energy. A χ^2 test shows that such a nonlinear relationship is as valid as the linear fit. Our data suggests a falling off from linearity at low energies. Nonlinearity in the opposite direction is clearly excluded. Theoretical support to a possible nonlinear phase shift dependence has been given by Cini *et al.*,[7] who have shown from dispersion relations that crossing symmetry implies that $(\alpha_1-\alpha_3)/\eta$ must be an odd function of ω.

At present the experimentally measured value of 1.5 ± 0.1 for the Panofsky ratio is in disagreement with the value 2.5 ± 0.4 calculated from a linearly extrapolated value $(\alpha_1-\alpha_3)/\eta=0.27$, the photoproduction cross section on hydrogen of $(1.43\pm 0.06)\times 10^{-28}$ cm^2 and the π^+/π^- production ratio of 1.30 ± 0.05 in deuterium. In order to retain the validity of the pion-proton dispersion relations with a fixed value for $f^2=0.08$, a change in the magnitude of the zero-energy value of α_3/η requires an equal change in the magnitude of α_1/η. A reduction of the zero-energy extrapolation of α_3 by a slight amount would, therefore, go far to remove the present internal disagreement among the parameters of low-energy pion physics.

VI. ACKNOWLEDGMENTS

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π^+-p Interactions at 500 Mev*†

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The interaction of π^+ mesons with protons at an energy of 500 Mev has been studied in a hydrogen bubble chamber. Phase-shift analyses with S and P waves were made, and a near degeneracy was found between the Fermi and Yang solutions. When D waves were included, an additional ambiguity was found. The D-wave phase shifts are small, but they have a considerable effect on the other phase shifts. The cross section for single pion production is 2.85 ± 0.5 mb. The ratio $(p+0)/(n++)$ is $1.5_{-0.5}^{+1.5}$. The cross section leading to $p++-$ was found to be of the order of 30 μb.

I. INTRODUCTION

THE angular distribution of pion scattering by protons has been extensively studied at energies below about 310 Mev.[1,2] Experiments using hydrogen diffusion chambers were performed at 395 Mev[3] and in an energy range centered on 500 Mev.[4] Recently, preliminary results of the interactions of negative pions in a propane bubble chamber have been reported.[5] Angular distributions for positive and negative pion scattering from protons near 1 Bev have been obtained as a by-product of experiments on strange particle production.[6] Measurements of the total cross section for positive and negative pions up to 1.5 Bev have been published.[7-9]

The ambiguities which formerly existed in the phase-shift analyses of the pion elastic scattering at moderate energies have now been satisfactorily resolved.[10] For the positive pion scattering, at least, the agreement of all the experiments up to 500 Mev with the dispersion relations[11] is satisfactory. The principal questions to be answered by experiments on the π^++p interactions around 500 Mev are: the behavior of the S- and P-wave phase shifts,[12] the question of the presence of the D-wave

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§ Much of this work was done while the author was a National Science Foundation Predoctoral Fellow, and a guest of Brookhaven National Laboratory.

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[3] R. Margulies (private communication). Experiments were also performed at 258 and 294 Mev.

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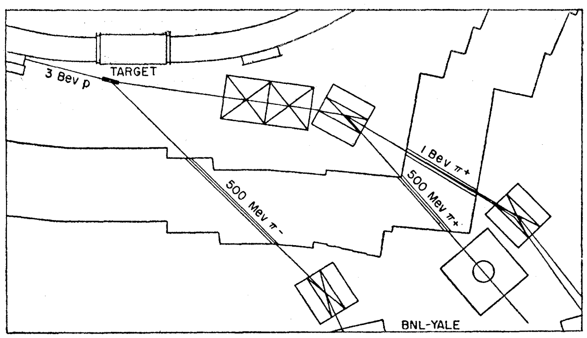

FIG. 1. The experimental arrangement at the Cosmotron.

phase shifts,[2] and the cross section for, and details of, production of additional pions by the incident pion.[4] Several calculations of the cross section for the last process have been made,[13–17] but rather disparate results were obtained. Most of them employ the Chew-Low method, which has also been established for this purpose from dispersion relations.[18]

It seemed worthwhile to secure improved statistics with a positive pion beam of well-defined energy, near 500 Mev, using a liquid hydrogen bubble chamber.

