Nuclear Levels of Cs^{133}†*

M. G. STEWART AND D. C. LU
Institute for Atomic Research and Department of Physics, Iowa State University, Ames, Iowa
(Received April 9, 1959; revised manuscript received December 28, 1959)

The gamma rays of Cs^{133} following the electron-capture decay of Ba^{133} were studied using a coincidence scintillation spectrometer. Gamma rays with the following energies (in kev) were observed: 386, 356, 301, 276, (220), 162, 82, 80, and 54. The energies (in kev) of the nuclear levels of Cs^{133} with their spins and parities are: ground state ($\frac{7}{2}+$), 82 ($\frac{5}{2}+$), 162 ($\frac{3}{2}+$), 383 ($\frac{3}{2}+$), and 438 ($\frac{1}{2}+$). The electron-capture transitions and their relative intensities are as follows: (a) to the 438-kev level, 76% (b) to the 383-kev level, 11% (c) to the 162-kev level, 13%. In the decay of Xe^{133}, $(1.5\pm1.0)\%$ of the beta transitions was observed to go to the 162-kev level in Cs^{133} and the remaining go to the 82-kev level.

I. INTRODUCTION

In recent years the electron-capture decay of Ba^{133} has been studied by several authors,[1–5] but the results have been somewhat conflicting. In the use of this isotope in our laboratory, it became apparent that there were additional unreported gamma rays. Thus it was decided to reinvestigate the decay of Ba^{133}. While this investigation was in process, the report by Crasemann *et al.*[4] was published which also reported two of the new transitions. In addition, the reports by Gupta *et al.*[3] and Koicki *et al.*[5] were published which are more in agreement with our results.

II. SCINTILLATION SPECTRUM

A spectrum of the Cs^{133} gamma rays following the decay of Ba^{133} is shown in Fig. 1. The source was produced by the reaction Ba^{132} $(n,\gamma)Ba^{133}$ at the Oak Ridge National Laboratory using unenriched $BaCl_2$. The data were taken with a 2×2-inch NaI(Tl) crystal, a Dumont-6363 photomultiplier tube, and a RCL-256 channel analyzer, with a source to crystal distance of 10 cm. In Fig. 1, the two main groups at the high energy end can be resolved into four separate gamma rays with energies of 386, 356, 301, and 276 kev; the line shape for a single gamma ray was determined from the 323-kev gamma ray in the decay of Cr^{51}. The bump at 190 kev is probably the Compton edge associated with the 356-kev gamma ray, and the bump at 220 kev may be the Compton edge for the 386-kev gamma ray, but it may also be a weak gamma ray as will be shown later. The scattering peak is at 155 kev, and the very weak line at 115 kev is probably the sum of the Cs K x-ray and the strong line at 82 kev. The line at 53 kev is mostly the iodine K x-ray escape peak associated with the 82-kev gamma ray. For an 82 kev gamma ray, the ratio of the escape peak to the photopeak should be about 6.7%.[6] However, the spectrum of the region of 0–100 kev, shown by the curve through the solid circles in Fig. 2, indicates that the escape peak is approximately 11% of the 82-kev peak. The higher than normal ratio suggests that there may also be a gamma ray of approximately 53 kev. Since a 0.044-inch copper absorber would attenuate a 53-kev gamma ray about five times as much as it would attenuate an 82-kev gamma ray, it is evident that in a comparison of two spectra, one with the absorber and one without, a change in the ratio of the escape peak to the 82-kev photopeak would indicate the presence of an additional gamma ray. The low-energy spectra with and without the absorber as well as their difference are plotted in Fig. 2. The difference spectrum gives a line at 54 kev. The position at which the sum of two Cs K x-rays would appear (61.6 kev) is also indicated, so that it is apparent that the difference in the two spectra is not due to the summing of these x-rays.

III. COINCIDENCE SPECTRA

The coincidence spectra were taken with the 256-channel analyzer gated by the output of a single-channel analyzer. The resolving time of the combination is 0.2 microsec. The spectrum from the 2×2 inch crystal was fed into the 256-channel analyzer, and this counter we shall refer to as the "spectrum" counter. The single channel analyzer was used to select a given energy gamma ray from a $1\frac{1}{2}\times1$-inch NaI(Tl) crystal and this counter we shall refer to as the gating counter. Figure 3 shows a spectrum of the gamma rays that are in coincidence with the 82-kev gamma ray. For comparison, a plot of the singles spectrum is shown on the same figure. The two spectra are normalized to the same number of counts at 356 kev.

It is seen that (1) the 386-kev gamma ray is not in coincidence with the 82-kev gamma ray; (2) the 356-

† A preliminary report of this work was presented at the May, 1958 American Physical Society meeting in Washington, D. C. [Bull. Am. Phys. Soc. 3, 208 (1958)].

* Contribution No. 691. Work was performed in the Ames Laboratory of the U. S. Atomic Energy Commission.

1 R. W. Hayward, D. D. Hoppes, and H. Ernst, Phys. Rev. **93**, 916(A) (1954).

2 M. Langevin, Compt. rend **238**, 1310 (1954); and **240**, 289 (1955); Ann. Phys. **1**, 57 (1956).

3 R. K. Gupta, S. Jha, M. C. Joshi, and B. K. Madan, Nuovo cimento **8**, 48 (1958).

4 B. Crasemann, J. G. Pengra, and I. E. Lindstrom, Phys. Rev. **108**, 1500 (1957).

5 S. D. Koicki, A. M. Mijatovic, and J. M. Simic, Bull. Inst. Nuclear Sci. Boris Kidrich (Belgrade) **8**, 1 (1958).

6 P. R. Bell, in *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (Interscience Publishers, Inc., New York, 1955), p. 155.

