (31). In the least favorable circumstances we find an upper limit to the p-wave contribution of less than 1%.

Referring now to Fig. 1 it is clear that the exclusion principle will no longer be manifested as a sharp cutoff on the range of intermediate momenta. However, to simplify our calculations we have computed the total volume excluded by the spheres (2a) and (2c), and from this computed an effective Fermi momentum $k_F^{\text{eff}}(a,k_c)$. It is likely that our treatment of the excluded regions in momentum space will tend to underestimate the enhancement factors since we expect $u(k)$ to fall off with increasing k. We have redone the calculations of IV only for the value $a=k_F/2$, i.e., at the Fermi surface. For this latter calculation we treat the screening restriction (3) in the same fashion as in III. No estimate of the errors involved in this approximation were made, but we again expect these to be small.

It is then possible to average the enhancement factor $|\Psi(0)|^2$ over the Fermi distribution of electron momentum. We assumed that $\Psi(0)$ varies linearly with electron momentum in performing this average. Table II contains the information relevant to the annihilation of an electron at the Fermi surface, as well as the final lifetimes T taking into account the variation of annihilation rates with electron momentum.

We have not corrected the lifetimes for the errors due

TABLE II. Wave function at the Fermi surface and annihilation lifetimes.

r	0.6667	1.0000	1.2500
k_{eff}/k_f	1.069	1.129	1.179
r_{eff}	0.6236	0.8860	1.060
α_0	2.38	3.85	5.05
α_1	−0.47	−2.14	−5.03
α_2	−0.05	0.51	3.09
α_3	−0.03	−0.15	−1.07
α_4	...	−0.006	0.82
α_5	...	...	−0.23
α_6	...	...	+0.08
$T(10^{-10}$ sec)	1.22	1.51	1.76

to the approximate treatment of the screening. Such a correction would appear to increase the lifetime by about 6% for $r=1.25$ and leave unaltered the lifetime for $r=0.667$.

VI. CONCLUSIONS

One may sum up the experimental situation on positron annihilation in metals by saying that the lifetimes are of the order of 1.5×10^{-10} sec over a wide range of metals.[9] This is in good agreement with our calculations. The size of the errors quoted in the experimental lifetimes varies between 0.3×10^{-10} and 0.6×10^{-10} sec.[9] Hence the variation of lifetime with metals that we predict is certainly not ruled out. It is hoped that more accurate experiments will check this point more carefully.

Moreover the work on the two photon angular correlation from the decay[10] has indicated that the distribution of the total momentum of the annihilating pair is very nearly that of the Fermi distribution of valence electrons. Since our calculations predict a roughly constant enhancement factor across the Fermi sea, we are in agreement with this angular correlation data.

We see that both the exclusion principle and screening are indeed important features in the calculation of a positron-electron correlation function in an electron gas. If one ignored both of these but took into account the pure electron-positron Coulomb interaction one would have found far too many electrons annihilating near zero momentum, and too few near the Fermi surface. The exclusion principle by itself reduces the annihilation rate at zero momentum and enhances the rate near the Fermi surface. The latter increase can be understood by considering a perturbation calculation involving only the inhomogeneous term in (7). The energy denominator in this term would be sometimes negative sometimes positive in the pure Coulomb case. The exclusion principle eliminates that region of relative momenta in which the energy denominator is negative. The destructive interference amongst the various relative momentum contributions. to the amplitude $\Psi(0)$ is not permitted by the exclusion principle. This is an effect distinct from the suppression of scattering which is a restriction on the configuration wave function at $r=\infty$.

However, the exclusion principle by itself is not sufficient to account for the observed data. The screening reduces the annihilation rates to more reasonable values and is especially important in smoothing out the variation of the Sommerfeld rate with electron density.

Finally we would like to say something about the effect on our results of the long-range interactions. Ferrell[1] has concluded that all plasmon effects have a negligible influence on annihilation rates. However, since we are using a momentum cutoff on the plasmon spectrum appreciably higher than that of Ferrell, there is some reason to believe the long range interactions may play a larger part in our calculations. For this reason we intend to consider this problem in more detail later.

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