II. EXPERIMENTAL ARRANGEMENT

Three bubble-chamber experiments, in addition to this experiment, were performed simultaneously, using the same target. The arrangement is shown in Fig. 1. The 3-Bev proton beam of the Cosmotron was brought out of the machine within a time of a few microseconds, with the aid of the rapid beam ejector.[19] The protons struck a polyethylene target 1 in. wide and 20 in. long. Particles emerging at about a 7° angle to the proton beam were focused by a pair of 12-in. aperture quadrupoles (adjusted for the 1-Bev pion beam) and deflected by a 36-in. long analyzing magnet. Positive particles, deflected through 41°, passed through a 4-in. diameter channel in a lead collimator set in the concrete shielding wall and into the bubble chamber. By means of current-carrying wire measurements, it was found that the central momentum of the beam was 635 Mev/c with a spread of $\pm 3\%$.

The hydrogen bubble chamber used in the experiment measured 6 in. (in length) by $2\frac{3}{4}$ in. by 3 in. and was built by the Brookhaven Cloud Chamber Group.[20] No magnetic field was used. About twenty-five beam particles were visible in each photograph. The ratio of minimum-ionizing particles to protons was found to be 1.1/1.

III. SCANNING AND MEASUREMENT OF THE EVENTS

A. Scanning

About twenty thousand pictures of good quality were scanned, by looking along each track in two views of the chamber, and recording any deviations or scatterings.

It has been found that, in propane, the true bubble density along a track varies as $1/\beta^2$, where β is the velocity of the particle.[21] In the present experiment the protons should have a bubble density of about 3.3 times that of the pions, if the same law holds for hydrogen. This was found to be the case, as will be discussed below.

This difference in bubble density causes the pions and protons to have a quite different appearance in the photographs. The scanners were told to record all events, and to indicate whether these events were thought to be caused by incident pions or protons. A physicist examined all the events. All the events which the scanners considered pions and in addition all events which the physicist considered pions or doubtful, were then measured. (Of the events measured, about 1% were found to be protons.)

About one-half of the pictures were scanned twice, by different scanners. The efficiency was found as a function of the scattering angles in the usual approximation where the probability for one scanner to find an event which the other missed is considered to be (efficiency of the first)×(1−efficiency of the other).

B. Measurements

The polar angle θ and the azimuthal angle ϕ for the secondary tracks in each event were measured by reprojecting the two stereoscopic photographs of the chamber through the same lenses used to take them upon a screen mounted in gimbal rings. The accuracy, for a typical event, was about $\frac{1}{2}°$ in θ, and 3° in ϕ. Since the photographs were taken in liquid hydrogen, with index of refraction 1.085,[22] but reprojected in air, the measured angles had to be used to calculate the true angles. The resulting corrections were generally rather small, never exceeding 2.3°.

Estimates of the bubble density were made at the time of measurement. Since the bubble density was used to analyze the events which were possible examples of pion production, more accurate values were needed for these events. In pictures with such events, measure-

[13] E. Kazes, Phys. Rev. **107**, 1131 (1957).
[14] S. Barshay, Phys. Rev. **103**, 1102 (1956).
[15] J. Franklin, Phys. Rev. **105**, 1101 (1957).
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[20] Bolze, Morris, Rahm, Rau, Shutt, Thorndike, and Whittemore, Rev. Sci. Instr. **29**, 297 (1958).
[21] Willis, Fowler, and Rahm, Phys. Rev. **108**, 1046 (1957).
[22] L. Stevenson, University of California Radiation Laboratory Report UCRL 4310-03 M13A (unpublished). The operating point of the chamber was at a vapor pressure of about 59 psia. *Note added in proof.*—It has been found [R. R. Ross, UCRL4320-12M4 (unpublished)] that a more accurate value of the index is 1.097. This would change the average pion scattering angle by about 0.1°.

ments of the lengths of the gaps between bubbles were made by means of a scale in a microscope eyepiece, for two minimum-ionizing tracks, as well as for the tracks involved in the event.