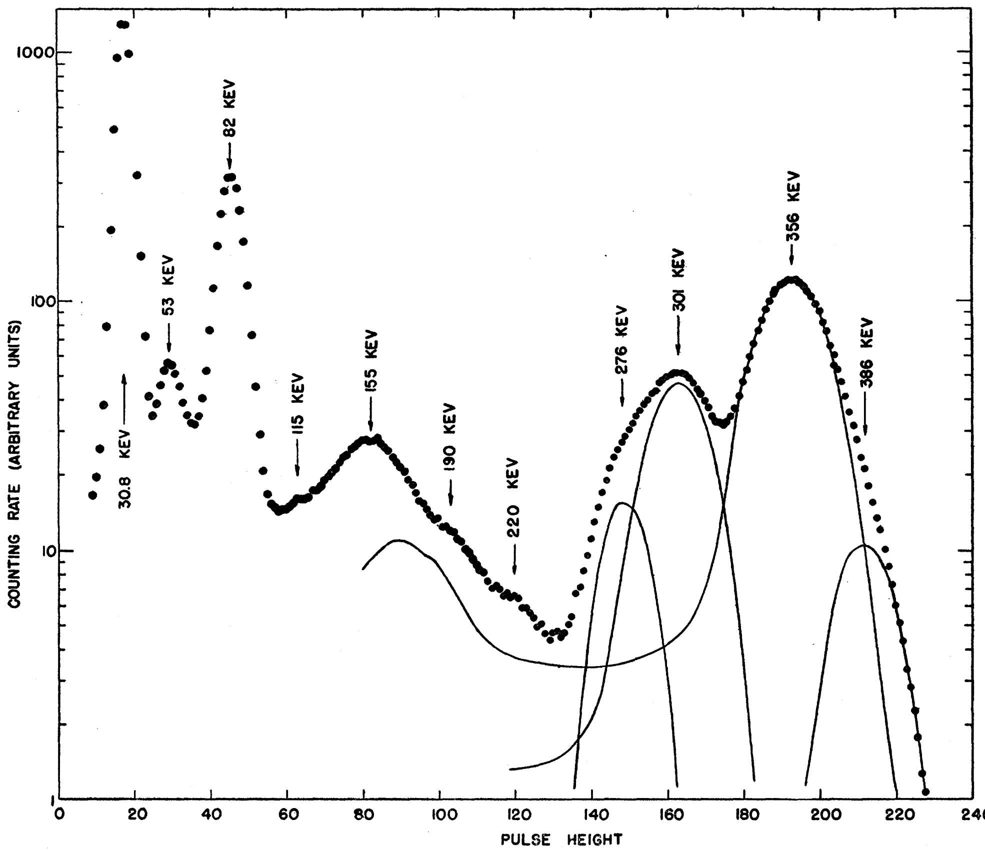

FIG. 1. Spectrum of the gamma rays following the decay of Ba^{133}.

kev and 301-kev gamma rays are in coincidence with it; (3) the 276-kev gamma ray is in coincidence with it and is enhanced with respect to the 301- and 356-kev gamma rays; and (4) there are two gamma rays at about 82 kev which are in coincidence.

In the coincidence spectrum, the line which was at 82 kev in the singles spectrum has shifted to about 81 kev. Since in the coincidence spectrum this line is composed of equal numbers of the two gamma rays, the less intense gamma ray has an energy of about 80 kev. Hence, the reason for the enhancement of the 276-kev line is that it must be in coincidence with both the 80- and 82-kev gamma rays. The last peak is the Cs K x-ray which must be in coincidence with all of the gamma rays since Ba^{133} decays by electron capture.

The position at which a 220-kev gamma ray would appear is also shown in Fig. 3. The statistics are not good enough to say definitely that a line of this energy does exist, however, the counting rate in this region of the spectrum appears somewhat higher than one might expect from that of the three higher energy gamma rays. In this spectrum there can be no confusion with the Compton edge for the 386-kev gamma ray since this line is not in coincidence with either the 80- or 82-kev gamma rays.

Coincidences with the 383-kev, 356-kev, and 301-kev gamma rays show that these gamma rays are in coincidence with nothing higher than the 82-kev gamma ray. It is difficult to tell whether or not they are in coincidence with the 54-kev gamma ray, since it is so weak and falls on top of the escape peak for the 82-kev gamma ray. Furthermore, the 386-kev gamma ray cannot be selected cleanly in the coincidence gating channel without accepting an appreciable amount of the 356-kev gamma ray (see Fig. 1).

Coincidences with the 276-kev gamma ray reveal a peak at 82 kev and one at 162 kev. This is shown in Fig. 4. The source was placed close to the crystal (solid angle $\sim 50\%$) and a 0.016-inch copper absorber was placed over the crystal to keep out the Cs x-rays. It is shown in Appendix I that the peak at 162 kev cannot be entirely due to the summing of the 80- and 82-kev gamma rays, but must be partly due to a gamma ray with an energy of 162 kev.

