Millimeter Wave Absorption in Superconducting Aluminum. II. Calculation of the Skin Depth

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The skin depth in superconducting aluminum is calculated from the measured frequency dependence of the surface resistance through the Kronig-Kramers integral transforms. At absolute zero, it is found that the skin depth δ is independent of frequency at low frequencies but begins to increase at higher frequencies. The maximum rate of increase of δ occurs when the photon energy equals the gap energy, $h\nu=3.2kT_c=\mathcal{E}_g$; at this point $\delta(h\nu=\mathcal{E}_g)/\delta(h\nu=0)\approx1.12$. The maximum value of δ occurs at $h\nu\approx4kT_c$. The superconducting penetration depth λ [i.e., $\delta(h\nu=0)$] is found to vary approximately as $\lambda(t)=\lambda(0)(1-t^4)^{-\frac{1}{2}}$, with $\lambda(0)=5.15\times10^{-6}$ cm and $t\equiv T/T_c$. The effects of changes in the skin depth have been eliminated from the determination of the energy gap by calculation of the real part of the complex conductivity, σ_r. The energy gap values deduced from the behavior of σ_r differ only slightly from the results obtained directly from the surface resistance measurements.

I. INTRODUCTION

IN the preceding paper[1] (hereafter referred to as I) the surface resistance ratio of a superconductor was measured as a function of temperature and of frequency. It was noted that the accurate determination of the energy gap as a function of temperature requires information about the variation of the electromagnetic skin depth with frequency. The complex skin depth δ may be defined by

$$\delta\equiv\frac{1}{H(0)}\int_0^\infty Hdy, \tag{1}$$

where $H(0)$ is the magnetic field at the surface of the metal and H is the field at a distance y from the surface. It then follows that, so long as $H(\infty)=0$,

$$\delta=\frac{-icE(0)/H(0)}{\omega}\equiv\frac{icZ}{4\pi\omega}, \tag{2a}$$

where $E(0)$ is the electric field at the surface, ω is the angular frequency, and Z is the surface impedance. The skin depth so defined may be divided into a real and an imaginary part; $\delta=\delta_r-i\delta_i$, and since $Z=R+iX$

$$\delta_r=cX/4\pi\omega, \tag{2b}$$

$$\delta_i=cR/4\pi\omega. \tag{2c}$$

The quantities δ_i and δ_r are not independent, but are related by the Kronig-Kramers integral transforms[2]

$$\delta_r(\omega)=\frac{2}{\pi}\int_0^\infty\frac{\omega'\delta_i(\omega')}{\omega'^2-\omega^2}d\omega'+\delta_r(\infty), \tag{3a}$$

and

$$\delta_i(\omega)=-\frac{2\omega}{\pi}\int_0^\infty\frac{\delta_r(\omega')}{\omega'^2-\omega^2}d\omega'. \tag{3b}$$

It is the purpose of this paper to evaluate the real part of the skin depth $\delta_r(\omega)$ from the measured values of surface resistance ratio $r\equiv R/R_n$ through Eqs. (2c) and (3a). The limiting value, $\delta_r(0)$, is the superconducting penetration depth, λ. It will be shown that, as a consequence of the energy gap, the skin depth (at least at the lower temperatures) rises with increasing frequency, reaching a maximum at photon energies somewhat greater than the energy gap and then falls toward the value of the skin depth in the normal state. Finally, we will show that the estimate of the temperature dependence of the energy gap is only slightly modified (from that obtained in I) when the effects of penetration have been eliminated.

II. METHOD OF CALCULATION

In Fig. 6 of I, the frequency dependence of the microwave absorption is given at a number of temperatures. Unfortunately, these curves only cover the range $0.64\leq h\nu/kT_c\leq3.9$. In order to evaluate the real part of the skin depth, δ_r, from Eq. (3a) it is necessary to know δ_i (and thus the surface resistance) at all frequencies. Therefore, we have found it necessary to extrapolate our results to both higher and lower

[1] M. A. Biondi and M. P. Garfunkel, preceding paper [Phys. Rev. **116**, 853 (1959)].