Very short gaps are missed; therefore we choose a length x_0, such that a gap of length x_0 would surely be recorded. If N is the number of such gaps, of total length $\sum x_i$ (x_i measured between the estimated centers of the bubbles defining the gap), then the true bubble density as given by the maximum likelihood method may be shown to be

$$m=N/(\sum x_i-Nx_0). \tag{1}$$

On measuring all the tracks in a group of photographs, it was found that the values for m in each photograph fell into two groups corresponding to minimum ionizing tracks and protons, and that the ratio of the values of m for these groups was consistent with $m\propto 1/\beta^2$. The values of m for tracks in an event were normalized by means of the minimum-ionizing tracks, corrected for the foreshortening due to projection upon the film plane, and used to determine the values of β according to Eq. (1). For minimum-ionizing tracks several centimeters long, the standard deviations of the m value distribution was about 20%.

C. Criteria of Acceptance

There were two reasons for setting rather strict conditions for acceptance of events in this experiment. The scattering cross section rises by an order of magnitude as the energy of the pion decreases to the resonance energy, and beam pions which interact in the nuclei of the chamber wall or collimator probably tend to emerge near this energy. Clearly, it is important to be sure that a scattered pion is a beam pion, and that the tracks will be of sufficient length to make accurate measurements. Moreover, it is desirable to make restrictions which will ensure a high scanning efficiency.

The following were the criteria for the acceptance of an event:

1. Figure 2 shows the distribution of the angle ϕ for elastic scattering events. The plane perpendicular to the optic axis corresponds to $\phi=90°$. The interval of ϕ around 0° shows a significantly smaller scanning efficiency. Scattering events in this interval were not used in the analysis.
2. The direction of the incoming particle was required to be within 2° of the average beam direction.
3. The vertex of the event had to be 2 cm or more from the beginning of the visible region, along the beam direction, but more than 1 cm from the outgoing end.
4. It was required that each outgoing track from an event be at least 1 cm long, if it did not stop. (Because of the finite cross section of the chamber, this condition introduces a small bias as a function of the angle θ_π. This bias was computed to be negligible except for the interval $\bar{\theta}_\pi>150°$, in the center-of-mass system.)

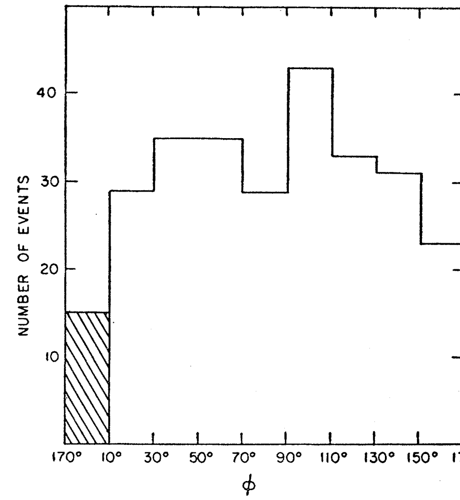

FIG. 2. The number of elastic scatterings found as a function of the azimuthal angle ϕ.

5. Events classified as elastic scatterings were required to fit the kinematic relations for the correct momentum within errors of $2\frac{1}{2}°$ in the angles. This excludes events of pion energy below about 400 Mev.
6. If the scattering angle is so small that there is no visible recoil, the scanning efficiency is small, and there may be confusion with π-μ decays. For this reason, only events with $\theta_\pi>9°$ in the laboratory were used.

The application of these criteria reduced the number of events from about 600 to 265.

IV. RESULTS

A. Elastic Scattering Angular Distribution

A total of 228 elastic scattering events was found which satisfied all the criteria outlined above. They were divided into eight 20° intervals in $\bar{\theta}_\pi$ (the pion angle in the center-of-mass system), including angles from 13.7° to 170°. When determined in the way described in III, the efficiency for finding these events proved to be 0.82 for the interval $13.7\leqslant\bar{\theta}_\pi\leqslant 30°$, 0.96 for $30°\leqslant\bar{\theta}_\pi\leqslant 150°$ and 0.89 for $150°\leqslant\bar{\theta}_\pi\leqslant 170°$. The number of events was divided by the efficiency, and also by the geometrical factor mentioned in Sec. III, which is 1.07 for the $\bar{\theta}_\pi=160°$ interval, and approximately 1.00 for the other intervals.