IV. THE DECAY SCHEME

Figure 5 is a decay scheme which is consistent with the experimental results. It is shown in Appendix IV that there must be about 13% of the electron capture transitions going directly to the 162-kev state, 11% to the 383-kev state, and 76% to the 438 state. The relative intensities of the transitions in Cs^{133}, listed in Table I, are estimated from a knowledge of the number

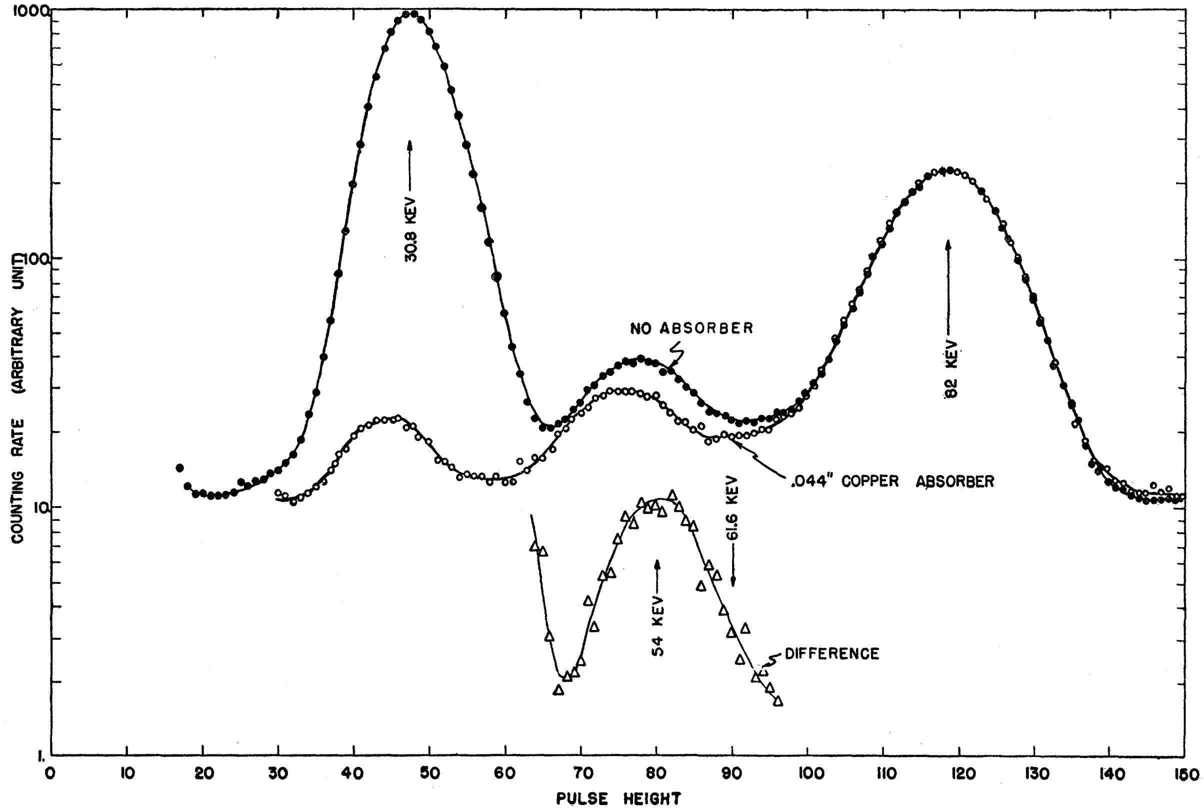

FIG. 2. The low-energy spectra of the gamma rays following the decay of Ba^{133}. The curve drawn through the solid circles is the spectrum with no absorber, and the curve through the open circles is the spectrum with a 0.044-inch copper absorber. The difference between the two spectra is shown by the triangular points.

of counts in each photopeak, the detection efficiency,[7] the peak to total ratio,[8] and the conversion coefficient[9] for each transition. The ratio of the intensities of the 301–82 kev cascade gamma rays to the 386-kev cross-over gamma ray is 2.7 ± 0.4, and that of the 80–82 kev cascade to the 162-kev cross-over is 3.8 ± 1.2. These values agree with the values of 2.5 and 4.5, respectively, obtained by Fagg[10] from the Coulomb excitation of Cs^{133}.

V. SPIN ASSIGNMENTS AND MULTIPOLARITIES OF THE GAMMA RAYS

The ground state of $_{55}Cs^{133}$ has a measured spin of $\frac{7}{2}$,[11] in agreement with the shell-model prediction of $g\frac{7}{2}$ for the 55th proton.[12] The next four single particle levels predicted by the shell model have the assignments of $d\frac{5}{2}$, $d\frac{3}{2}$, $s\frac{1}{2}$, and $h11/2$. The ground state of $_{56}Ba^{133}$ has a spin of $\frac{1}{2}+$,[13] which agrees with the shell-model pre-

TABLE I. Relative intensities of the transitions in Cs^{133}.

Gamma ray	Energy (kev) and character[a]	Relative gamma-ray intensity	Theoretical $K+L+M$ conversion coefficient[b]	Relative total transition intensity[c]
γ_1	386 ($E2$)	10	0.020	10 ± 1
γ_2	356 ($E2$)	100	0.026	100
γ_3	301 ($M1$)	27	0.046	28 ± 3
γ_4	276 ($M1$)	8	0.058	8 ± 1
γ_5	220 ($M1$)	$\sim$0.3	0.10	$\sim$0.3
γ_6	162 ($E2$)	2[d]	0.33	3 ± 1[e]
γ_7	82	55[d]	1.77 ± 0.05[f]	154 ± 5[g]
γ_8	80 ($M1$)	9[d]	1.74	26 ± 3[h]
γ_9	54 ($M1$)	3[d]	5.4	19 ± 6[i]

[a] See Sec. V.
[b] See footnote 9.
[c] The errors are estimated errors and are not statistical.
[d] Calculated from columns 4 and 5.
[e] Calculated from the ratio of the 80- to 162-kev transitions. See Appendix I.
[f] I. Bergström, Arkiv Fysik **5**, 191 (1952); I. Bergström, S. Thulin, A. H. Wapstra, and B. Aström, Arkiv Fysik **7**, 255 (1954).
[g] Calculated from the number of transitions feeding the 82-kev level. See Appendix II.
[h] Calculated from the ratio of the 80- to 82-kev transitions. See Appendix II.
[i] See Appendix III.

