# Contribution of Nucleon Currents to Radiative Muon Capture by a Proton*

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The observables for the reaction $\mu^-+p\rightarrow n+\nu+\gamma$ are calculated for a general $(pn\mu\nu)$ coupling including all terms of order $(\mu/M)$ and leading terms of order $(\mu/M)^2$, where $\mu$ is the meson mass and $M$ the nucleon mass. The nucleon anomalous magnetic moments are handled by the inclusion of a simple Pauli term and all other virtual pion effects are omitted here. The $(\mu/M)$ terms change the radiative capture rate by as much as 30%. Parity-nonconserving effects are decreased only slightly.

THE interest in radiative muon capture arises from the fact that only limited information about the coupling constants is available from the ordinary capture.[1] Unlike $\beta$ decay, in which the relativistic electron momentum distinguishes between $S$ and $V$ and between $A$ and $T$ couplings, the ordinary capture proceeds from a state in which the muon is essentially at rest; therefore, to make these distinctions, terms of order $v/c$ in the nucleon must be measured.[2] In addition, it is extremely difficult to observe parity nonconservation via the correlation between muon spin and neutron momentum, because (1) the residual polarization of the muon after cascading down to the $1s$ state is at most about 20%, and (2) the asymmetry is very small for $A-V$ or $A+V$ couplings.

Explicit expressions for observations on the processes

$$\mu^-+p\rightarrow n+\nu \tag{1}$$

and

$$\mu^-+p\rightarrow n+\nu+\gamma \tag{2}$$

have been given by Huang, Yang, and Lee.[3] They consider only diagram (1a) for process (2); it then follows from Cutkosky's theorem[4] that the spectrum, circular polarization, and the angular correlation (with respect to the neutrino) of the photon are the same as in the nonradiative emission of a positive lepton in the extreme relativistic limit $(v\rightarrow c)$. For diagram (1a), it is also easy to show that the asymmetry of the photons with respect to the muon spin is determined by their circular polarization, so that if the muon polarization is known, the asymmetry measurement is an alternative to the circular polarization measurement. Thus, reaction (2) proves valuable for making the distinctions which are hard to make using (1). However, (1a) is not the only diagram by means of which reaction (2) proceeds. As pointed out by Obayashi and Sakita,[5] the nucleon current contributions, even though of order $\mu/M$, can be quite important because of the large anomalous magnetic moments of the nucleons. It is these contributions which are calculated here.

We assume the same local weak interaction as in reference 3, with left-handed neutrinos, and treat the nucleons as simple Dirac particles with anomalous magnetic moments included by means of a Pauli term. The processes considered are shown in the four Feynman diagrams of Fig. 1. In the results we include all terms of order $\mu/M$ ($\mu$=muon mass, $M$=nucleon mass), including those in the density of states. We also include the biggest terms in $(\mu/M)^2$, those arising from diagrams (1c) and (1d) including the interference between them. These are included in order to illustrate the effects of $(\mu/M)^2$ terms, although some terms only slightly smaller may have been ignored.

The square of the matrix element for reaction (2) summed over initial proton and final neutron spins is proportional to

$$\begin{aligned}A_R+B_R\boldsymbol{\sigma}\cdot\hat{k}+C_R\hat{p}_\nu\cdot\hat{k}+D_R\hat{p}_\nu\cdot\boldsymbol{\sigma}+E_R\boldsymbol{\sigma}\cdot\hat{k}\hat{p}_\nu\cdot\hat{k}\\+F_R\boldsymbol{\sigma}\times\hat{p}_\nu\cdot\hat{k}+A_L-B_L\boldsymbol{\sigma}\cdot\hat{k}-C_L\hat{p}_\nu\cdot\hat{k}+D_L\hat{p}_\nu\cdot\boldsymbol{\sigma}\\+E_L\boldsymbol{\sigma}\cdot\hat{k}\hat{p}_\nu\cdot\hat{k}-F_L\boldsymbol{\sigma}\times\hat{p}_\nu\cdot\hat{k},\end{aligned}$$

where $R$, $L$ refers to the circular polarization of the

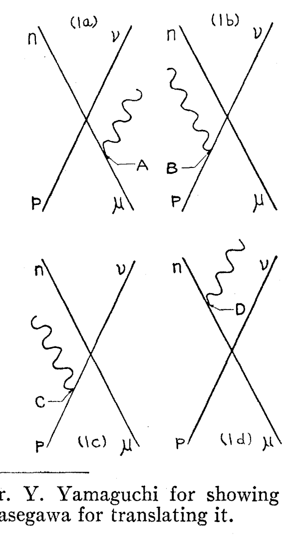


FIG. 1. Feynman diagrams for processes considered in this paper. The interactions are $A$: $-e\gamma^\mu A_\mu$; $B$: $+e\gamma^\mu A_\mu$; $C$: $-(\mu_pe/2M)\times\sigma^{\mu\nu}F_{\mu\nu}$; $D$: $-(\mu_ne/2M)\times\sigma^{\mu\nu}F_{\mu\nu}$.