A preliminary phase-shift analysis was made by the method described below, excluding the interval $\bar{\theta}_\pi=20°$, and the resulting curve used for two purposes. The first interval was corrected for the number of events missing due to the small-angle cutoff, at $\bar{\theta}_\pi=13.7°$. Also the Coulomb interference with this preliminary set of phase shifts was computed.[23] This had the effect of raising the differential cross section in the 20° interval by 7% for that set of phase shifts. Since the effect is small and

[23] F. Solmitz, Phys. Rev. 94, 1799 (1954).

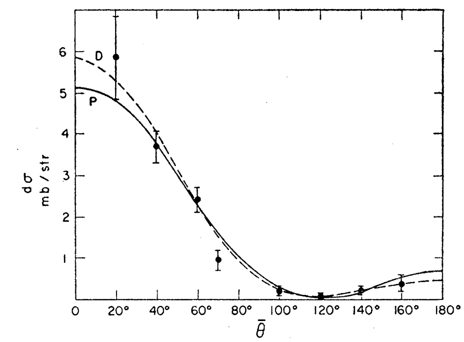

FIG. 3. The angular distribution for elastic scattering. The curve P includes powers of $\cos\bar{\theta}_\pi$ up to $\cos^2\bar{\theta}_\pi$, curve D includes powers up to $\cos^4\bar{\theta}_\pi$.

not highly sensitive to the phase shifts, the Coulomb interference may be taken into account with good accuracy by multiplying the number of events in the 20° interval by 0.93. The resulting differential scattering cross section is shown in Fig. 3. The errors shown are the standard deviations due to statistics only, except for the 20° point, where the error has been increased somewhat, because of the various corrections made.

The absolute cross-section scale was determined in the following manner. The contamination of muons and electrons in the pion beam was not well known; therefore it was thought more advisable to use the published information on the total cross section than to rely on the value obtained in this experiment, which is presented in the Appendix. The values of the total cross section from counter work[8] at 450 and 550 Mev were interpolated to give a value at $E_\pi=500$ Mev:

$$\sigma_{\text{total}}=20.0\pm2.5 \text{ mb}.$$

The ratio of elastic to inelastic events determined in this experiment was used to obtain the total elastic cross section

$$\sigma_{\text{el}}=17.2\pm3 \text{ mb}.$$

A least-squares fit of the curves

$$d\sigma/d\Omega=\sum_{n=0}^{N} a_n \cos^n\bar{\theta}_\pi, \tag{2}$$

to the experimental points was made, with weights inversely proportional to the errors. Values of 2, 3, and 4 were used for N. The total elastic scattering cross section obtained by integrating the curve was set equal to 17.2 mb. The a_n and the diagonal elements of their error matrix are given in Table I. The curve for $N=2$ is labelled "P" on Fig. 3, that for $N=4$ is labelled "D".

TABLE I. Coefficients in $d\sigma/d\Omega=\Sigma^N a_n \cos^n\bar{\theta}_\pi$, in mb/sterad.

N	a_0	a_1	a_2	a_3	a_4
2	0.585 ± 0.06	2.27 ± 0.14	2.36 ± 0.20		
3	0.482 ± 0.05	1.98 ± 0.11	2.66 ± 0.15	0.711 ± 0.23	
4	0.446 ± 0.05	1.91 ± 0.18	2.62 ± 0.34	0.549 ± 0.35	0.218 ± 0.50

B. Phase-Shift Analysis

None of the previous experiments, up to 525 Mev, has been found to require the introduction of D-wave phase shifts.[4] It should be noticed that the coefficient of $\cos^3\bar{\theta}_\pi$ may be zero. Therefore an analysis first in terms of S- and P-wave phase shifts will be made, and then the effect of the D waves discussed.

