

Calculations of Neutron Capture Cross Sections*

C. MOSSIN-KOTIN,[†] B. MARGOLIS,[‡] AND E. S. TROUBETZKOY[§]
Columbia University, New York, New York

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Neutron capture cross sections are calculated for a number of target nuclei using the statistical theory of nuclear reactions. These calculations are compared with experiment. The different shapes of these capture cross sections as a function of energy are explained in terms of the effect of higher incident neutron partial waves and the competition from inelastic scattering. Using the measured capture sections it is seen that considerable information can be extracted concerning properties of the target and compound nuclei.

INTRODUCTION

THE statistical theory of nuclear reactions has proved very successful in predicting neutron cross sections in the energy region up to a few Mev for heavy target nuclei. The general theory has been presented by Wolfenstein¹ and specialized for the case of inelastic scattering processes by Feshbach and Hauser² and for capture processes by Margolis.³ A refinement of the theory has been made by Dresner⁴ which is especially important when one has to consider only a small number of neutron channels.

In the statistical theory, nuclear reactions are described following the Bohr assumption. There is a probability for compound nucleus formation and a subsequent independent probability for decay through the various open channels. It is assumed that at any energy there are compound states of all angular momenta and both parities available, these being of high density and of random phases so that one can neglect interference terms.

The purpose of this paper is to make a detailed comparison with experiment of calculations of neutron capture cross sections. The theory involved has been described in detail in reference 3 and in papers by Rae, Margolis, and Troubetzkoy⁵ and Lynn and Lane.⁶

THEORY AND DISCUSSION

Since the appropriate formulas have been written down in detail in the references given above,^{3,5} we will content ourselves here with a qualitative formula. The cross section for neutron capture is essentially of the form

$$\sigma = \sigma_c \Gamma_\gamma / \Gamma,$$

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[†] On leave from University of Buenos Aires, Argentina. Fellow of the American Association of University Women.

[‡] Present address: Physics Dept., Ohio State University, Columbus, Ohio.

[§] Now at Nuclear Development Corporation, White Plains, New York.

¹ L. Wolfenstein, *Phys. Rev.* **82**, 690 (1951).

² W. Hauser and H. Feshbach, *Phys. Rev.* **86**, 366 (1952).

³ B. Margolis, *Phys. Rev.* **88**, 327 (1952).

⁴ L. Dresner, *Proceedings of the Columbia University International Conference on Neutron Interactions with Nuclei*, 1957, Columbia University Report CU-175, TID-7547 (unpublished), p. 71.

⁵ Rae, Margolis, and Troubetzkoy, *Phys. Rev.* **112**, 492 (1958).

⁶ A. M. Lane and J. E. Lynn, *Proc. Phys. Soc. (London)* **A70**, 557 (1957).

where σ_c is the cross section for compound nucleus formation, Γ_γ is the capture width and Γ the total width for decay of the compound nucleus. The quantitative formulas involve sums over contributions from the different partial waves.

Figures 1, 2, and 3 show theoretical fits to the measurements of Johnsrud, Silbert, and Barschall⁷ for the target elements As⁷⁵, Mo¹⁰⁰, Ag¹⁰⁷, W¹⁸⁶, and Au¹⁹⁷. This group has measured capture cross sections for a great many target nuclei using activation techniques. The cases considered were selected for analysis for several reasons. They cover a considerable range of the periodic table, by and large the parameters needed to calculate the cross sections are known, and the cross sections show a variety of shapes as a function of energy.

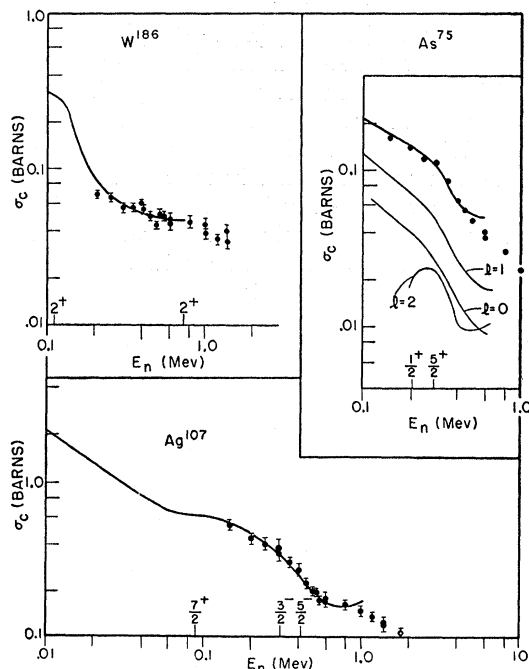


Fig. 1. Calculated neutron capture cross sections for As⁷⁵, Ag¹⁰⁷, and W¹⁸⁶ together with measurements of Johnsrud, Silbert, and Barschall.⁷ The energies, spins, and parities of excited states of the target are given along the energy abscissa. For As⁷⁵ the partial wave cross sections up to $l=2$ are also plotted.

⁷ Johnsrud, Silbert, and Barschall, preceding paper [*Phys. Rev.* **116**, 927 (1959)].

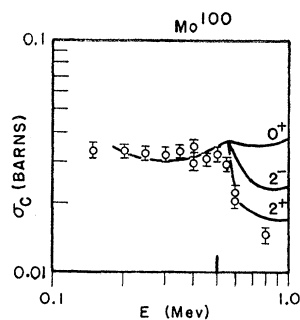


FIG. 2. Calculated neutron capture cross section for Mo^{100} together with measurements of reference 7. The different branches of the curve above neutron energy $E=500$ kev correspond to different choices of spin and parity for the excited state near 500 kev.