[2] A. B. Pippard, *Advances in Electronics and Electron Physics*, edited by L. Marton (Academic Press, Inc., New York, 1954), Vol. 6, pp. 4, 5. In this paper it is pointed out that the skin depths defined above have a simple physical interpretation only for fields which decay exponentially. However, even when the penetration is not exponential, if $\delta_i=0$ (as occurs in many of the cases of interest in the present paper), δ_r is the "effective" extinction distance. Even when $\delta_i\neq0$, it can be shown that δ_r is still sufficiently large to dominate and thus remains a good approximation to the extinction distance. We use δ_r throughout the paper in this sense. *Note added in proof.*—To avoid confusion, we point out that our subscripts r and i refer to the real and imaginary parts, respectively, of the skin depth S, as defined in Eq. (1). On the other hand, in Pippard's notation the subscript r refers to the resistive and i to the inductive part of the skin depth, as defined by Eqs. (2b) and (2c). Thus, our δ_r is the same as Pippard's δ_i, and vice versa.

frequencies. In two cases this is a simple task. These are the extrapolation of the lowest temperature curves to zero frequency, and the extrapolation of the highest temperature curves to infinite frequency. For the low-temperature curves the values of the surface resistance ratio are already so small at $h\nu/kT_c=0.64$ that their contribution to the integral in Eq. (3a) at lower frequencies is negligible and is unlikely to cause any error. Similarly, the high-temperature curves at $h\nu/kT_c=3.9$ are already so close to $r=1$ that it is unlikely that any appreciable error will be introduced in extrapolating these curves to the value $r=1$. In this case, however, there is the possibility of some difficulty if the curve were to go appreciably *above* $r=1$ and then approach unity at values of $h\nu/kT_c$ several decades higher.[3] This does not seem to be very likely from the appearance of the aluminum data. Furthermore, the experimental results of Richards and Tinkham[4] on other superconductors give no evidence for such behavior. For the other cases, namely, the low-frequency extrapolation of the high-temperature curves and the high-frequency extrapolation of the low-temperature curves, we have investigated the sensitivity of the results to the particular extrapolation that we use.

In Fig. 1 we display the curves for $r\equiv R/R_n$ as a function of $h\nu/kT_c$ that are used in Eqs. (2c) and (3a) to determine δ_r. The solid lines are from the data in I, while the dashed lines represent the extrapolations. For the extrapolations to $h\nu/kT_c=0$ the curves are drawn so that they join the data smoothly and go to zero at $T=0$.[5] For the extrapolations to $h\nu/kT_c=\infty$ the form of the curves was made consistent with the observed shapes of the higher temperature curves.

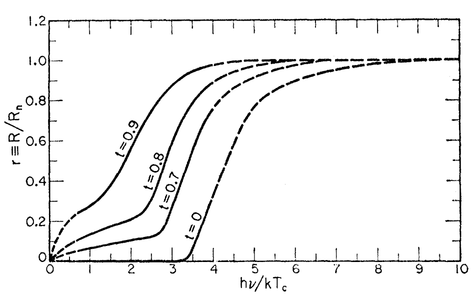

FIG. 1. Surface resistance ratio r as a function of the reduced photon energy $h\nu/kT_c$ at various reduced temperatures. The solid lines represent the measured values from I and the dashed lines are the extrapolations to higher and lower frequencies.

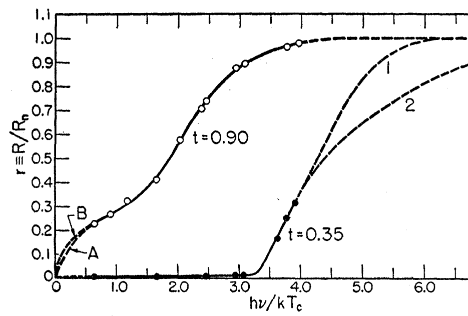

FIG. 2. Surface resistance ratio r as a function of reduced photon energy $h\nu/kT_c$ for those cases in which the skin depth is most sensitive to the form of the extrapolations. The dashed curves marked A, B and 1, 2 represent reasonable limits for the extrapolations at these temperatures.