The differential cross section is given by

$$d\sigma/d\Omega=\lambda\!\!\!^{-2}(|f_\alpha|^2+|f_\beta|^2), \tag{3}$$

where $\lambda\!\!\!^-$ is the wavelength of the pion in the center-of-mass system and

$$f_\alpha=a_S+(a_{P1}+2a_{P3})\cos\bar{\theta} \\ +(2a_{D3}+3a_{D5})(3\cos^2\bar{\theta}-1)/2, \tag{4}$$

$$f_\beta=(a_{P3}-a_{P1})\sin\bar{\theta}e^{i\phi}+(a_{D5}-a_{D3})3\cos\bar{\theta}\sin\bar{\theta}e^{i\phi}, \tag{5}$$

where

$$a_{LJ}=(\eta_{LJ}e^{2i\alpha_{LJ}}-1)/2i. \tag{6}$$

In order to conform to the general usage, the phase shifts α_{LJ} will be identified by indices representing twice the isotopic spin and twice the total angular momentum. With a positive pion and a proton we have only a $\frac{3}{2}$ isotopic spin state. To avoid ambiguities the

TABLE II. S- and P-wave phase shifts. All $\eta_{LJ}=0.981$.

	α_3	α_{31}	α_{33}
I	$-29.0°$	$-25.6°$	163.0°
II	$-29.0°$	$-14.2°$	157.3°

S-wave phase shift is written α_3, and the D-wave phase shifts δ_{33} and δ_{35}. The η_{LJ} are the absorption coefficients, in terms of which the pion production cross section is

$$\sigma_{\text{inel}}=\pi\lambda\!\!\!^{-2}\sum_{L,J}[(2J+1)/2](1-\eta_{LJ}^2). \tag{7}$$

It is not possible to evaluate the η_{LJ} from the data in this experiment, but a constraint is placed on the η_{LJ} by Eq. (7). Various assumptions which satisfy Eq. (7) were investigated, and it was found that it is not possible to produce substantial changes in the phase shifts by changes in the η_{LJ} consistent with Eq. (7), hence most of the computations were done by assuming all η_{LJ} equal, which, with the σ_{inel} found in this experiment, makes $\eta_{LJ}=0.98$. (The effect of an extreme assumption may be seen in solution "a" and "b" of Table III.) The phase shifts obtained when $\delta_{33}=\delta_{35}=0$, and when the coefficients for $N=2$ in Table I were used are given in Table II. There are two sets, I and II, corresponding to the Fermi-Yang ambiguity, but they are almost degenerate, and it is difficult to determine with certainty which set corresponds to the Fermi solution at lower energies. Figure 4 shows α_{31} from the S- and P-wave analyses of the scattering at intermediate energies, as a function of $\bar{\eta}^3$, where $\bar{\eta}$ is the meson momentum in units of the meson mass in the center-of-mass system. The straight line is an extrapolation of the low-energy dependence found by Chiu and Loman,[10] $\alpha_{31} - 9.1° \bar{\eta}^3$. The points at the higher energies seem to fall near this line, as does our solution I, which is somewhat more likely, therefore, to correspond to the Fermi solution.

When an attempt is made to find solutions including D waves, the large errors in a_3 and a_4 must be taken into account. To present a summary of the many fits obtained with different values of a_3 and a_4, it is convenient to show the solutions as labelled points on a plot of δ_{33} and δ_{35} (Fig. 5). Reference may then be made to Table III to find all the corresponding phase shifts and the values of a_3 and a_4. There are many possible D-wave solutions, but effort has been concentrated on the one which most nearly resembles the S- and P-wave solution. There is one solution of this sort which gives values of a_3 and a_4 most nearly equal to the values of

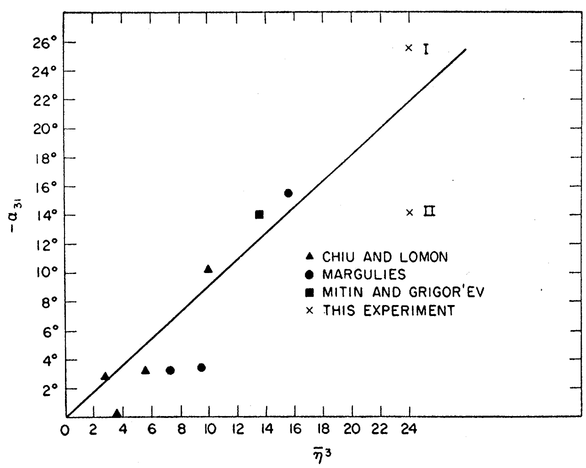

FIG. 4. Results for α_{31} of analyses at lower energies where the Yang and Fermi solutions are distinct. The line is the low-energy dependence suggested by Chiu and Lomon.[10]

Table I, solution "a." The value of a_4 is not equal to the experimental value but it is well within the error given in Table I. The solutions obtained by changing a_3 and a_4 by approximately the error listed in Table I. are given in Table III.

