Lifetimes of 2^+ Rotational States

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The mean lives of the rotational 2^+ states of nuclei in the region $150 \leqslant A \leqslant 186$ have been measured with a pulsed proton beam timing device. The presence of prompt proton bremsstrahlung requires more elaborate analysis of the time distributions than the customary determination of center of gravity shift. In order to achieve the necessary accuracy and freedom from drift, a beam switching procedure was introduced, alternating the beam between two targets and therefore allowing accurate comparisons to be made. A measurement of the mean life of B^{10}, carried out as a check, yielded results in good agreement with previous measurements. A method for utilizing the lifetime measurements for accurate determinations of $B(E2)$ values is discussed.

INTRODUCTION

THE purpose of the measurements described here is to obtain a comprehensive and accurate survey of lifetimes of the 2^+ rotational states in even-even nuclei in the region $150 \leqslant A \leqslant 186$. Previous work in this region consisted of measurements of delayed coincidences in γ-γ cascades following β decay.[1] In the present experiments the 2^+ levels were excited by a pulsed proton beam and the delayed γ emission after the proton pulse was measured. The uniformity in procedure in this type of experiment greatly increases the accuracy and reliability of the measurements as far as comparisons between different nuclei are concerned.

EXPERIMENTAL PROCEDURE

The nuclei investigated were isotopes of Nd, Sm, Gd, Er, Dy, Hf, and W. The targets were made of compressed powder of the oxides of the materials, about 25 mg/cm^2 thick. The 2^+ levels were excited by Coulomb excitation with 2.8 Mev protons from a 3-Mev High Voltage Engineering Van de Graaff accelerator.

The beam pulsing apparatus is shown schematically in Fig. 1. It consisted of an electrostatic quadrupole lens and a 7-Mc/sec horizontal oscillating field which sweeps the beam across a 1.8 mm slit at a distance of 7 meters. The beam spot was about 1 mm wide and beam pulses of 0.75×10^{-9} sec could be produced with a mean current of 0.05 μa. The total current on the slit was about 7 μa. The γ radiation was detected by a 2×2 in. plastic scintillator bonded to a 6810A RCA photomultiplier, and the time between pulses in the multiplier and a fixed phase of the oscillator was translated into pulse height by a "delay to pulse-height converter."[2]

The pulses from the converter were fed into a 99-channel pulse-height analyzer that was gated by a single-channel analyzer which selected pulses corresponding to a fixed energy dissipated in the scintillator.

The pulse distributions were compared with distributions correspondent to prompt radiation emitted by targets of odd-A nuclei (Ta, Ho) which have states with mean lives of the order of 5×10^{-11} sec.

A serious source of inaccuracy in measurements of this nature is the shift in the pulse distribution produced by drifts in the electronic equipment. This difficulty was overcome, or rather avoided, by an arrangement which allowed both targets, producing "delayed" and "prompt" radiation, to be bombarded at essentially the same time; the two targets were placed one above the other and a vertical electrostatic field was employed to alternate the beam twice a second between them. At the same time the multichannel analyzer, which was divided into two parts, was switched to record in either the first or the last fifty channels.

With this arrangement the only effect of instrumental drift was to cause a general broadening of the pulse distribution, manifesting itself as a deterioration of the time resolution. It was found that the time scale was shifted up to 3×10^{-10} sec during a run of two hours. The effect of this shift on the resolution was quite unimportant whereas in a long-term comparison measurement it could have caused serious errors.

During each cycle of the oscillator the target is hit twice, so that one should observe two peaks in the pulse distribution separated by 71×10^{-9} sec. However, if the beam is not exactly centered on the slit (for zero deflection voltage), the timing is distorted. To overcome this difficulty the beam was swept in an ellipse, with part of the oscillator voltage supplied to the horizontal plates. In this way the targets were hit only once during a period and on the return path the beam struck a thick piece of lead.

The energies of the transitions investigated varied from 70 to 130 kev, and the gating pulse was set accordingly at 12 to 40 kev, close to the Compton edge. The overall time resolution varied from 2.2×10^{-9} sec for the highest energies to 3×10^{-9} sec for the lowest energies. A typical run is shown in Fig. 2 in which the pulse distribution from Sm^{152} is compared with that of Ta^{181} taken simultaneously.