In order to obtain a measure of the sensitivity of the results to the form of the extrapolations, several alternatives were tried. Figure 2 shows curves of r *vs* $h\nu/kT_c$, giving various possibilities for extrapolation, at those temperatures for which the skin depth is most sensitive to the details of the extrapolation. We shall show from the results that the extrapolation to low frequencies has a rather large effect on the penetration, while the extrapolation to high frequencies has a relatively small effect.

In Eq. (2c) we see that the quantity $\delta_i(\omega)$ which must be used in Eq. (3a) to obtain $\delta_r(\omega)$ is proportional to the surface resistance, R. Since the measurements give $r\equiv R/R_n$, it is necessary to determine R_n. We have no independent measurements of R_n; therefore we have had to rely on the measurement of Faber and Pippard[6] at a frequency of 1200 Mc/sec. The theory of the anomalous skin effect in normal metals[7,8] was then used to obtain the frequency dependence of R_n. As mentioned in I, the conditions of the experiment were not completely in the extreme anomalous limit. Furthermore, it turns out that at these frequencies we are approaching the region where retardation effects become important; that is, where the electromagnetic field changes appreciably while an electron traverses the skin depth. Thus we see that the relevant parameter $\omega\delta/v_0\approx 0.07$ (v_0 is the Fermi velocity) is not sufficiently small compared to unity for a negligible retardation effect.[9]

In order to determine the effects of the above deviations from the extreme anomalous limit we use two different means of determining the frequency dependence of R_n. The first (Case I) assumes that we are in the extreme anomalous limit. Then, we have from Reuter and Sondheimer,[7] for diffuse reflection of the

[3] The calculations from the theory (reference 18) predict that near $t=1$ there should be a small rise of r above 1 for the higher frequencies. However, this is too small either to detect in the experiment or to cause any errors in this analysis. (See also reference 30 of I.)

[4] P. L. Richards and M. Tinkham, Phys. Rev. Letters **1**, 318 (1958).

[5] The form of the curves we have used at *very* low frequencies is probably wrong, because one would expect the approach to zero to be much more rapid, having zero slope at $h\nu=0$. Since we do not have any good theoretical basis for the precise behavior, we used the simplest shape that goes to zero at $h\nu=0$.

[6] T. E. Faber and A. B. Pippard, Proc. Roy. Soc. (London) **A231**, 336 (1955).

[7] G. E. H. Reuter and E. H. Sondheimer, Proc. Roy. Soc. (London) **A195**, 336 (1948).

[8] R. B. Dingle, Physica **19**, 311 (1953).

[9] In the remainder of the paper the extreme anomalous limit is taken to imply that the ratio of the skin depth to the free path $\delta/l\ll 1$ and that $\omega\delta/v_0\ll 1$.

electrons at the surface,

$$R_n=\frac{(\sqrt{3}\pi)^{\frac{1}{3}}\omega^{\frac{2}{3}}}{c^{\frac{4}{3}}(\sigma_n/l)^{\frac{1}{3}}},\tag{4}$$

where σ_n is the dc conductivity of the metal and l is the electron mean free path. σ_n/l is then obtained from the results of Faber and Pippard at 1200 Mc/sec. The second means of determining R_n is to use the detailed theory of the anomalous skin effect of Dingle[8] (referred to hereafter as Case II). In this theory it is necessary to introduce certain constants of the metal, namely, γ, the coefficient of the linear term in the electronic specific heat; σ_n; and σ_n/l.