The fact that the spin flip scattering is very small, and that it may result from both the P and D waves, is responsible for double solutions, such as "f" and "g."

The D-wave phase shifts of Table III seem rather small compared to the results of Mukhin at 307 Mev,[2] but are in agreement with Margolis[3] and Blevins *et al.*[4]

C. Dispersion Relations

The dispersion relations for the real part of the forward scattering amplitude derived from the requirement of causality by Goldberger, Miyazawa, and Oehme[24] require knowledge of the $\pi^+ - p$ and $\pi^- - p$

[24] Goldberger, Miyazawa, and Oehme, Phys. Rev. **99**, 986 (1955).

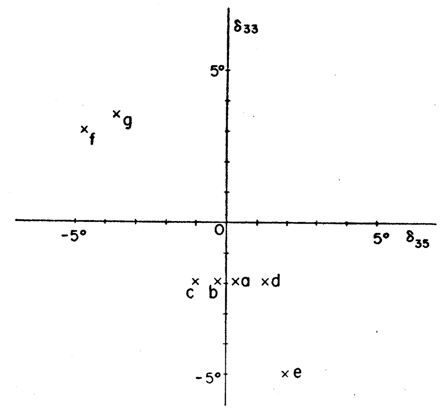

FIG. 5. A plot identifying the various phase-shift solutions given in Table III by the two D-wave phase shifts, δ_{33} and δ_{35}.

total cross sections at all energies. Sternheimer[11] has used the experimental cross sections and assumptions about the behavior at high energies to evaluate Re[$f(0°)$]. He has also obtained Im[$f(0°)$] from the optical theorem and the same total cross sections, finally giving curves of $(d\sigma_{el}/d\Omega)(0°)$. It is found that, at 500 Mev, this quantity is mostly due to the Re[$f(0°)$] and varies quite rapidly with energy. The value at 500±15 Mev is

$$\frac{d\sigma}{d\Omega}(0°) = 5.2 \pm 0.8 \text{ mb/sterad.}$$

The value given by the fit corresponding to curve "P" is

$$\frac{d\sigma}{d\Omega}(0°) = 5.2 \pm 0.5 \text{ mb/sterad.}$$

and that given by curve "D" is

$$\frac{d\sigma}{d\Omega}(0°) = 5.85 \pm 0.6 \text{ mb/sterad.}$$

Either fit agrees satisfactorily with the prediction.

D. Single-Pion Production

Thirty-eight events were found for which the measurements were consistent with the reactions

$$\begin{aligned} \pi^+ + p &\rightarrow n + \pi^+ + \pi^+ \quad (n++) \\ &\rightarrow p + \pi^+ + \pi^0 \quad (p+0). \end{aligned}$$

These events were recognized by their lack of coplanarity and failure to fit the elastic-scattering kinematics. The measurements of the velocity of the secondary particles were used to calculate the mass of the neutral particle, for a given assumption of the masses of the emergent particles. The measured velocities were changed, within their errors, to produce a fit. Somewhat more than a quarter of the events were found to fit both final-state assignments.

TABLE III. S-, P-, and D-wave phase-shift solutions. All $\eta_{LJ}=0.981$ except for solution "b", which corresponds to solution "a" except $\eta_S=0.816$, the other $\eta_{LJ}=1.00$.