In order to calibrate any timing device a controllable known delay must be introduced in some part of the system. In the present experiment, calibrating delays

[1] A. W. Sunyar, Phys. Rev. 98, 653 (1955).
[2] The converter was of the type described by L. E. Beghian *et al.* Rev. Sci. Instr. 29, 753 (1958).

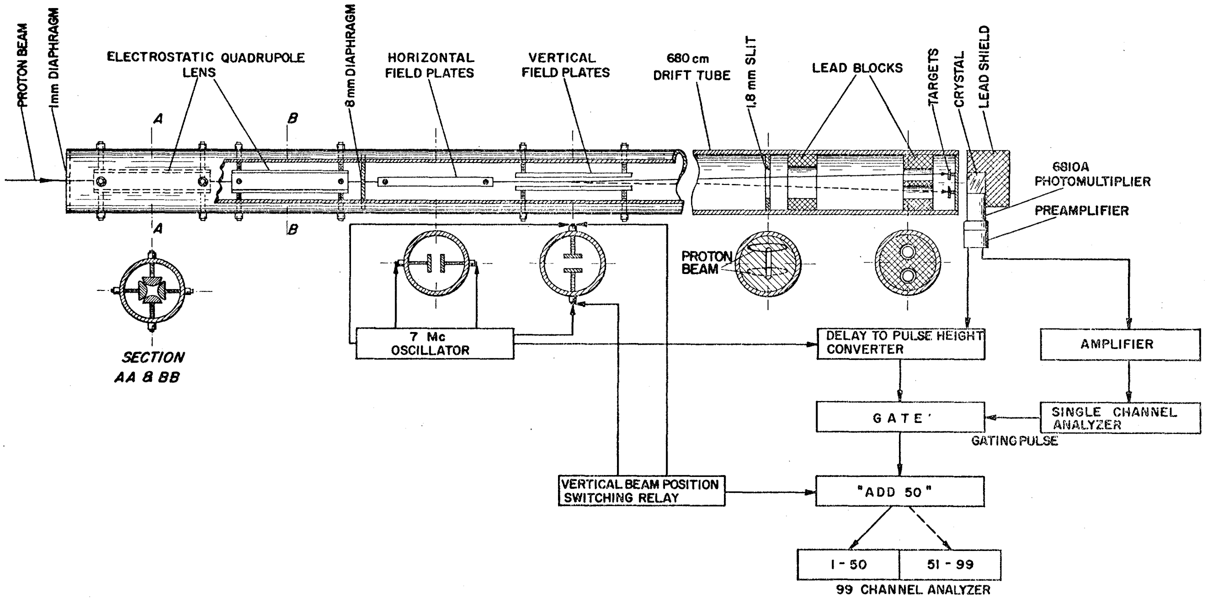

FIG. 1. Schematic drawing of the apparatus.

were introduced by two different methods: by varying the length of cable between the counter and the converter, and by varying the energy of the protons, thereby changing the time between the oscillator phase and the moment of impact of the protons on the target. The shifts in the pulse distribution produced by the two types of delay were found to be compatible. The delay per channel was measured as $(0.451\pm0.015)\times10^{-9}$ sec.

EVALUATION OF MEAN LIVES

The most widely accepted procedure for evaluating lifetimes in low-resolution measurements (i.e., if the resolution is not small compared to the measured mean life) is to determine the mean life τ from the relation

$$\tau=M_1(f)-M_1(\bar{f}), \tag{1}$$

Where $M_1(f)$ is the center of gravity of the time distribution of pulses from the source which is being investigated, and $M_1(\bar{f})$ is the center of gravity of the pulse distribution from a source emitting prompt radiation. This procedure is valid only if the delayed radiation corresponds to a pure exponential decay. In our case the decay was not purely exponential because the proton bremsstrahlung contributes an appreciable component of prompt radiation; the fraction of prompt radiation was measured by the methods described in this paragraph and was found to vary between 0.25 and 0.75. Even in cases where relation (1) seems to be applicable, a determination of τ based on this relation alone is not very satisfactory because one has no check of internal consistency.

