Scattering of 3.7–25 Mev Positive Pions by Hydrogen*

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The Columbia University hydrogen bubble chamber was used to investigate the π^+-p scattering cross section in a laboratory energy range from 3.7 to 25 Mev. A total of 950 events were measured, of which 338 were caused by incident pions that would have come to rest in the chamber. Treating the small p-wave and large Coulomb contributions as known, the s-wave phase shift is found to deviate from a linear dependence on momentum only by one and a half standard deviations.

I. INTRODUCTION

SOME attention is still focused on apparent inconsistencies in the parameters of low-energy pion physics. In particular, the numerical value of the difference of the s-wave pion-nucleon amplitudes $(\alpha_3-\alpha_1)/\eta$ extrapolated to zero energy is different when determined either from a combination of the ratio of negative to positive pion photoproduction[1,2] and the Panofsky ratio[3] or determined directly from an analysis of pion-nucleon scattering.[4] Although numerous theoretical attempts have been made, by relaxing isotopic spin conservation[5,6] or modifying the momentum dependence of the phase shifts[7] to resolve this and other discrepancies, it was thought instructive to re-examine experimentally the scattering of pions in an energy region not previously treated with great statistical accuracy.[8]

In this paper we will report on a hydrogen bubble chamber measurement which will yield values of the s-wave phase shift α_3 at 5 c.m. momenta from 0.2 to 0.55 $p/\mu c$ each with an average statistical accuracy of 13%. In order to extract this data, we will assume a knowledge of the p-wave phase shifts, to which, at this energy, the experimentally determined cross sections are not particularly sensitive. It is worth noting that in this region counter experiments are difficult to perform because the scattered particles have very short ranges. At 24 Mev, the highest point covered by this experiment, our experimental values overlap and are in agreement with counter measurements.[9]

II. METHOD

A. Experimental Arrangement

A momentum-analyzed 60-Mev π^+ beam from the Nevis cyclotron was degraded in energy and brought to rest in a 12-in. diameter, 6-in. deep liquid hydrogen bubble chamber.[10] On the average about 8 stopped mesons were photographed per pulse every 2 seconds. The collimation of the beam was such that over 90% of the flux was located at least 2 cm from the glass wall of the chamber with an rms scattering angle of 7°. A field of 8000 Gauss at right angles to the average beam direction served to identify the particle's charge.

B. Event Identification and Measurement

All tracks having kinks of greater than 30° were examined when the film was reprojected on scanning tables. This angle was chosen so that the majority of forward Coulomb and π-μ decay in flight contributions were eliminated from consideration. To be accepted as a π^+-p scattering, an event had to satisfy either/or of the following criteria:

(a) The incoming track scattered at an angle greater than 50° projected and there was a visible proton recoil of 0.03 cm or longer. This category covers all events above 13 Mev and those of energy greater than 5 Mev with angle larger than 90°.

(b) No proton recoil is seen, but the alleged scattering is followed by the characteristic decay signature of the pion. This is defined to be a visible μ^+ of decay angle greater than 5° in one of three views and of length less than 1.3 cm, followed by a positron whose length is at least 0.5 cm. It is clear that the above criteria introduce biases. One of these, the 50° angle cutoff, will be discussed in detail later. Other biases arising from tracks leaving the fiducial region, or not being otherwise identified, can be shown by geometric considerations to be on the average no greater than a few percent. While

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1 Beneventano *et al.*, *Proceedings of the CERN Symposium on High-Energy Accelerators and Pion Physics, Geneva, 1956* (European Organization of Nuclear Research, Geneva, 1956), Vol. 2, p. 259.
2 S. Penner, thesis, University of Illinois, 1956 (unpublished).
3 Fischer, March, and Marshall, Phys. Rev. **109**, 533 (1958); E. L. Koller and A. M. Sachs, Bull. Am. Phys. Soc. Ser. II, **4**, 24 (1959).
4 J. Orear, Nuovo cimento **4**, 856 (1956).
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7 M. Cini *et al.*, Nuovo cimento **10**, 243 (1958).
8 For a summary of experimental data see G. Puppi, *1958 Annual International Conference on High-Energy Physics at CERN*, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958), p. 39.
9 Barnes *et al.*, Atomic Energy Commission Report NYO-2170 (unpublished); W. Johnson and M. Camac, Atomic Energy Commission Report NYO-2169 (unpublished).
10 F. Eisler *et al.*, Nuovo cimento **10**, 468 (1958).