The values of the constants are as follows: $\sigma_n/l=2.24\times10^{23}$ esu,[6] $\gamma=1.36\times10^{3}$ ergs/cm^3-deg^2,[10] and $\sigma=2.25\times10^{20}$ esu.[1] These quantities can then be used to define the effective mass m, the number density n, and also the Fermi velocity v_0 by

$$n=\left(\frac{3}{8\pi}\right)^{\frac{1}{2}}\frac{h^{\frac{3}{2}}}{e^3}\left(\frac{\sigma_n}{l}\right)^{\frac{3}{2}}=1.79\times10^{23}\ \text{cm}^{-3},$$

$$m=\left(\frac{3}{8\pi}\right)^{\frac{1}{2}}\frac{eh^{\frac{3}{2}}}{(\pi k)^2}\frac{\gamma}{(\sigma_n/l)^{\frac{1}{2}}}=1.37\times10^{-27}\ \text{g},\tag{5}$$

$$v_0=\left(\frac{\pi k}{e}\right)^2\frac{\sigma_n/l}{\gamma}=1.34\times10^{8}\ \text{cm/sec}.$$

With these values we now determine R_n as a function of frequency from Dingle's tables.[8]

While Dingle's theory (as well as the Reuter-Sondheimer theory) is, strictly speaking, only applicable for spherical Fermi surfaces, and aluminum probably has a very distorted Fermi surface, it is believed that the experimental condition of random orientation of the sample's crystal faces relative to the electromagnetic field make the application of the theory a reasonable approximation. With these assumptions, the integral of Eq. (3a) has been calculated numerically (with a high speed digital computer) for Case II at several temperatures and for Case I at $t=0$. The constant term in Eq. (3a), $\delta_r(\infty)$ is zero for Case I since $\delta_r\propto\omega^{-\frac{1}{3}}$. In Case II, however, $\delta_r(\infty)$ is the skin depth of a free electron gas in the extreme retardation region, where $\omega\delta/v_0\gg1$; i.e.,

$$[\delta_r(\infty)]_{\text{Case II}}=(mc^2/4\pi ne^2)^{\frac{1}{2}}.\tag{6}$$

With the values of n and m given above, we obtain $[\delta_r(\infty)]_{\text{Case II}}=1.54\times10^{-6}$ cm.

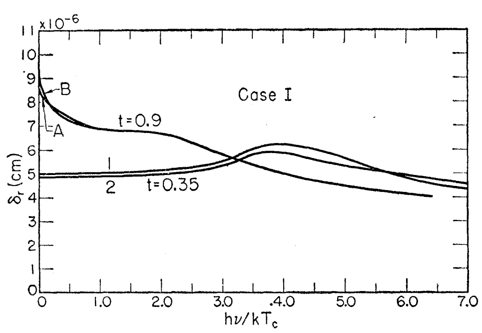

FIG. 3. The real part of the skin depth δ_r as a function of the reduced photon energy $h\nu/kT_c$ for the various extrapolations of Fig. 2, indicating the sensitivity of δ_r to the details of the extrapolations of the surface resistance curves.

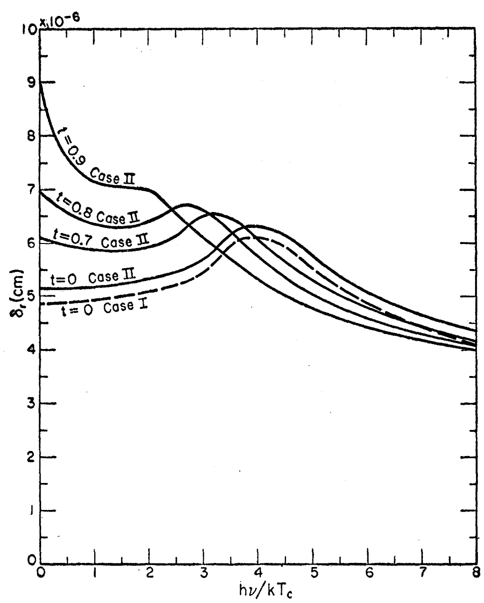

FIG. 4. The real part of the skin depth δ_r as a function of the reduced photon energy $h\nu/kT_c$ derived from the surface resistance curves of Fig. 1. The dashed curve at $t=0$ is calculated assuming that the surface resistance in the normal state R_n has the frequency dependence given in the extreme anomalous limit (Case I), while the solid curves are calculated assuming that R_n has the frequency dependence given by the detailed theory of Dingle (Case II).