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[1] A good review is given by H. Primakoff, Revs. Modern Phys. **31**, 802 (1959).

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[3] Huang, Yang, and Lee, Phys. Rev. **108**, 1340 (1957).

[4] R. Cutkosky, Phys. Rev. **107**, 330 (1957).

[5] H. Obayashi and B. Sakita, University of Nagoya, 1955. We do not know whether this has been published. We are grateful to Dr. Y. Yamaguchi for showing us this paper and to Dr. H. Hasegawa for translating it.

photon. Here, the caret indicates a unit vector, $\boldsymbol{\sigma}$ is the expectation value of the muon spin, $\mathbf{p}_\nu$ is the neutrino momentum, and $\mathbf{k}$ is the photon momentum.

For each coefficient $A$, $B$, $C$, etc. we write

$$A_{R,L}=A_{R,L}^{(1)}+A_m;\quad B_{R,L}=B_{R,L}^{(1)}+B_m,\quad \text{etc.},$$

where the subscript $m$ refers to the "pure magnetic-moment terms," diagrams (1c) and (1d), which do not distinguish between $L$ and $R$. The relation between the photon circular polarization and its asymmetry with respect to the muon spin, which holds for all but the pure magnetic terms, is expressed by

$$A_{R,L}^{(1)}=B_{R,L}^{(1)},\quad C_{R,L}^{(1)}=D_{R,L}^{(1)}+E_{R,L}^{(1)}.$$

The relations between the coefficients and the coupling constants are then:

$$\begin{aligned}
A_R^{(1)}=\;&3|C_A|^2+|C_V|^2+|C_V+C_A|^2(|\mathbf{k}|/M)+|C_V-C_A|^2(|\mathbf{p}_\nu|/M)-(\mu/M)\,\mathrm{Re}[C_A^*(C_S-C_P+3C_V-3C_A)\\
&-C_V^*(C_V-C_A)]-(\mu/M)(\mu_p-\mu_n)\,\mathrm{Re}(C_A^*C_F+C_V^*C_G)+(\mu/M)(\mu_p+\mu_n)2\,\mathrm{Re}(C_A^*C_G),\\
D_R^{(1)}=\;&(\mu/M)\,\mathrm{Re}[C_A^*(C_S-C_P)]+(\mu/M)(\mu_p-\mu_n)\,\mathrm{Re}(C_A^*C_F)+(\mu/M)(\mu_p+\mu_n)\,\mathrm{Re}(C_A^*C_G),\\
E_R^{(1)}=\;&|C_V|^2-|C_A|^2+|C_V+C_A|^2(|\mathbf{p}_\nu|/M)+|C_V-C_A|^2(|\mathbf{k}|/M)+(\mu/M)(|C_V|^2-|C_A|^2)\\
&-(\mu/M)(\mu_p-\mu_n)\,\mathrm{Re}(C_V^*C_G)-(\mu/M)(\mu_p+\mu_n)\,\mathrm{Re}(C_A^*C_G),\\
F_R^{(1)}=\;&(\mu/M)\{\mathrm{Im}[C_A^*(C_S-C_P)]+(\mu_p-\mu_n)\,\mathrm{Im}(C_A^*C_F)+(\mu_p+\mu_n)\,\mathrm{Im}(C_A^*C_G)\},\\
A_L^{(1)}=\;&3|C_T|^2+|C_S|^2+\mathrm{Re}[C_T^*(2C_T+C_S+C_P)](|\mathbf{k}|/M)-\mathrm{Re}[C_T^*(-2C_T+C_S+C_P)](|\mathbf{p}_\nu|/M)\\
&-(\mu/M)\,\mathrm{Re}[C_T^*(C_S+C_P-4C_T-C_V-C_A)+C_S^*(2C_T+C_V+C_A)]\\
&-(\mu/M)(\mu_p-\mu_n)\,\mathrm{Re}(C_T^*C_F+C_S^*C_G)+(\mu/M)(\mu_p+\mu_n)2\,\mathrm{Re}(C_T^*C_G),\\
D_L^{(1)}=\;&(\mu/M)\{\mathrm{Re}[C_T^*(2C_T+2C_V+2C_A+C_S+C_P)]+(\mu_p-\mu_n)\,\mathrm{Re}(C_T^*C_F)+(\mu_p+\mu_n)\,\mathrm{Re}(C_T^*C_G)\},\\
E_L^{(1)}=\;&-|C_T|^2+|C_S|^2-\mathrm{Re}[C_T^*(2C_T+C_S+C_P)](|\mathbf{p}_\nu|/M)+\mathrm{Re}[C_T^*(-2C_T+C_S+C_P)](|\mathbf{k}|/M)\\
&-(\mu/M)\{\mathrm{Re}[(C_S^*+C_T^*)(2C_T+C_V+C_A)]+(\mu_p-\mu_n)\,\mathrm{Re}(C_S^*C_G)+(\mu_p+\mu_n)\,\mathrm{Re}(C_T^*C_G)\},\\
F_L^{(1)}=\;&(\mu/M)\,\mathrm{Im}\{[C_T^*(2C_T+C_S+C_P+2C_V+2C_A)]+(\mu_p-\mu_n)(C_T^*C_F)+(\mu_p+\mu_n)(C_T^*C_G)\},\\
A_m=\;&(\mu^2/4M^2)\{(\mu_p-\mu_n)^2|C_F|^2+[2(\mu_p+\mu_n)^2+(\mu_p-\mu_n)^2]|C_G|^2\},\\
B_m=\;&(\mu^2/4M^2)\{[(\mu_p+\mu_n)^2+(\mu_p-\mu_n)^2]|C_G|^2-(\mu_p-\mu_n)(\mu_p+\mu_n)2\,\mathrm{Re}(C_F^*C_G)\},\\
C_m=\;&(\mu^2/4M^2)\{[(\mu_p+\mu_n)^2+(\mu_p-\mu_n)^2]|C_G|^2+(\mu_p-\mu_n)(\mu_p+\mu_n)2\,\mathrm{Re}(C_F^*C_G)\},\\
D_m=\;&(\mu^2/4M^2)[(\mu_p+\mu_n)^2|C_G|^2-(\mu_p-\mu_n)^2|C_F|^2],\\
E_m=\;&(\mu^2/4M^2)[(\mu_p-\mu_n)^2-(\mu_p+\mu_n)^2]|C_G|^2,\\
F_m=\;&(\mu^2/4M^2)(\mu_p-\mu_n)(\mu_p+\mu_n)2\,\mathrm{Im}(C_F^*C_G),
\end{aligned}$$