	α_3	α_{31}	α_{33}	δ_{33}	δ_{35}	a_3 (mb/sterad)	a_4 (mb/sterad)
a	$-27.3°$	$-15.0°$	$157.3°$	$-2.0°$	$0.3°$	0.55	-0.004
b	$-30.8°$	$-20.5°$	$160.4°$	$-2.0°$	$-0.2°$	0.55	0.01
c	$-30.0°$	$-16.0°$	$159.0°$	$-2.0°$	$-1.0°$	0.94	0.07
d	$-25.5°$	$-15.0°$	$156.5°$	$-2.0°$	$1.3°$	0.26	-0.05
e	$-29.3°$	$-19.0°$	$161.8°$	$-5.0°$	$2.0°$	0.55	-0.26
f	$-24.2°$	$-7.5°$	$155.3°$	$3.0°$	$-4.6°$	0.55	-0.24
g	$-29.5°$	$-17.9°$	$161.4°$	$3.5°$	$-3.7°$	0.55	-0.26

To obtain the ratio of elastic to inelastic events, the number of elastic events must be corrected for the excluded region of ϕ. Since the inelastic events do not in general lie in a plane, there is no excluded region of ϕ, and the scanning efficiency may be different. On the basis of the limited statistics in the rescanning (three events), the scanning efficiency seems to be about 90% of that for the elastic events.

Applying these corrections, the result is

$$\sigma_{1\pi}/\sigma_{\text{total}}=0.143\pm0.025,$$

or, since $\sigma_{\text{total}}=20.0\pm2.5$ mb, from Sec. IV A,

$$\sigma_{1\pi}=2.85\pm0.5 \text{ mb}, \quad \text{and} \quad \sigma_{\text{el}}=17.2\pm3 \text{ mb}.$$

The single-pion production cross section agrees approximately with the calculation of Kazes.[13]

The center-of-mass momentum and angle for each particle for the 16 cases of $p+0$ and the 11 cases of $n++$ identified with reasonable certainty, are shown on the scatter plot Fig. 6. For the ambiguous cases, points plotted for both assumptions appear in Fig. 7. It should be noticed that most of the ambiguous cases involve a slow track and a very fast one. In such cases, it is difficult to determine whether the heavy track is a pion or a proton. However there is only a narrow momentum band such that a pion has a high bubble density but has a range long enough to escape from the chamber. (None of the particles stopped.) It seems somewhat unlikely to find so many events in such a small phase space. Probably, then, no more than half of the ambiguous events are $n++$, but they could be all $p+0$. These considerations may be combined with the expected statistical errors to give the charge-state ratio:

$$(p+0)/(n++)=1.5_{-0.5}^{+1.5},$$

which is to be compared to $\simeq0.6$ predicted by the most recent meson-theory calculation.[13] The isobar model, which is expected to be valid only for higher energies, predicts 6.5 for this ratio.[25]

A close comparison of Figs. 6 and 7 shows that if the ambiguous events are actually all of one of the two states, or if they are mixed so that not all of one state comes from a small area of Fig. 7, the general features of Fig. 6 are hardly affected. It is then possible to draw conclusions from Fig. 6 with as much confidence as the limited statistics will admit. The only feature, however, which may be considered striking is the division of the π^+ from $n++$ into a fast and a slow group. This feature is also present in the data of Blevins *et al.*[4] Another characteristic of their data which they point out is a pronounced backward peaking of the neutrons. This can also be seen in Fig. 6, but it is not statistically significant.

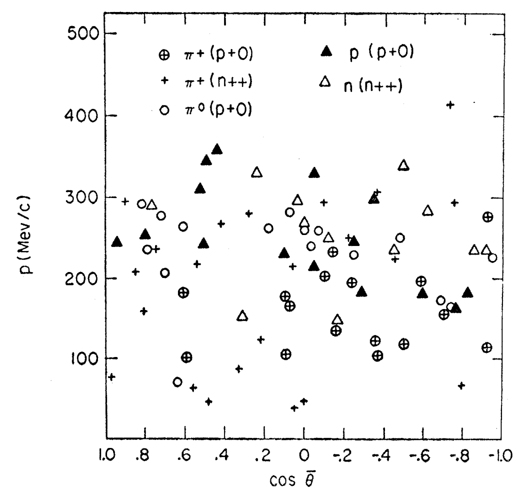

FIG. 6. A scatter plot showing the center-of-mass angle and momentum of each particle from a pion production event identified as to charge state. The points indicate the type of particle, and the final charge state, as shown in the legend.