III. RESULTS

(a) The Skin Depth

The sensitivity of the results to the various extrapolations proposed in Fig. 2, and to the particular forms of the frequency dependence of R_n that we have chosen is illustrated in Figs. 3 and 4. Figure 3 shows the values of $\delta_r(\omega)$ obtained for the various cases of Fig. 2, using the R_n of Eq. (4) (Case I). It is evident that over most of the frequency range there is very little difference between the different extrapolations used in each case. For the low-temperature curves ($t\equiv T/T_c=0.35$) the difference is insignificant for frequencies below about $h\nu/kT_c=4$. For the high-temperature cases ($t=0.9$) the only important difference occurs near $h\nu/kT_c=0$. Unfortunately, this region is important in determining the zero frequency intercept of $\delta_r(\omega)$, which is the

[10] N. E. Phillips, *Proceedings of the Fifth International Conference on Low-Temperature Physics, Madison, Wisconsin, 1957* (University of Wisconsin Press, Madison, Wisconsin, 1958), p. 414.

superconducting penetration depth, λ. This sensitivity to choice of extrapolation introduces a sizeable uncertainty in the deduced temperature dependence of the superconducting penetration depth at the higher temperatures.

Figure 4 is a plot of the derived values of $\delta_r(\omega)$ for Case I and Case II. The curves at $t=0$ have essentially the same shapes but differ by about 5% in absolute value. This difference is not unexpected since, strictly speaking, Case I is only approximately valid under experimental conditions. In both cases, the absolute accuracy (but not the variation with frequency) depends not only on the accuracy of the results of I, but also on the accuracy of the value of σ_n/l obtained from Faber and Pippard.

In Fig. 4 we see that $t=0$, as we increase frequency the penetration stays essentially constant at low frequencies, i.e., until the photon energy approaches the value of the energy gap $\mathcal{E}_g(0)$. It then increases, having its maximum slope[11] at $h\nu=\mathcal{E}_g(0)$, and reaches a maximum value at photon energies somewhat larger than the gap. It has, in the meantime become greater than the normal state skin depth, and as frequency is increased further, δ_r approaches the value for the skin depth in the normal state. This can be understood in the following way. At low frequencies the shielding by the superconductor resembles that of a perfect conductor or of a free electron gas and thus is independent of frequency. As the frequency increases, the approach to an absorption region (i.e., at energies exceeding the gap) is anticipated by an increase in the skin depth. When the photon energy exceeds the gap, electrons are excited across the gap, and these shield in much the same way as electrons in the normal state. With further increases in frequency more and more electrons are excited across the gap until the penetration is indistinguishable from that in the normal state.

The curves at the higher temperatures have much the same behavior except that there is also a characteristic initial decrease of δ_r with increasing frequency. This is the result of the frequency dependent penetration associated with the "normal" electrons, i.e., those that have been *thermally* excited across the energy gap. One might expect that, as $h\nu/kT_c$ approaches zero, there is a frequency below which the penetration associated with the thermally excited electrons is so great that it does not affect the total penetration, and thus there should no longer be any frequency dependence. In fact, the rapid frequency dependence that is shown at low frequencies is due entirely to our method of extrapolation to zero frequency. If we extrapolate our absorption results so that, at low frequencies, $R=C\omega^2$ then, for the proper choice of C, δ_r is independent of frequency at low frequencies.[12]

The change in skin depth at $t=0$ between $h\nu=0$ and $h\nu=\mathcal{E}_g$ has been predicted from a microscopic model by Khalatnikov and Abrikosov[13] as $[\delta(\mathcal{E}_g)/\delta(0)]_{\mathrm{Th}}=1.16$. This is in essential agreement with our results which give $[\delta(\mathcal{E}_g)/\delta(0)]_{\mathrm{exp}}=1.12$.