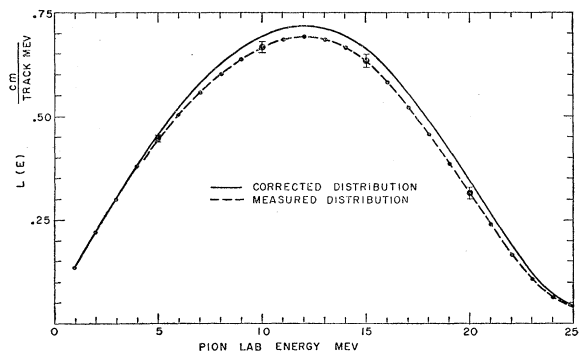

FIG. 1. Differential track length distribution of stopping flux.

it is true that some of these biases could have been entirely eliminated by choosing a much smaller fiducial region, this would also have eliminated about one-third of the events, thereby increasing the already much larger statistical error.

Within the criteria established, each of two independent scanners found greater than 93% of the total number of events seen. An examination of events missed did not show that the scanning efficiency was either energy or angle dependent. A third scanning, specifically for low-energy events which might at first be suspect, yielded no new results. In this experiment, we have therefore regarded the over-all scanning efficiency as $98\pm2\%$.

The projected coordinates, angles and ranges of event tracks were measured on a scanning machine and transformed to the space variables with the aid of an IBM-650 computer. The center-of-mass energy and angle were then found from kinematic charts. An event is overdetermined when both recoil proton and pion ranges are available. Inconsistencies in an overdetermined event were never more than 5% in energy and 8° in angle.

C. Flux, Intensity, and Energy Distribution

The total number of pions which stop in the chamber fiducial region was determined by direct count on every tenth picture. A stopping pion is one which exhibits a decay as previously defined. A check on flux count showed only two errors of interpretation per one thousand tracks.

The cross section determination requires the knowledge of the differential track length distribution as a function of energy. Expressed differently, one needs to know the amount of target thickness within a differential energy interval. Once the length distribution of an unbiased sample of pion tracks is known by measuring each track with a curved ruler from the point at which it enters the fiducial region to the point where it decays, the flux distribution can be calculated. Three thousand track lengths were measured in projection and related to space lengths by an average chamber magnification.[11] The uncorrected flux distribution, shown in Fig. 1, was obtained using the differential range-energy relation of Aron, Hoffman, and Williams.[12] This distribution must be augmented by folding in the flux contribution due to pion lengths of pions which decay in flight and are not accepted under the counting criteria. Such a correction, never exceeding several percent, may be made from the known mean lifetime of the pion and the uncorrected path lengths. Biases due to chamber edge effects enter the flux measurement and have been computed. They are significant primarily for very long tracks, that is, for incident pion energies greater than 23 Mev.

The operating conditions of the Columbia chamber are such that the density of liquid hydrogen is 0.0566 ±0.0003 g/cc. This value was deduced from a direct measurement of 1200 muon ranges and a logarithmic interpolation of the Aron, Hoffman, and Williams relations.

III. ANALYSIS

A. Projected Angle Bias

As mentioned before, scatterings with *projected laboratory angles* ψ smaller than 50° are not accepted. This means that not all scatterings, even if they have a *space angle* θ larger than 50°, are accepted. The probability of detecting such a scattering is called the forward-angle cutoff bias, $W(\theta)$, and can be calculated from purely geometric considerations provided the dip angle distribution of the incident beam tracks is known. To illustrate, if we define the polar angle of scattering to be θ and the azimuthal angle ϕ, then

$$\tan\psi=\tan\theta\cos\phi\geq\tan50°.$$

Assuming that the scattering distribution is independent of the azimuthal angle one finds that the acceptance region is specified by

$$-\frac{\tan50°}{\tan\theta}\geq\sin\phi\geq\frac{\tan50°}{\tan\theta}.$$

Accordingly, the acceptance fraction for the case of no dip is

$$W(\theta)=\frac{2}{\pi}\cos^{-1}\left[\frac{\tan50°}{\tan\theta}\right].$$

This function is the solid line of Fig. 2 now plotted against the cosine of the center-of-mass scattering angle.

A more involved type of calculation can be made for various angles of dip. The dip angle distribution of beam tracks was found to be Gaussian, symmetric

[11] For specific detail of this and other procedures, see E. W. Jenkins, thesis, Columbia University, 1959 (unpublished).

[12] Aron, Hoffman, and Williams, University of California Radiation Laboratory Report UCRL-121 (2nd rev.), 1949 (unpublished).

about zero degrees, with a standard deviation of 7°. Therefore, the weighted function $W(\theta)$ shown by the dotted line was used.