[7] R. L. Heath, Philips Petroleum Company, Report IDO-16408, 1957 (unpublished).
[8] Estimated from R. L. Heath, reference 7, and P. R. Bell, reference 6, p. 139.
[9] M. E. Rose, *Internal Conversion Coefficients* (Interscience Publishers, Inc., New York, 1958); L. A. Sliv and I. M. Band, Leningrad Physico-Technical Institute Report, 1956 [translation: University of Illinois Report 57ICCK1 (unpublished)]; Leningrad Physico-Technical Institute Report, 1958 [translation: University of Illinois Report 58ICCL1 (unpublished)].
[10] L. W. Fagg, Phys. Rev. **109**, 100 (1958).
[11] J. E. Mack, Revs. Modern Phys. **22**, 64 (1950).
[12] M. G. Mayer and J. H. Jensen, *Elementary Theory of Nuclear Shell Structure* (John Wiley and Sons, Inc., New York, 1955).
[13] R. D. Hill, G. Scharff-Goldhaber, and M. McKeown, Phys. Rev. **84**, 382 (1951).

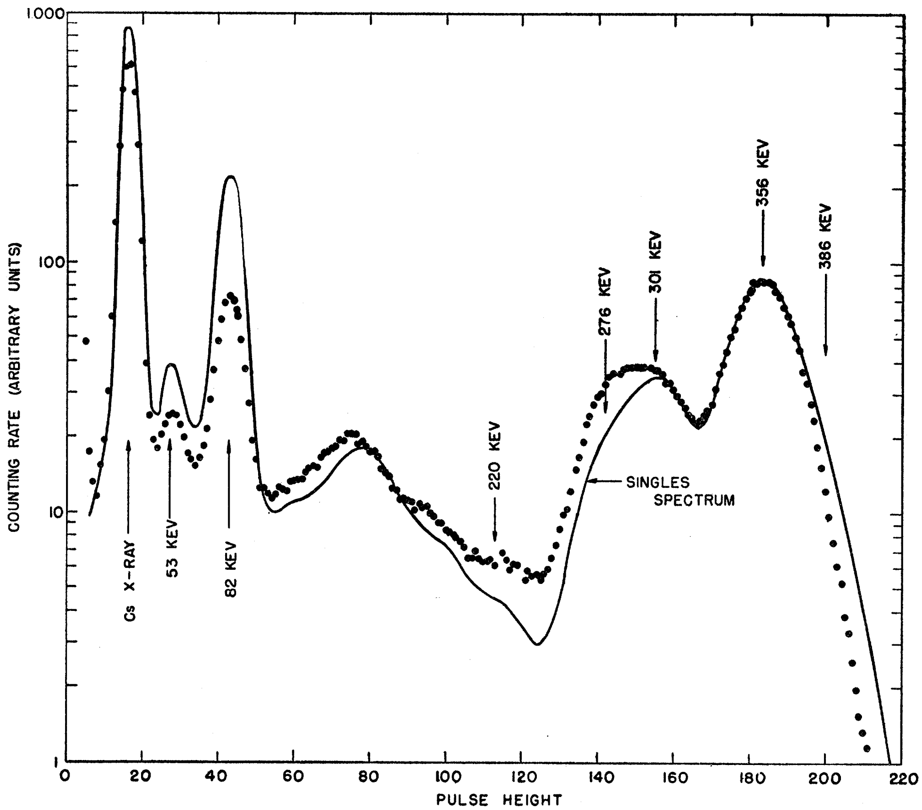

FIG. 3. Spectrum of the gamma rays, following the decay of Ba^{133}, that are in coincidence with the 82-kev gamma ray. For comparison, the solid curve shows the singles spectrum which has been normalized to the coincidence spectrum at 356 kev.

diction of $s\frac{1}{2}$, and decays by electron capture to the 162-, 383-, and 438-kev levels in Cs^{133}. The $h11/2$ state will not be reached in the decay of Ba^{133} since a transition to this state would be highly forbidden. For the first excited state, an assignment of $d\frac{5}{2}$ would be in agreement with the known decay of Xe^{133}.[14] The fact that the 82-, 162-, and 383-kev levels can be reached by Coulomb excitation[10] indicates that the spins of these levels must not differ by more than two units from the ground-state spin. Furthermore, since the 438-kev level cannot be reached by Coulomb excitation,[10] it probably has a spin differing by at least three units from the ground-state spin or else the $E2$ transition probability for the excitation of this state is abnormally small.

Since it is expected that all the states reached in this decay scheme will have plus (+) parity, the electron capture transitions will be either allowed ($\Delta I=0, \pm 1$; no) or second forbidden ($\Delta I=\pm 2$; no). For an allowed transition, the logft value should be about 3–5[15] and for a second forbidden transition about 10. If the Q value for the Ba^{133} decay is about 700 kev,[16] the transitions to the 438-, 383-, and 162-kev states have logft values of 7.8, 8.8, and 9.2, respectively. This Q value, however, is estimated from beta-decay systematics and is not a measured value. The true Q value cannot be less than 474 kev, since the 438-kev level in Cs^{133} is fed by at least some K capture from the Ba^{133} decay,[17] and it probably is not higher than 1.0 Mev, which is 40% higher than the estimated value. One may then set limits on the ft-values.