As stated earlier, the intercept at zero frequency is just the superconducting penetration depth λ. This quantity has been determined by Faber and Pippard[6] by direct measurements of the surface reactance X [see Eq. (2b)]. Unfortunately, the method they used only enables them to obtain differences in the penetration depth as a function of temperature and thus they needed a detailed law for the temperature dependence in order to obtain the absolute magnitude of the penetration. The law that has been used was derived from the Gorter-Casimir two-fluid model; namely,

$$\lambda(t)=\lambda(0)(1-t^4)^{-\frac{1}{2}}. \tag{7}$$

The measurements of the difference between the penetration in the normal state and in the superconducting states were plotted against $(1-t^4)^{-\frac{1}{2}}$. This should give a straight line with slope $\lambda(0)$. In this way the absolute magnitude of λ was determined.

Recently, Schawlow and Devlin[14] have found that there are appreciable departures from this law, Eq. (7), in superconducting tin. This invalidates the method of determining λ and in fact it has been shown by Miller[15] that the microscopic theory leads one to expect that there should be an error of $\sim 10\%$ in the penetration depth when it is determined in this way. Thus, the value of $\lambda(0)=4.92\times10^{-6}$ cm obtained for aluminum by Faber and Pippard[6] may be in error. From Fig. 4, Case II, we obtain $\lambda(0)=5.15\times10^{-6}$ cm (for the less realistic Case I we get 4.9×10^{-6} cm). It is difficult to estimate the accuracy of our value because we do not know how accurately the normal state surface resistance of Faber and Pippard applies to our sample, nor how the application of Dingle's theory for an idealized metal affects the results when applied to a real metal. Furthermore, there are the errors of extrapolation of our results and also the uncertainty in the electronic specific heat constant, γ. We estimate that our result is accurate to $\sim\pm3\%$.

It is of some interest to compare the temperature

[11] In fact, if there is a discontinuity in the slope of the r *vs* $h\nu/kT_c$ curves at the gap edge, the slope of the curve of δ_r *vs* $h\nu/kT_c$ becomes infinite at that point. Our experiment has not been able to establish a discontinuity in the slope of the absorption curves.

[12] There is an interesting point here with regard to the theory of Bardeen, Cooper, and Schrieffer [Phys. Rev. **108**, 1175 (1957)]. In their theory the density of states falls from *infinity* at the edge of the gap and slowly approaches the normal state value at energies far removed from the gap. If the density of states does not go to infinity in a real superconductor, but goes to some large, approximately constant value at energies very close to the gap edge then the surface impedance would be expected to be proportional to the square of the frequency at low frequencies. A similar problem has been discussed by L. C. Hebel and C. P. Slichter [Phys. Rev. **107**, 901 (1957)].

[13] Khalatnikov and Abrikosov, Advances in Phys. **8**, No. 29, 45 (1959).

[14] A. L. Schawlow and G. E. Devlin, Phys. Rev. **113**, 120 (1959).

[15] P. Miller, Phys. Rev. **113**, 1209 (1959).

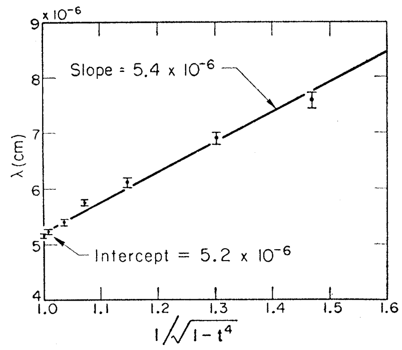

FIG. 5. The superconducting penetration depth λ as a function of $(1-t^4)^{-\frac{1}{2}}$. The highest point corresponds to $t=0.85$. From the Gorter-Casimir two-fluid model a straight line, with slope equal to the intercept at $(1-t^4)^{-\frac{1}{2}}=1$, is expected.