B. Maximum Likelihood Phase Shift Analysis

The expression for the center-of-mass π^+-p differential scattering cross section as a function of the three $T=\frac{3}{2}$ phase shifts is[6,13,14]

$$\frac{d\sigma}{d\Omega}=\lambda\!\!\!^{-2}\left\{\left|\alpha_3+(2\alpha_{33}+\alpha_{31})\cos\theta-\frac{\alpha}{1-\cos\theta}\right|^2+(\alpha_{33}-\alpha_{31})^2\sin^2\theta\right\}, \quad (1)$$

where α_3 is the phase shift for the s-wave and α_{33} and α_{31} are p-wave phase shifts for $J=\frac{3}{2}$ and $\frac{1}{2}$, respectively. The Coulomb parameter, α, is given by $\alpha=e^2/\hbar v$, where v is the relative velocity of the particles. $\lambda\!\!\!^-$ is the reduced wavelength of the pion. This expression is valid for nonrelativistic pion velocities and small phase shift angles. The π^+-p events measured in this experiment form a continuous rather than discrete distribution in energy and angle. The most efficient use is made of this type of data by performing the analysis by the maximum likelihood method.

Following Anderson,[15] the probability for finding an event in this experiment within an angular interval $\Delta x_i=2\pi\sin\theta_i\Delta\theta_i$ and a momentum interval $\Delta\eta_i$, where η_i is the pion momentum in the center-of-mass system in

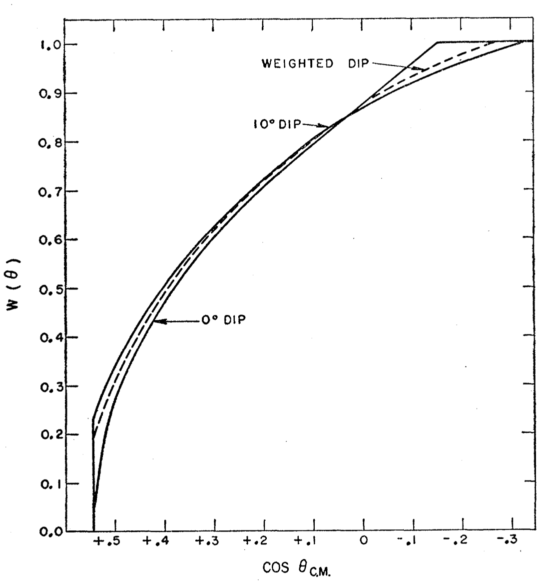

FIG. 2. Probability of detecting an event at forward angles, $W(\theta)$.

[13] L. Van Hove, Phys. Rev. 88, 1358 (1952).
[14] F. T. Solmitz, Phys. Rev. 94, 1799 (1954).
[15] H. L. Anderson and W. C. Davidon, Nuovo cimento 5, 1238 (1957).

units of $m_\pi c$, is

$$\frac{d\sigma_i}{d\Omega}f(\eta_i)W(x_i)\Delta x_i\Delta\eta_i, \quad (2)$$

where $W(x_i)$ is the 50° cutoff bias, $f(\eta_i)$ is the corrected flux distribution of Fig. 1 in the proper units. The probability for not finding an event is one minus the previous expression. The probability for finding a given distribution of N events having x_i, η_i is the product of finding N events in the interval Δx_i, $\Delta\eta_i$ times the probability of not finding these events in all other intervals:

$$L=\prod_i^N \frac{d\sigma_i}{d\Omega}f(\eta_i)W(x_i)\Delta x_i\Delta\eta_i \times\prod_{j\neq i}\left(1-\frac{d\sigma_j}{d\Omega}f(\eta_j)W(x_j)\Delta x_j\Delta\eta_j\right), \quad (3)$$

which, in the limit of small intervals becomes

$$L=\prod_i^N \frac{d\sigma_i}{d\Omega}f(\eta_i)W(x_i)dx_id\eta_i \times\exp\left(-\int\int\frac{d\sigma}{d\Omega}f(\eta)W(x)dxd\eta\right). \quad (4)$$

It is more convenient to work with the logarithm of the likelihood. By performing the angular integration and dropping constant terms, we have

$$\log L=\sum_{i=1}^N \ln\frac{d\sigma_i}{d\Omega}W(x_i)-\int_{\eta_1}^{\eta_2}\sigma_{\text{tot}}f(\eta)d\eta, \quad (5)$$

where σ_{tot} is the total weighted cross section from 50° to 180°.