For the transition to the 438-kev state, the lower limit for the logft value for K capture may be made arbitrarily small by adopting a Q value that approaches an energy of 474 kev. The upper limit is 8.5. For the transition to the 383-kev state, the limits are 7.4 to 9.4, and for the transition to the 162-kev state, the limits are 8.7 to 9.6.

Since the logft value for the transition to the 438-kev state is less than 8.5, this is an allowed transition. This, in conjunction with the Coulomb excitation data, makes this a spin $\frac{1}{2}$ state. The transition to the 383-kev state has a logft value between 7.4 and 9.4 and is, presumably, an allowed rather than a second forbidden transition. Again in conjunction with the Coulomb excitation data this assigns a spin of $\frac{3}{2}$ to the 383-kev state. The logft value for the transition to the 162-kev state is between 8.7 and 9.6. This is close to the logft value for second forbidden transitions. However, of the known second

[14] Nuclear Data Sheets, National Academy of Sciences—National Research Council NRC—58-6-49, 1958 (National Research Council, Washington, D. C., 1958).
[15] Reference 12, p. 129.
[16] K. Way and M. Wood, Phys. Rev. 94, 119 (1954).
[17] Even neglecting the absorption in the crystal mounting, one cannot account for all of the K x-rays unless it is assumed that there is K capture to the 438-kev state.

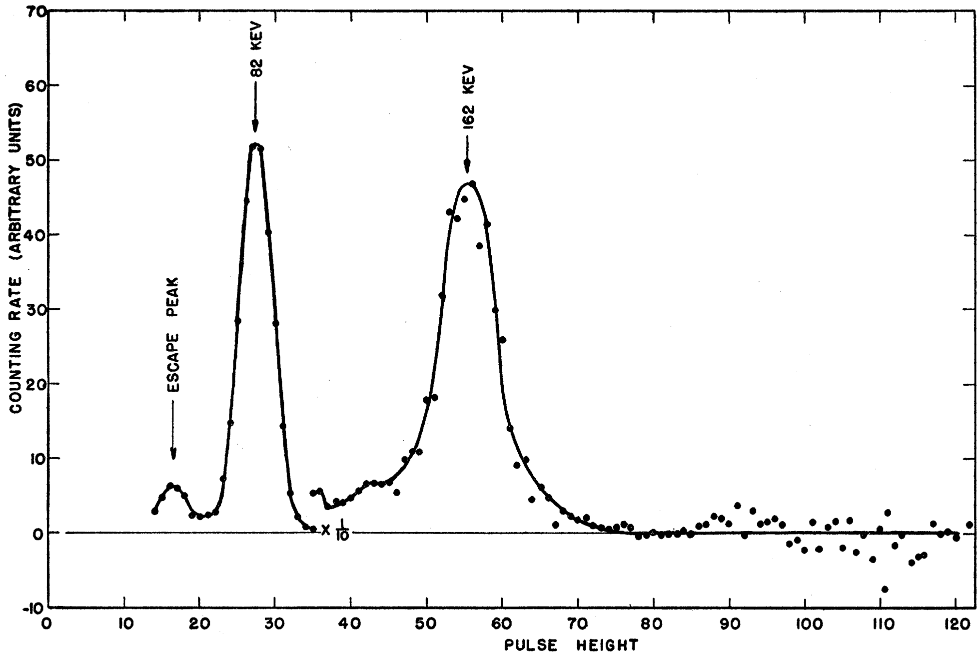

FIG. 4. Spectrum of the gamma rays, following the decay of Ba^{133}, that are in coincidence with the 276-kev gamma ray.

forbidden transitions, the lowest log*ft* value is 10.8 in the decay of Fe^{59}.[18] Thus it seems probable that the transition to the 162-kev state involves a spin change of one rather than of two units. This would assign a spin of $\frac{3}{2}$ to the 162-kev state in Cs^{133}.

If the 438-kev state has a spin of $\frac{1}{2}$, this is not in disagreement with either the shell model or the Coulomb excitation data. If it has a spin of $\frac{3}{2}$, then the $E2$ transition probability for the excitation of this level must be retarded and the predicted $s\frac{1}{2}$ state must lie still higher. It seems not unreasonable to suppose that the 438-kev state does indeed have a spin of $\frac{1}{2}$. If single particle configurations are given to the levels in Cs^{133}, then we must choose between the 383- and 162-kev levels for the $d\frac{3}{2}$ assignment. If it is given to the 162-kev level, then one would expect the $d\frac{3}{2}$ ground state of Xe^{133} to decay to both the 82- and 162-kev levels with about the same log*ft* value. However, from the gamma-ray intensity measurements of Pleasonton,[19] the log*ft* value for the beta transition to the 162-kev level has been estimated[20] to be 7.2 while the transition to the 82-kev state has a log*ft* value of 5.6. Thus, it seems likely that the $d\frac{3}{2}$ assignment should be given to the 383- and not to the 162-kev level. A determination of the intensity of the beta transition to the 162-kev level in Cs^{133} was also made in this laboratory, but since there is some contamination from the 164-kev gamma ray of Xe^{131m}, it was decided to observe the coincidence counting rate between the 80- and 82-kev gamma rays rather than the relative intensities of the gamma rays. (See Sec. VI.)