dependence of λ that we obtain with that of Eq. (7). In Fig. 5 we plot λ (the intercept at $h\nu/kT_c=0$ for Case II) against $(1-t^4)^{-\frac{1}{2}}$. Because of the increasing uncertainty in the intercept as temperature increases, we have shown values to $t=0.85$. To the estimated accuracy (error limits are indicated by horizontal lines), the points lie on a straight line. If Eq. (7) is valid, the slope and intercept should yield the same value of $\lambda(0)$. From the line determined by a least-squares fit, we obtain a slope of 5.4×10^{-6} cm and an intercept of 5.2×10^{-6} cm. This is as close agreement as we can expect. The accuracy is not sufficiently great to permit observation of the deviations from Eq. (7) that have been reported by Schawlow and Devlin.[14]

(b) Determination of the Complex Conductivity and the Energy Gap

The evaluation of the skin depth was initially motivated by the desire to eliminate the effects of penetration from the experimental determinations of the temperature dependence of the energy gap. These determinations are based on the observation of an absorption edge in the proper bulk property of the superconductor.

In the London formulation[16] of the theory of superconductivity the current density was taken as the sum of a normal component, j_n, and a superconducting component, j_s. The London equations may be written in the form

$$j=j_n+j_s=(\sigma_r-ic^2/4\pi\omega\lambda)E, \tag{8}$$

where σ_r is the conductivity of the normal fluid and λ is the superconducting penetration depth. Glover and Tinkham[17] have generalized this formulation by defining the complex conductivity in the superconducting state as the ratio of electric field to current density,

$$\sigma\equiv j/E\equiv\sigma_r-i\sigma_i, \tag{9}$$

where the London concept, that σ_r is the conductivity of the normal electrons, has been dropped. Thus, a sudden increase in absorption would be reflected in the real part of the conductivity, σ_r, without any of the complications of penetration. It remains to evaluate σ_r as a function of frequency from the absorption data.

The Mattis and Bardeen[18] formula relating the surface impedance in the superconducting state, Z, to that in the normal state, Z_n, is given in terms of the conductivities for the extreme anomalous limits[19]:

$$Z/Z_n=(\sigma_r/\sigma_n-i\sigma_i/\sigma_n)^{-\frac{1}{3}}. \tag{10}$$

Although the conditions of our experiment are not completely in the extreme anomalous region, we have seen that the results for the skin depth are not much affected by assuming extreme anomalous behavior. For this reason, a calculation of σ_r from Eq. (10) should be accurate enough to determine the threshold for direct excitation and thus the energy gap.

The assumption of extreme anomalous behavior gives for the normal state, $Z_n=R_n(1+i\sqrt{3})$. For the superconducting state, X is evaluated from δ_r by Eq. (2b). Substituting these quantities into Eq. (10), we obtain σ_r/σ_n and σ_i/σ_n as functions of temperature and frequency.

Figure 6 is a plot of σ_r/σ_n *versus* reduced photon energy for several temperatures. The knee of each curve is taken to indicate the energy at which the absorption edge occurs, the limits of uncertainty being indicated by the arrows.

The rapid rise in σ_r/σ_n as $h\nu/kT_c$ becomes small is of some interest. It is this behavior which prevents the observed absorption from varying as the square of the frequency at low temperatures, as would be expected from a simple two-fluid model. This variation of σ_r/σ_n is probably a consequence of a rapid decrease in the

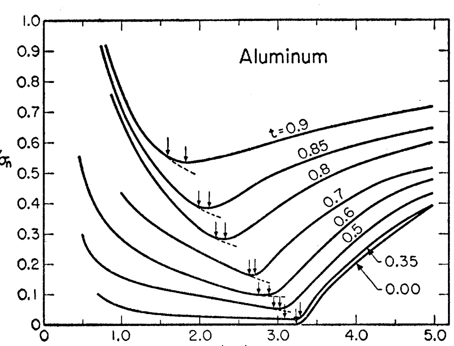

FIG. 6. The real part of the conductivity ratio σ_r/σ_n as a function of reduced photon energy $h\nu/kT_c$ at various reduced temperatures. The knee of each of these curves occurs at the energy of the absorption edge at that temperature. The arrows indicate the estimated limits of the uncertainty.