The above formula has been programmed for an IBM 650 computer. All events were grouped into five momentum intervals. Although it is possible to solve for all three phase shifts simultaneously, at these low energies the contribution of the p-wave to the cross section is very small. We therefore assume the p-wave phase shifts known from higher energy experiments[9] to be

$$\alpha_{33}=0.210\eta^3 \quad\text{and}\quad \alpha_{31}=-0.0427\eta^3.$$

Within each of the five momentum intervals the s-wave phase shift was assumed to have linear momentum dependence. The results quoted in the next section are the maxima of the likelihood calculations.

In our experiment, the likelihood functions were found to be approximately Gaussian in shape, therefore, the logarithm of these functions will be parabolic. The error quoted for the phase shifts is then the halfwidth of the parabolic curve at an ordinate differing from the maximum by 0.5. This error is clearly one standard statistical deviation in the phase shift but is related to the number of experimental events only by

TABLE I. Experimentally determined *s*-wave amplitudes.

Momentum η	No. of events	α_3/η	% Stat. error	% Other errors	*S*-wave nuclear amplitude α_3^N/η
0.20–0.27	45	−0.069	23.8	4.3	−0.076±0.019
0.27–0.34	69	−0.087	11.5	4.3	−0.094±0.012
0.34–0.41	100	−0.106	11.3	4.4	−0.112±0.014
0.41–0.48	94	−0.112	8.5	4.6	−0.118±0.011
0.48–0.54	30	−0.107	15.2	9.1	−0.112±0.020

the phase shift's functional dependence on the cross section. A concerted effort was made to keep the other errors, both systematic and random, to a small fraction of the statistical error.

IV. RESULTS

Table I shows results of the computation in column 3. Column 4 denotes the above-mentioned statistical error.

Van Hove[13] has shown that the phase shifts defined by formula (1), to which we have fitted out data, are not strictly nuclear in nature. A pure nuclear phase shift is defined as one which describes the scattering in the absence of the Coulomb field. At low pion energies especially, the *s*-wave amplitude α_3 is modified by Coulomb effects, a modification in addition to the pure Coulomb term $\alpha/(1-\cos\theta)$. This arises from the diminution of the pion wave function near the origin by the repulsive electric field. We have chosen to correct our experimental numbers using the model of Van Hove. Within a radius r_0, the Coulomb potential has been assumed to be negligible compared to the nuclear potential. Outside this radius, the wave function is dependent on a point Coulomb potential. By matching the wave function at r_0, a functional relation is established between the experimentally determined phase shift α_3 the Coulomb increment to this phase shift, and the nuclear phase shift α_3^N by

$$\alpha_3=\alpha_3^N+\alpha[C+\log(2kr_0)-\mathrm{Ci}(2kr_0)\cos(2\alpha_3^N)+\mathrm{si}(2kr_0)\sin\alpha_3^N],$$

where C is Euler's constant$=0.5772$, k is the pion wave number, Ci is the cosine integral, and si is the sine integral.

Column 6 lists the corrected phase shift values with their total error. The radius r_0 used for the correction was assumed to be 0.7×10^{-13} cm. This value is the radius at which the Coulombic potential is thought to be modified.[16] If we use the Compton wavelength of the pion $h/m_\pi c=1.4\times10^{-13}$ cm, then the numbers of column 6 should be increased by 4%.

As a check on our method, we used a value for α_3 of -0.103η and the usable remainder of the 950 measured events to determine the *p*-wave amplitudes. These events were caused by high-energy pions that would not have come to rest in the chamber and are therefore unrenormalizable to the flux by our technique. Nevertheless, they have an angular distribution from which the information may be extracted by an analogous likelihood calculation. Table II shows the results of this procedure. Because two variables are involved, the errors quoted are related to the size of a skew error ellipse. Coulomb corrections have not been made. The value of α_{33} so obtained, when plotted on a Chew-Low[17] basis yields a number for the unrenormalized coupling constant $f^2=0.081\pm0.007$ for $\omega_0=2.17$.

TABLE II. Experimentally determined *p*-wave amplitudes.

Incident energy (Mev)	No. of events	α_{31}/η^3	α_{33}/η^3
10–20	175	−0.06 ±0.07	0.208±0.051
20–30	250	−0.046±0.036	0.213±0.043
30–40	95	−0.061±0.042	0.190±0.050