With the above spin assignments, the 386-, 356-, and 162-kev gamma rays must be electric quadrupole radiation, while all the rest must be predominantly $M1$ radiation. The $M1$ gamma rays could well have some $E2$ admixture since it appears that they are all retarded to some extent. For example, according to estimates for

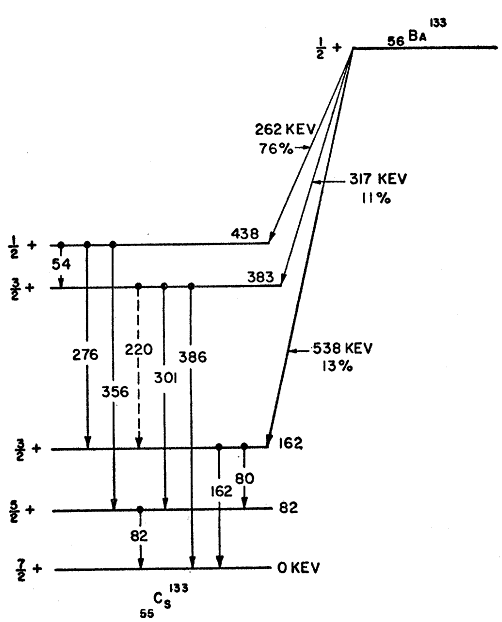

FIG. 5. Proposed decay scheme for Ba^{133}.

[18] Eugene Feenberg, *Shell Theory of the Nucleus* (Princeton University Press, Princeton, 1955).

[19] F. Pleasonton, Oak Ridge National Laboratory Report ORNL-2430, 1958 (unpublished), p. 46.

[20] Nuclear Data Sheets, National Academy of Sciences-National Research Council, NRC 58-6-49 and 58-6-46, 1958 (National Research Council, Washington, D. C., 1958).

single particle transitions,[21] the 276-kev $M1$ transition should be about 600 times more intense than the 356-kev $E2$ transition, while it is, in fact, about 12 times weaker. Table II shows a listing of the approximate retardation factors for the $M1$ transitions assuming the $E2$ transitions from the same level proceed at a normal rate.

VI. THE DECAY OF Xe^{133}

The 5.3-day Xe^{133} source was obtained from fission products from the Oak Ridge National Laboratory. Because the source also contained some Xe^{131m} which decays to the ground state of Xe^{131} from the 12-day, 164-kev isomeric state, it was thought that the observation of the coincidence counting rate between the 80- and 82-kev gamma rays would give a better determination of the intensity of the 266-kev beta group from Xe^{133} than would a direct measurement of the intensity of the 162-kev gamma ray. It is shown in Appendix V that the fraction of the beta transitions leading to the 162-kev state in Cs^{133} is 0.015 ± 0.01 of the total. This value agrees with that given in reference 20.

VII. DISCUSSIONS

With the spin assignments of $\frac{7}{2}$, $\frac{5}{2}$, $\frac{3}{2}$, $\frac{3}{2}$, $\frac{1}{2}$, all the electron capture transitions are allowed ($\Delta I=0, \pm1$; no) yet they all have rather large logft values. This might be explained when one considers the configurations of the particles undergoing the transitions. For example, consider the transition from the $s\frac{1}{2}$ ground state of Ba^{133} to the 438 kev, spin $\frac{1}{2}$ state in Cs^{133}. Presumably, this is also an $s\frac{1}{2}$ state. The spin of the ground state of Ba^{133} arises from the configuration of the odd neutron ($s\frac{1}{2}$), and it appears as though this would be an $s\frac{1}{2}\rightarrow s\frac{1}{2}$ transition which should have had a normal logft value. However, it is not the odd neutron, but one of the protons that undergoes a transition. Since Ba^{133} has fifty-six protons, and fifty is a magic number, the last six protons will determine the proton configuration. If the 438-kev state in Cs^{133} has a configuration determined by an odd proton in the $s\frac{1}{2}$ state, then the part of the Ba^{133} ground-state wave function that would contribute to the transition would have to come from configurations containing ($s\frac{1}{2}$) terms, e.g., $(g\frac{7}{2})^4(s\frac{1}{2})^2$. However, such terms may represent only a small fraction of the Ba^{133} wave function, so one would expect the total transition to be retarded.

The transition to the 383-kev state in Cs^{133} is retarded even more and can be explained if one attributes the $\frac{3}{2}$ spin of this level to an unpaired $d\frac{3}{2}$ proton. Then the electron capture transition would involve a $d\frac{3}{2}$ proton decaying to an $s\frac{1}{2}$ neutron and would be l-forbidden.

The transition to the 162-kev state has a still higher logft value, but it was seen that the beta transition from the $d\frac{3}{2}$ ground state of Xe^{133} to this level also has a high logft value. The spin of this level then cannot be attributed to an unpaired $d\frac{3}{2}$ proton, but must arise from some more complicated configuration.

TABLE II. Retardation factors for the $M1$ transitions in Cs^{133}.

Energy of the $M1$ transition (kev)	Energy of the $E2$ transition from the same level (kev)	Observed ratio of $M1$ to $E2$ gamma-ray intensities	Calculated ratio[a]	$M1$ retardation factor[b]
276	356	0.08	620	7800
54	356	0.03	5	200
301	386	2.7	590	220
220	386	0.03	105	3500
80	162	3.8	830	200

[a] See footnote 21.
[b] Assuming the $E2$ transition from the same level proceeds at a normal rate.

[21] S. A. Moszkowski, in *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (Interscience Publishers, Inc., New York, 1955), Chap. XIII, p. 391.