[16] F. London, *Superfluids* (John Wiley & Sons, Inc., New York, 1950), Vol. 1, Chap. B.

[17] R. E. Glover, III, and M. Tinkham, Phys. Rev. **108**, 243 (1957).

[18] D. C. Mattis and J. Bardeen, Phys. Rev. **111**, 412 (1958).

[19] For superconductors, the additional condition $\lambda(0)/\xi_0\ll1$ (where ξ_0 is the superconducting coherence length) is required to describe the extreme anomalous limit.

density of states as energy is increased beyond the high side of the gap.[12]

The energy gap-temperature curve deduced from Fig. 6 is shown in Fig. 7. The dashed curve is that obtained in I. As expected, differences between the two curves only exist at the higher temperatures. Considering the uncertainty in the deduced values, the differences between the two curves are barely significant.

IV. SUMMARY AND CONCLUSION

The superconducting penetration depth for aluminum at absolute zero has been determined to be $\lambda(0)=5.15\pm0.15\times10^{-6}$ cm, and the temperature dependence is shown to be approximately that given by the Gorter-Casimir two-fluid model, $\lambda=\lambda(0)(1-t^4)^{-\frac{1}{2}}$. However, the accuracy is not sufficiently great to distinguish between this law and that observed in superconducting tin by Schawlow and Devlin.[14] We note that the observed high frequency absorption properties lead to the characteristic small penetration depth of low-frequency magnetic fields.[20] In fact, the Meissner effect (i.e., dc behavior) has been shown to follow from the microwave and infrared absorption properties.[21]

In the superconducting state at $t=0$, the skin depth is independent of frequency at low frequencies, increases as the photon energy approaches the gap energy, has a maximum just above the gap, and falls toward the normal state penetration at high frequencies. This is in sharp contrast to the skin depth for a normal metal which falls monotonically as frequency increases (starting at infinity at $h\nu=0$). In aluminum the fractional change in skin depth between $h\nu=0$ and $h\nu=\mathcal{E}_g(0)$ is $[\delta(\mathcal{E}_g)/\delta(0)]_{t=0}\approx1.12$.

This method for determining the penetration depth λ has certain advantages over the other methods that have been used for bulk metals. In particular, it gives absolute values of the penetration depth rather than differences in values, such as are obtained in the methods of Faber and Pippard,[6] and Schawlow and Devlin.[14] On the other hand, the present method requires measurements over an extensive frequency range, which complicates the experimental techniques.

[20] This effect was also pointed out by R. A. Ferrell and M. Tinkham, Phys. Rev. Letters 2, 331 (1959), from an analysis of thin-film results.

[21] R. A. Ferrell, Bull. Am. Phys. Soc. Ser. II, 4, 225 (1959).

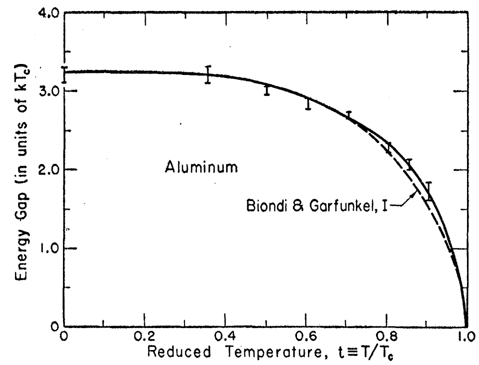

FIG. 7. The temperature variation of the energy gap in superconducting aluminum. The values are derived from the real part of the conductivity ratio (Fig. 6) with the limits of uncertainty shown by the horizontal lines. The solid line is the best curve through these points, forced to go to zero at $t=1$. The dashed curve is that deduced from the surface resistance data, as given in I.

The determination of the temperature dependence of the energy gap has been improved by the elimination of one of the sources of uncertainty, i.e., the variation of the skin depth near the absorption edge. It has been shown that this effect is only significant at the higher temperatures ($t>0.7$) and even there is only slightly greater than the probable error of the derived curve.

ACKNOWLEDGMENTS

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