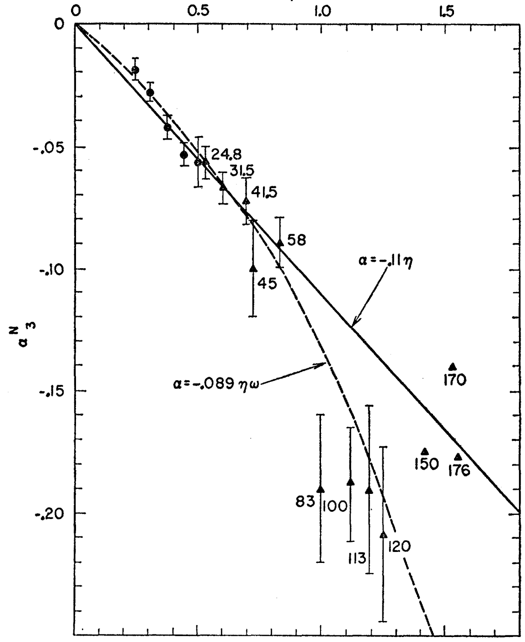

FIG. 3. A plot of the *s*-wave phase shift α_3 against c.m. momentum. The solid circles are obtained from the present work; for other points, see reference 8, p. 43.

V. DISCUSSION

In Fig. 3 our experimental values for the *s*-wave phase shift have been plotted. We have tried to fit this data to the functional dependence, $\alpha_3^N=0.11\eta$, given by Orear.[4] A χ^2 test of the data does not exclude such a functional dependence. If a linear relationship were to be assumed, a best straight-line fit to our data would be $\alpha_3^N=0.104\pm0.006\eta$.

We have also compared our data to an assumed functional dependence $\alpha_3^N=0.089\eta\omega$, ω being the total energy of the π^+-p system excluding the proton rest

[16] E. E. Chambers and R. Hofstadter, Phys. Rev. **103**, 1454 (1956).

[17] G. F. Chew and F. E. Low, Phys. Rev. **101**, 1570 (1956).

energy. A χ^2 test shows that such a nonlinear relationship is as valid as the linear fit. Our data suggests a falling off from linearity at low energies. Nonlinearity in the opposite direction is clearly excluded. Theoretical support to a possible nonlinear phase shift dependence has been given by Cini *et al.*,[7] who have shown from dispersion relations that crossing symmetry implies that $(\alpha_1-\alpha_3)/\eta$ must be an odd function of ω.

At present the experimentally measured value of 1.5 $\pm$0.1 for the Panofsky ratio is in disagreement with the value 2.5$\pm$0.4 calculated from a linearly extrapolated value $(\alpha_1-\alpha_3)/\eta=0.27$, the photoproduction cross section on hydrogen of $(1.43\pm0.06)\times10^{-28}$ cm^2 and the π^+/π^- production ratio of 1.30$\pm$0.05 in deuterium. In order to retain the validity of the pion-proton dispersion relations with a fixed value for $f^2=0.08$, a change in the magnitude of the zero-energy value of α_3/η requires an equal change in the magnitude of α_1/η. A reduction of the zero-energy extrapolation of α_3 by a slight amount would, therefore, go far to remove the present internal disagreement among the parameters of low-energy pion physics.

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π^+-p Interactions at 500 Mev*†

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The interaction of π^+ mesons with protons at an energy of 500 Mev has been studied in a hydrogen bubble chamber. Phase-shift analyses with S and P waves were made, and a near degeneracy was found between the Fermi and Yang solutions. When D waves were included, an additional ambiguity was found. The D-wave phase shifts are small, but they have a considerable effect on the other phase shifts. The cross section for single pion production is 2.85$\pm$0.5 mb. The ratio $(p+0)/(n++)$ is $1.5_{-0.5}^{+1.5}$. The cross section leading to $p++-$ was found to be of the order of 30 μb.

I. INTRODUCTION

THE angular distribution of pion scattering by protons has been extensively studied at energies below about 310 Mev.[1,2] Experiments using hydrogen diffusion chambers were performed at 395 Mev[3] and in an energy range centered on 500 Mev.[4] Recently, preliminary results of the interactions of negative pions in a propane bubble chamber have been reported.[5] Angular distributions for positive and negative pion scattering from protons near 1 Bev have been obtained as a by-product of experiments on strange particle production.[6] Measurements of the total cross section for positive and negative pions up to 1.5 Bev have been published.[7-9]

The ambiguities which formerly existed in the phase-shift analyses of the pion elastic scattering at moderate energies have now been satisfactorily resolved.[10] For the positive pion scattering, at least, the agreement of all the experiments up to 500 Mev with the dispersion relations[11] is satisfactory. The principal questions to be answered by experiments on the π^++p interactions around 500 Mev are: the behavior of the S- and P-wave phase shifts,[12] the question of the presence of the D-wave

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§ Much of this work was done while the author was a National Science Foundation Predoctoral Fellow, and a guest of Brookhaven National Laboratory.

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11 R. M. Sternheimer, Phys. Rev. **101**, 384 (1956). See also reference 8.

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