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Elastic Scattering of Low-Energy Electrons by Argon†

B. Kivel
AVCO-Everett Research Laboratory, Everett, Massachusetts

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The partial-wave Schrödinger equation has been integrated numerically for a potential adjusted so that the predicted elastic scattering is in agreement with measurements by Ramsauer and Kollath. The cross section at zero energy is found to be 8×10^{-16} cm^2 which is appreciably above the minimum value of 0.3 $\times10^{-16}$ cm^2 at 0.4 ev. The high value at zero energy, which results from the tail of the polarization force, will be reduced at densities of one atmosphere and above, because the field is cut off by neighboring atoms. Application of the method to predict the eigenvalues of the excited bound states of potassium indicates the validity of the static potential and that the exchange force decreases with increasing angular momentum.

THE Ramsauer-Townsend minimum in the elastic scattering of electrons by argon has been measured with nearly monoenergetic electrons by Ramsauer and Kollath.[1,2] The effect was computed by Holtsmark[3] using a partial-wave solution of the Schrödinger equation with a Hartree-atom potential plus a polarization force. The angular distribution of the scattering has also been measured by Ramsauer and Kollath,[4] from which one concludes that the phase shift for angular momentum $l=1$ is zero at about[5] 1.15 ev (symmetric distribution around 90°). We have used this information to extrapolate the measurements to zero energy.

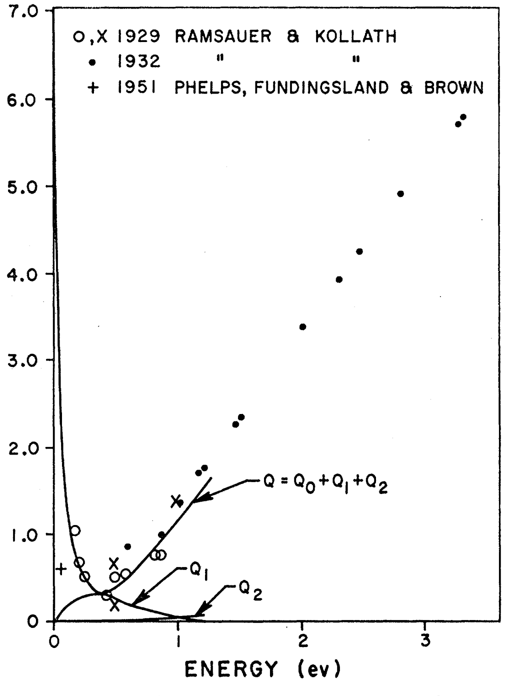

Fig. 1. Comparison of total elastic scattering experiments[1,4,9] with the semiempirical theoretical prediction Q. Also shown are partial cross sections for $l=1$ adjusted so that $Q_1=0$ at 1.2 ev and for $l=2$ adjusted so that $Q_2=Q_0/20$ at 1.2 ev, which give the best fit to the angular distribution.[5] The large value of the cross section at very low energy ($Q=7.5\times10^{-16}$ cm^2 at zero energy) results from the tail of the polarization force and will be reduced at densities of one atmosphere and above.

Our calculation differs only slightly from that of Holtsmark. We use the Hartree-Fock atomic charge distribution[6]: an exchange potential proportional to the atomic electron charge density to the $\frac{1}{3}$ power,[7,8] and a polarization potential $[p/(r^2+d^2)^2]$. We use[3] $p=11$ and find $d^2\doteq2.5\doteq\langle r_{3s}^2\rangle$ gives a good fit to the data where $\langle r_{3s}^2\rangle$ is the mean value of r^2 for the bound $3s$ state. It is expected that the magnitude of the exchange potential depends on l and, indeed, that required for $l=1$ is about $\frac{1}{2}$ of that for $l=0$.