APPENDICES

In the subsequent discussions, the following notation will be used:

$N_0(E)$ = Total number of photons of energy E (in kev) emitted per sec by the sample.

$\eta_s(E)$ = Over all probability that a photon of E kev from the sample be detected by the single channel gating counter. It depends upon the solid angle of the counter, the detection efficiency of the counter, the attenuating material in front of the counter, and the single-channel analyzer window setting. However, it has not taken into account the fact that some counts may be lost due to summing if another photon enters the detector in coincidence.

$\eta_m(E)$ = Over-all probability that a photon of E kev from the sample be recorded under the photopeak by the multichannel analyzer. Like $\eta_s(E)$, it has not taken into account the effect due to summing.

$N_{E'}(E)$ = Number of photons of E kev recorded under the photopeak per sec by the multichannel analyzer, in coincidence with the detection of a photon of E' kev by the coincidence gating counter.

α_E = Total internal conversion coefficient of the transition of E kev.

$I_E=N_0(E)[1+\alpha_E]$, total intensity of the transition of E kev.

$\delta_E=I_E/\sum_{E'}I_{E'}$, the branching ratio of the transition of E kev. The summation is over all transitions originating from the same level.

$N_m(E)=N_0(E)\eta_m(E)$.

APPENDIX I. INTENSITY OF THE 162-kev TRANSITION

The existence of a 162-kev cross-over transition, as well as its branching ratio, can be deduced from Fig. 4, which is the coincidence spectrum taken with the single channel gating counter set to accept pulses in the range from 261 to 278 kev. At this setting, a considerable fraction of the counts in the gating counter are due to the 301- and the 356-kev gamma rays. Therefore, the total number of counts under the 81-kev peak in Fig. 4 represents the sum of $N_{276}(80)+N_{276}(82)+N_{301}(82)+N_{356}(82)$. The counts under the peak at 162 kev represent the sum of $N_{276}(162)+N_{276}(80+82)$; the last term is the rate of triple coincidences in which both the 80- and the 82-kev cascading photons enter the "spectrum" counter; these coincidence rates can be written as follows:

$$N_{276}(80)=N_0(276)\eta_s(276)\delta_{80}(1+\alpha_{80})^{-1}\cdot\eta_m(80)W\cdot(1-F_{82})\cdot(1-F_k),$$

where W is a factor to account for angular correlation effects; it is equal to unity in this case since the spectrum counter subtends a solid angle of 2π. F_{82} is a correction term which takes into account the possibility that a count would be lost to $N_{276}(80)$ due to the occurrence of a triple coincidence in which either the photon or the electron conversion x-ray from the 82-kev transition is detected by either of the two counters; F_k is a similar term for the K-capture x-rays. Similarly,

$$N_{276}(82)=N_0(276)\eta_s(276)\times\delta_{80}(1+\alpha_{82})^{-1}\eta_m(82)\cdot(1-F_{80})(1-F_k),$$

$$N_{276}(162)=N_0(276)\eta_s(276)\times\delta_{162}(1+\alpha_{162})^{-1}\eta_m(162)\cdot(1-F_k),$$

$$N_{356}(82)=N_0(356)\eta_s(356)(1+\alpha_{82})^{-1}\eta_m(82)\cdot(1-F_k),$$

$$N_{301}(82)=N_0(301)\eta_s(301)(1+\alpha_{82})^{-1}\times\eta_m(82)\cdot(1-F_{54})(1-F_k),$$

$$N_{276}(80+82)=N_0(276)\eta_s(276)\delta_{80}(1+\alpha_{80})^{-1}(1+\alpha_{82})^{-1}\times\eta_m(80)\eta_m(82)\cdot(1-F_k).$$

In this measurement $(1-F_{80})\simeq(1-F_{82})\simeq0.87$. Also, since $\eta_m(80)\simeq\eta_m(82)$, we obtain, as the ratio of the counts under the 162-kev peak to that under the 81-kev peak,

$$R=\frac{N_{276}(162)+N_{276}(80+82)}{N_{276}(80)+N_{276}(82)+N_{301}(82)+N_{356}(82)}=\frac{\eta_m(80)+\delta\cdot(1+\alpha_{82})(1+\alpha_{80})(1+\alpha_{162})^{-1}\eta_m(162)/\eta_m(80)}{(1+\alpha_{80})(1-F_{80})+(1+\alpha_{82})(1-F_{82})+A(1+\alpha_{80})(1+\delta)},$$

where $A=[N_0(356)\eta_s(356)+N_0(301)\eta_s(301)(1-F_{54})]/N_0(276)\eta_s(276)$ and $\delta=\delta_{162}/\delta_{80}$. From Fig. 1, A is deduced to be 0.8 ± 0.2. Using the values $\eta_m(80)=0.33\pm0.03$, $R=0.17\pm0.02$, and values of α_E's from Table I, we obtain $\delta=0.15\pm0.03$. This gives $\delta_{80}=0.87\pm0.03$ and $\delta_{162}=0.13\pm0.03$.