where $\mu_p$ and $\mu_n$ are the anomalous proton and neutron magnetic moments in units of nuclear Bohr magnetons, respectively, and

$$C_F=C_S+C_V,\quad C_G=C_A+C_T.$$

We note that in the "pure magnetic terms" only those with a factor $(\mu_p-\mu_n)^2$ have any appreciable magnitude.

In order to carry out integrations over the spectrum it is necessary to express each of the coefficients $A_R$, $A_L$, $B_R$, etc. in the form

$$A=A^0+(|\mathbf{k}|/M)A^k+(|\mathbf{p}_\nu|/M)A^p,$$

where the third term is the term in $A$ proportional to $|\mathbf{p}_\nu|$, the second is the term proportional to $|\mathbf{k}|$, and $A^0$ stands for all the other terms. For the observations discussed in reference 3 we find:

1. The capture rate for reaction (1) is[2]

$$\tau_{\mathrm{cap}}^{-1}=\Delta^2\xi/2\pi^2a^3,$$

with

$$\begin{aligned}
\xi=\;&(|C_F|^2+3|C_G|^2)(1-2\Delta/M)+(\Delta/M)\{|C_V-C_A|^2\\
&-\mathrm{Re}[C_T^*(-2C_T+C_S+C_P+3C_V-3C_A)\\
&+C_A^*C_P-C_S^*C_V]\},
\end{aligned}$$

where $a$ is the Bohr radius of the mesonic $1s$ orbit and $\Delta$ is the energy available, which is equal to $\mu$ if the binding energy and proton-neutron mass difference are ignored.

2. The capture rate for reaction (2) yielding right ($R$)- or left ($L$)-handed photons is

$$\tau_{\mathrm{rad}\;R,L}^{-1}=(\alpha\eta_{R,L}/12\pi\xi)\tau_{\mathrm{cap}}^{-1},$$

where $\alpha=1/137$,

$$\begin{aligned}
\eta_{R,L}=\;&A_{R,L}^0[1-(9\Delta/5M)]+(\Delta/M)[3A_{R,L}^p/5\\
&+2A_{R,L}^k/5-2\lambda C_{R,L}^0/5],
\end{aligned}$$

$\lambda=+1$ (right-handed photons),
$\quad=-1$ (left-handed photons).