A perturbation method has been used to determine the effect of small changes in the potential. The basic approximation for relating two solutions (ψ and ϕ) for potentials which differ by ΔU is

$$(\psi\phi'-\phi\psi')_r=\int_0^r \Delta U\psi\phi dr\doteq\int_0^r \Delta U\phi^2 dr.$$

The results are shown in Fig. 1. Values of Q_0 which give a good fit to the Ramsauer-Kollath data of 1929 are (in units of 10^{-16} cm^2) 7.5 at zero energy, 4.0 at $E=0.02$ ev, 0.55 at 0.15 ev, and 1.4 at 1.2 ev. Q_1 has a maximum value of about 0.3 and is uncertain by about 25% mainly because of the uncertainty of energy at which $Q_1=0$. The low-energy prediction is appreciably above earlier microwave and swarm measurements[9,10] at room temperature and Holtsmark's result at zero energy. It is in agreement with recent measurements by Pack and Phelps[11] using a drift tube which indicates an E^{-1} energy dependence between 77°K and 300°K.

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Calculation for $l=2$ at 1.2 ev gives $Q_2=0.3\times10^{-16}$ cm^2 and a phase shift consistent with the angular distribution data. It is larger by a factor 4 than the best fit to the angular distribution obtained by Holtsmark[5] which has been used in Fig. 1. However, the higher value gives a better fit to the total cross section.

The cross section in the neighborhood of room temperature and below is strongly increased by the tail of the polarization force. If this tail is truncated by the presence of neighboring atoms, then the cross section will be reduced. The effect is strongest at zero energy, where the cross section (in units of 10^{-16} cm^2) is reduced from 7.5 to 6 for a cutoff at $r=60$, to 3 at $r=40$, and to 1 at $r=25$. This effect may explain the small value observed by Whalin who made measurements at 1 atmosphere but not that of Phelps, Fundingsland, and Brown which was made at a pressure of 1 mm of Hg.

We have applied the same method to predicting the eigenvalues of potassium using $p=8.5$.[12] Essentially the same potential fits the eigenvalues for 4*s*, 6*s*, 8*s*, and 10*s*. (The exchange force magnitude required is $\sim5\%$ larger for 4*s* than the others.) This establishes the validity of the static potential. The result is insensitive to the shape of polarization and exchange forces inside the atom because of the dominance of the Coulomb field. As in argon, a weaker ($\sim25\%$) exchange force was required for $l=1$ (6*p*) than for $l=0$ (6*s*).

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Energy Dependence of Fast-Neutron Activation Cross Sections*

A. E. Johnsrud,† M. G. Silbert,‡ and H. H. Barschall
University of Wisconsin, Madison, Wisconsin

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Fast-neutron capture cross sections of 24 nuclides ranging from $A=51$ to $A=197$ have been measured by an activation method, in the neutron energy region from 0.15 to 6.2 Mev. The neutron energy spreads were of the order of 0.1 Mev so that cross sections averaged over many energy levels of the compound nucleus were measured. Activities induced in samples by fast and thermal neutrons were compared. The relative neutron flux in the fast- and thermal-neutron activations was determined with a U^{235} fission counter. A knowledge of the energy dependence of the U^{235} fission cross section and of the thermal-neutron activation cross sections allows calculation of the fast-neutron activation cross sections.

I. INTRODUCTION

MEASUREMENTS of the cross section for the radiative capture of fast neutrons have furnished valuable information about nuclear structure such as the dependence of level spacing on excitation energy and on nucleon number, and the effects of closed shells and of even or odd numbers of protons and neutrons on level spacing. Several authors[1–3] have calculated fast-neutron capture cross sections averaged over resonances. Most recently Lane and Lynn,[4] and Rae, Margolis, and Troubetzkoy[5] have calculated the energy dependence of the capture cross sections of U^{235}, U^{238}, and Th^{232} for neutron energies below 1 Mev. These calculations gave cross sections in good agreement with observations. There are, however, not many other isotopes for which the energy dependence of the capture cross section has been measured over a wide energy range, and there are inconsistencies in the reported results. For these reasons the present experiments were undertaken to measure capture cross sections of intermediate and heavy nuclei averaged over many energy levels in the compound nucleus, for neutrons in the energy range from 0.15 to 6.2 Mev.

Three methods for measuring fast-neutron capture cross sections are currently widely used: (1) observations of induced radioactivities,[6,7] (2) observations of capture γ-rays,[8] and (3) sphere transmission experiments.[9]

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† Present address: Hughes Aircraft Company, Culver City, California.

‡ Present address: Los Alamos Scientific Laboratory, Los Alamos, New Mexico.

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