APPENDIX II. RELATIVE INTENSITIES OF THE 80- AND 82-kev TRANSITIONS

The relative intensities of the 80- and 82-kev transitions can be deduced from the singles spectrum and the spectrum in coincidence with the 82 (and 80) kev gamma rays in Fig. 3. Using same notations as before, we define

$$R_s\equiv\frac{N_m(80)+N_m(82)}{N_m(356)}=\frac{N_m(80)}{N_m(356)}\left(1+\frac{N_m(82)}{N_m(80)}\right),$$

$$R_c\equiv\frac{N_{82}(80)+N_{80}(82)}{N_{82}(356)}=2\frac{N_{82}(80)}{N_{82}(356)}=2\frac{N_m(80)}{N_m(356)}.$$

In the above formulation, effects due to summing have been neglected, since the solid angles subtended for this measurement were of the order of one percent. R_s and R_c are simply the ratios of the counts under the 81-kev peak to the counts under the 356-kev peak in the singles and coincidence spectrum, respectively, and can be directly measured from Fig. 3.

$$\frac{N_m(82)}{N_m(80)}=\frac{2R_s}{R_c}-1=5.8\pm0.7.$$

Thus, if $\eta_m(80)=\eta_m(82)$

$$\frac{I_{82}}{I_{80}}=\frac{N_m(82)}{N_m(80)}\cdot\frac{1+\alpha_{82}}{1+\alpha_{80}}=5.9\pm0.7.$$

Furthermore, since $I_{82}=I_{356}+I_{301}+I_{80}$, we get $I_{82}=154\pm5$ and $I_{80}=26\pm3$ on the basis of $I_{356}=100$ (and $I_{301}=28\pm3$, Table I).

APPENDIX III. RELATIVE INTENSITY OF THE 54-kev TRANSITION

In the spectrum of Fig. 2 the intensity of the peak at 54 kev is partly due to the escape peaks of the 80- and the 82-kev gamma rays. Therefore, the ratio of the number of counts under the 54-kev peak to the counts under the 80- and 82-kev peak is

$$R=\frac{N_m(54)+k[N_m(80)+N_m(82)]}{N_m(80)+N_m(82)}=\frac{N_m(54)}{N_m(80)+N_m(82)}+k,$$

where k is the escape-peak to photopeak ratio for an 80-kev gamma ray. Summing effects again have been neglected since the solid angle subtended was small.

When a 0.044-inch Cu absorber is inserted in front of the counter, the 54-kev and the 80-kev gamma rays are attenuated by different factors. If μ denotes the ratio of the attenuation factor for the 54-kev gamma ray to that for the 82-kev gamma ray, the ratio R then becomes

$$R'=\mu\cdot\frac{N_m(54)}{N_m(80)+N_m(82)}+k.$$

The measured values are: $R=0.109$ and $R'=0.073$ with an estimated error of $\pm 10\%$. The value of μ is calculated from known absorption coefficients,[22] using not the total cross section, but only the photoelectric plus the back-scattering cross sections. It was assumed that all forward scatterings involved energy losses that are small enough so that the scattered gamma ray appeared in the photopeak. For a 54-kev gamma ray, the back-scattered fraction is taken to be 42%; for 82 kev, 40%. This gives $\mu=0.18$. Solving the above equations gives $k=0.065\pm 0.018$ in agreement with the value of 0.067 obtained from the graphs of Bell in reference 6; and

$$\frac{N_m(54)}{N_m(80)+N_m(82)}=\frac{I_{54}(1+\alpha_{54})^{-1}}{I_{80}(1+\alpha_{80})^{-1}+I_{82}(1+\alpha_{82})^{-1}}\cdot\frac{\eta_m(54)}{\eta_m(80)}=0.044\pm 0.015.$$

For $\eta_m(54)/\eta_m(80)=0.94$ and values of α_E's taken from Table I, we obtain $I_{54}=19\pm 6$ on the basis of $I_{356}=100$.

APPENDIX IV. RELATIVE INTENSITIES OF THE ELECTRON-CAPTURE TRANSITIONS

If β_1, β_2, and β_3 are the branching ratios of the electron-capture transitions to the 438-, 383-, and 162-kev levels, we get

$$\frac{\beta_2}{\beta_1}=\frac{I_{386}+I_{301}+I_{220}-I_{54}}{I_{356}+I_{276}+I_{54}}=0.15\pm 0.06,$$

and

$$\frac{\beta_3}{\beta_1}=\frac{I_{162}+I_{80}-I_{220}-I_{276}}{I_{356}+I_{276}+I_{54}}=0.17\pm 0.04.$$

Therefore, $\beta_1=(76\pm 4)\%$, $\beta_2=(11\pm 4)\%$, and $\beta_3=(13\pm 3)\%$.

APPENDIX V. RELATIVE INTENSITIES OF THE BETA TRANSITIONS FROM Xe^{133}

If β_1 and β_2 are the branching ratios of the beta transitions from Xe^{133} to the 162- and 82-kev levels of Cs^{133}, respectively, then the coincidence counting rate between the 80- and the 82-kev gamma rays from the Xe^{133} sample is

$$N_c=N_{80}(82)+N_{82}(80)=2\cdot N_m(82)\beta_1\delta_{80}\times(1-\beta_1\delta_{162})^{-1}(1+\alpha_{80})^{-1}\cdot\eta_s(80),$$

whereas, the singles counting rate under the 82-kev peak is

$$N_m=N_m(82)+N_m(80)=N_m(82)\left(1+\beta_1\delta_{80}\frac{1+\alpha_{82}}{1+\alpha_{80}}(1-\beta_1\delta_{162})^{-1}\right).$$

The ratio of N_m/N_c is measured to be about 4500. Knowing that $\alpha_{82}=1.77$, $\alpha_{80}=1.74$, $\eta_s(80)=0.024$ for this measurement, and $\delta_{80}=0.87$, $\delta_{162}=0.13$ from Appendix I, we can solve for β_1 and obtain $\beta_1=0.015\pm 0.010$.

[22] C. M. Davison, in *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (Interscience Publishers, Inc., New York, 1955), Chap. II.