The neutron penetrabilities used in the calculations were taken from the tables of Beyster *et al.*⁸ who determine these by fitting to neutron scattering experiments with a diffuse-well optical model.

Table I lists the parameter $\Gamma_\gamma(B)/D(B)$ as determined from normalizing the calculated capture cross sections to the measurements of reference 7. Here $\Gamma_\gamma(B)$ is the radiation width at the separation energy, B , of a neutron and $D(B)$ is the level spacing at the same energy for compound levels that can be excited by s -wave neutrons. Also listed is $\Gamma_\gamma(B)$ as determined from slow neutron resonance measurements.⁹ If the radiation width is known then one can of course determine $D(B)$ which is also listed in Table I. It is to be noted that the value of Γ_γ/D extracted from the measurements⁷ depend somewhat on the functional dependence of the level spacing on angular momentum. This is taken to be of the form $D_J \propto (2J+1)^{-1} \exp[\alpha(2J+1)^2]$ following Lang and LeCouteur.¹⁰ The agreement of $D(B)$ as determined here with slow neutron results⁹ is very good.

The cross section measurements of reference 7 can be divided roughly into two classes, those that decrease monotonically (mainly the odd target nuclei) and those that are relatively flat or actually increase as energy increases (mainly even target nuclei). The increase or flatness in the cross sections is due to the onset of important contributions of partial waves with angular momentum greater than zero.

This increase or flatness will appear if the value

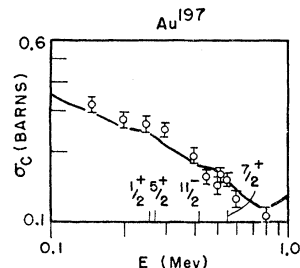


FIG. 3. Calculated neutron capture cross section for Au^{197} together with measurements of reference 7. In addition to the excited states indicated along the abscissa, there is a state with excitation 77 kev ($\frac{3}{2}+$).

⁸ Beyster, Salmi, Schrandt, and Walt, Los Alamos Report LA-2099 (unpublished).

⁹ *Neutron Cross Sections*, compiled by D. J. Hughes and J. A. Harvey, Brookhaven National Laboratory Report BNL-325 (Superintendent of Documents, U. S. Government Printing Office, Washington, D. C., 1955); D. J. Hughes and R. Schwartz, *ibid.*, Suppl. No. 1, 1957.

¹⁰ J. M. B. Lang and K. J. LeCouteur, Proc. Phys. Soc. (London) **A67**, 686 (1954).

of Γ_γ/D is small enough. Small values of this quantity imply strong contributions to the capture cross section from l values greater than zero.¹¹ An inspection of Table I provides a rough check on this rule. Mo^{100} and W^{186} have the smallest values of Γ_γ/D and their capture cross sections show flat regions. The behavior described above is of course modified by the onset of inelastic scattering which brings down the capture cross sections. The sudden drops in the cross sections as energy increases are of course due to the onset of inelastic scattering as one passes threshold. It is to be noted in Fig. 2 that one has a convincing check on the spin and parity of the excited state of Mo^{100} near 500 kev.

On the other hand, in the case of As^{75} it appears that the cross-section calculations are relatively insensitive to fine details of the level structure. As^{75} apparently has more low-lying levels than were put into the calculation when it was performed (see reference 7). It is to be noted from Fig. 2 that if the excited state of

TABLE I. Ratio of radiation width (assumed independent of spin of compound state) to level spacing at separation energy of a neutron. For odd-mass target nuclei $1/D=1/D_{i-1/2}+1/D_{i+1/2}$, where i is the spin of the target nucleus; for even-mass nuclei $D=D_i$. This ratio has been determined by fitting calculated capture cross sections to the measurements of Johnsrud, Silbert, and Barschall.^a Also listed are radiation widths from slow-neutron resonance measurements.^b The last column gives values of $D(B)$ determined using columns 2 and 3. In all cases where $D(B)$ is known from slow-neutron work^b there is agreement to within experimental error.

| Target | $10^3 \times \Gamma_\gamma(B)/D(B)$ | $\Gamma_\gamma(B)$ (10^{-8} ev) | $D(B)$ (ev) |
|-------------------|-------------------------------------|------------------------------------|-------------|
| As^{75} | 3.6 | 270 ± 50 | 76 |
| Mo^{100} | 1.86 | 260 ± 80 | 140 |
| Ag^{107} | 6.8 | 140 ± 25 | 21 |
| W^{186} | 0.59 | 46 ± 9 | 78 |
| Au^{197} | 5.8 | 125 ± 30 | 22 |

^a See reference 7.

^b See reference 9.

Mo^{100} near 500 kev had spin and parity 0^+ , the inelastic scattering competition would not be very important. It was not thought worthwhile to do other calculations on As^{75} since spins and parities of the levels involved are not known.

It is seen that the statistical theory of nuclear reactions predicts neutron capture cross sections in considerable detail if one has the necessary parameters: radiation width, binding energy, level spacing, target nucleus level structure and neutron penetrabilities. In the absence of complete knowledge it is possible to extract some of these quantities from measurements of capture cross sections as a function of energy. In addition to the type of information obtained above, it is in principle also possible to obtain, from an analysis of the experiments, information on the neutron penetrabilities, if all other quantities are known. An analysis of this type has already been made¹² in order to investigate p -wave neutron penetrabilities.

¹¹ E. P. Wigner, Am. J. Phys. **17**, 99 (1949).

¹² K. K. Seth, Proceedings of the International Conference on the Nuclear Optical Model, The Florida State University, Tallahassee, Florida, 1959 (unpublished).