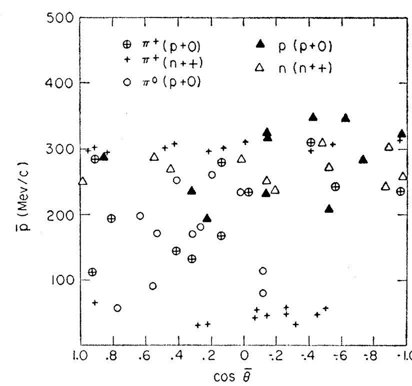

FIG. 7. A plot corresponding to Fig. 6 for the ambiguous pion production events, where each particle is plotted *twice*, for the two possible interpretations of the event.

E. Multiple Pion Production

One event found was an example of

$$\pi^+ + p \rightarrow p + \pi^+ + \pi^+ + \pi^-.$$

One of the π^+ stopped and decayed. This event was not

[25] S. Lindenbaum and R. M. Sternheimer, Phys. Rev. **105**, 1874 (1957).

in the restricted region of the chamber, and therefore it is to be compared with about 600 other pion events. The cross section for this reaction is, then, about 30 μb.

Two other inelastic events were found which would only fit single-pion production if the momentum were raised several hundred Mev but, they could fit the reactions

$$\pi^+ + p \rightarrow n + \pi^+ + \pi^+ + \pi^0,$$

or

$$\pi^+ + p \rightarrow p + \pi^+ + \pi^0 + \pi^0.$$

V. DISCUSSION

The S- and P-wave phase shifts at 500 Mev show reasonable continuations of the lower energy results and are little affected by the absorption due to inelastic processes. It is interesting to note that the S-wave phase shift, in all the various D-wave solutions, seems larger in magnitude than the value given by the Orear extrapolation,[26]

$$\alpha_3 = -6.3°\ \bar{\eta}, \quad \text{or} \quad \alpha_3 = -18.2° \text{ at } 500 \text{ Mev}.$$

This effect has been predicted by Drell *et al.*[12]

The best fits to the data seem to give rather small D-wave phase shifts, particularly in the light of the result of Mukhin *et al.*[2] who conclude that

$$\delta_{35} = -0.2\bar{\eta}^5,$$

for the data around 300 Mev. If the same energy dependence could be assumed at as high an energy as 500 Mev, we would conclude that

$$|\delta_{35}| < 0.08\bar{\eta}^5.$$

It has been suggested that their result is a statistical fluctuation.[4]

The present data are in agreement with the dispersion relations as evaluated by Sternheimer.[11] It has been pointed out[8] that it will be difficult to verify them with any accuracy at energies much higher than this, because the $\mathrm{Re}[f(0°)]$ soon becomes small compared to $\mathrm{Im}[f(0°)]$.

The calculation of Kazes[13] seems to be in approximate agreement with the results for the cross sections for $n++$ and $p+0$. An interesting momentum distribution of the pions from the former reaction has been pointed out. Only weak support can be given the suggestion that the pion production proceeds through a "stripping" of a virtual pion, resulting in a backward peaking of the nucleons.[4]

The cross section for multiple pion production has been found to be very small but finite at this energy.

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APPENDIX. MEASUREMENT OF THE TOTAL CROSS SECTION

The rolls of best quality were selected for an attempt to determine the total cross section. These rolls contain about one-third of the events. The minimum ionizing tracks in every tenth picture were counted. The path length thereby determined had to be corrected for the percentage of beam tracks not satisfying the criteria listed in Sec. III, found to be 30%. The number of events was corrected for the scanning efficiencies given in Sec. IV and for the events discarded because one track was too short (12%). The contamination of electrons and muons in similar beams was reviewed,[7,8] and the fraction of minimum ionizing tracks which were pions was taken as 0.86. The density of liquid hydrogen at the operating point is[22] 0.62 g/cm^3. The resulting value for the cross section is

$$\sigma_{\text{total}} = 17.0 \pm 3.4 \text{ mb},$$

where about one-half of the error results from uncertainties in the corrections which had to be applied. This value is consistent with the counter measurement.[8]

[26] J. Orear, Phys. Rev. **96**, 176 (1956).