3. The angular correlation between the $\gamma$ rays and the neutrino is given by

$$1+\gamma\cos\theta+\chi\cos^2\theta,$$

where

$$\begin{aligned}
\gamma\eta_1=\;&(C_R^0-C_L^0)[1-(9\Delta/5M)]\\
&+(\Delta/M)[3(C_R^p-C_L^p)/5+2(C_R^k-C_L^k)/5\\
&-6(A_R^0+A_L^0)/5],\\
\chi\eta_1=\;&-(6\Delta/5M)(C_R^0-C_L^0),
\end{aligned}$$

TABLE I. Observables in radiative muon capture for different couplings.

| | $A-V$ or $A+V$ $(\mu/M=0)$ | $A-V$ | $A+V$ | $V$ | $A$ |
|---|---|---|---|---|---|
| $\xi$ | 4.00 | 3.55 | 3.10 | 0.89 | 2.43 |
| $\eta_R$ | 4.00 | 4.89 | 2.74 | 0.99 | 2.79 |
| $\gamma$ | 0 | −0.060 | −0.073 | 0.81 | −0.41 |
| $\chi$ | 0 | 0 | 0 | −0.127 | 0.050 |
| $\tau_{\text{rad}}^{-1}/[\tau_{\text{cap}}^{-1}(\alpha/12\pi)]$ | 1.0 | 1.38 | 0.88 | 1.11 | 1.15 |
| $\eta_L$ | 0 | 0.066 | 0.066 | 0.033 | 0.033 |
| $\beta(=B)$ | 1.0 | 0.97 | 0.95 | 0.93 | 0.98 |

with

$$\eta_1=\eta_R+\eta_L-\chi\eta_1/3.$$

4. The circular polarization of the $\gamma$ ray is

$$\beta=(\eta_R-\eta_L)/(\eta_R+\eta_L).$$

5. The angular correlation between the $\gamma$ ray and the direction of muon spin is

$$1+B\cos\theta,$$

where $B=\beta$ in this approximation.

Numerical results for various combinations of $V$ and $A$ couplings are shown in Table I. The first column indicates the results for $(V-A)$ or $(V+A)$ if all $(\mu/M)$ terms are set equal to zero. The quantities $\xi$, $\eta_R$, and $\eta_L$ are to be multiplied by $|C_V|^2$ or $|C_A|^2$. We see that the nucleon current contributions can contribute as much as 30% to the relative probability of radiative capture and make an important difference between $A+V$ and $A-V$ couplings, as already noted in reference 5. In spite of this, the parity-nonconserving effects $\beta$ and $B$ differ from the maximum 1.0 by only about 5%. The reason for this is that the terms of order $(\mu/M)$ represent either interference between diagram (1a) and the others or nucleon recoil corrections to diagram (1a); since diagram (1a) yields only right-handed photons, all interference terms in the intensity must also correspond to right-handed photons. Thus it is only terms of order $(\mu/M)^2$ representing the square of the sum diagrams (1b) through (1d) that reduce the circular polarization $\beta$ below unity. Similar arguments hold for $B$. Since we have calculated only some of the $(\mu/M)^2$ terms, the magnitude of the deviation of $\beta$ and $B$ from unity is not given accurately; however, it is certainly small.

Application of these results is limited by the following observations:

(1) The present results hold for reaction (2), whereas actual experiments are contemplated using complex nuclei.[6] We believe that the present results may serve as a guide as to the magnitude of nucleon current effects in the case of complex nuclei, but that there may be appreciable uncertainties in the calculation for this case.

(2) Effects of virtual pions have not been included. It has been noted[1,7] that some reduction of the parity-nonconserving effects may arise from the induced pseudoscalar interaction. This reduction[7] is also very small, although somewhat greater than that found here. Definite conclusions about virtual pion effects, however, require inclusion of the currents of the virtual pions and of the nucleons. These effects are now being studied. Preliminary results indicate that when these currents are included, the reduction of the parity-nonconserving effects and the change in the photon spectrum may be appreciable.

[6] Our results are not true even for capture in hydrogen, since we have not included the consequences of the hyperfine coupling of the muon and proton, which polarizes the proton spin relative to the muon spin. The results given here hold strictly for capture by an unpolarized proton at rest, which we may consider as a model for a proton inside a spin-zero nucleus. The consequences of the hyperfine coupling on radiative capture in hydrogen have been discussed by Dye, Sen, Ho, and Tzu, Sci. Sinica (Peking) **8**, 423 (1959).

[7] J. Bernstein, Phys. Rev. **115**, 694 (1959).