In order to overcome these difficulties an analysis of higher moments of the distribution function may be introduced. An analysis of this nature was described by Bay.[3] For the problem at hand it was found expedient to change slightly the definition of the moments. We define for any normalized function $f(t)$:

$$\begin{aligned} M_1(f)&=\int_{-\infty}^{\infty} tf(t)dt,\\ M_2(f)&=\int_{-\infty}^{\infty} [t-M_1(f)]^2 f(t)dt, \\ M_3(f)&=\int_{-\infty}^{\infty} [t-M_1(f)]^3 f(t)dt. \end{aligned} \tag{2}$$

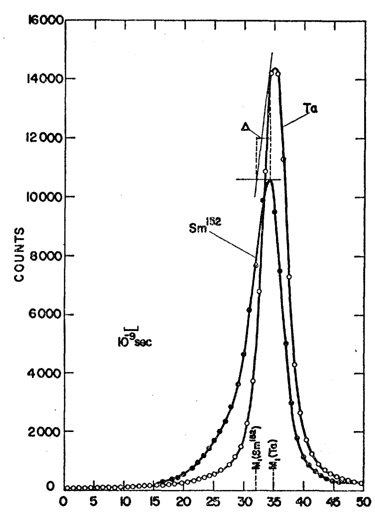

FIG. 2. Pulse distribution from a Sm^{152} target and a Ta^{181} target, normalized to the same number of total counts. $M_1(Sm^{152})$ and $M_1(T_a)$ are the centers of gravity of the two distributions. The significance of the segment Δ is explained in the text.

[3] Z. Bay, Phys. Rev. **77**, 419 (1950).

The time distribution $f(t)$ of the measured pulses may be written in the form.

$$f(t)=\int_{-\infty}^{\infty}\varphi(t')F(t-t')dt', \tag{3}$$

where $\varphi(t)$ is the decay function of the emitted radiation, and $F(t-t')$ is the transfer function of the apparatus (including finite time of production, spread in the crystal, photomultiplier, etc.). $\varphi(t)$ and $F(t-t')$ are taken to be normalized; $f(t)$ will then also be normalized.

The following relations hold between the moments of f, φ, and F:

$$M_i(f)=M_i(\varphi)+M_i(F), \quad i=1, 2, 3. \tag{4}$$

We shall also consider the distribution function $\bar{f}(t)$ corresponding to prompt radiation. In this case, $\varphi(t)=\delta(t-t_0)$ and therefore

$$\bar{f}(t)=F(t-t_0). \tag{5}$$

If we compare a distribution of decaying radiation f with a distribution of prompt radiation $\bar{f}$, produced at the same time t_0, we get the relations:

$$M_i'(\varphi)=M_i(f)-M_i(\bar{f}), \quad i=1, 2, 3, \tag{6}$$

where $M_i'(\varphi)$ is defined by

$$M_i'(\varphi)=M_i(\varphi)-t_0\delta_{i1}.$$

$M_i(f)$ and $M_i(\bar{f})$ can be determined from the measured functions f and $\bar{f}$. Relation (6) yields the moments $M_i'(\varphi)$ and through them the function φ is determined.

The precision with which φ is defined will increase with the number of moments in the analysis. However, the difficulty of measuring the moments increases with the order and one will therefore tend to keep the number of moments in analysis to a minimum.

For a pure exponential decay with mean life τ, $\varphi(t)$ is given by

$$\varphi(t)=\begin{cases}0 & \text{for} \quad t<t_0\\ (1/\tau)e^{-(t-t_0)/\tau} & \text{for} \quad t\geqslant t_0.\end{cases} \tag{7a}$$

If the radiation contains a prompt component as well as exponentially decaying radiation, in the ratio $(1-\eta)/\eta$, then $\varphi(t)$ will be given by:

$$\varphi(t)=\begin{cases}0 & \text{for} \quad t<t_0\\ (1-\eta)\delta(t-t_0)+(\eta/\tau)e^{-(t-t_0)/\tau} & \text{for} \quad t\geqslant t_0.\end{cases} \tag{7b}$$

For a pure exponential decay one gets from (7a): $M_1'(\varphi)=\tau$, and therefore Eq. (6) with $i=1$ yields the center-of-gravity relation (1). For a decay containing a prompt component one gets from (7b): $M_1'(\varphi)=\eta\tau$, and Eq. (1) is replaced by

$$\eta\tau=M_1(f)-M_1(\bar{f}), \tag{8}$$

which does not determine τ uniquely if η is not known. However, the second moment of (7b) is

$$M_2'(\varphi)=(2\eta-\eta^2)\tau^2, \tag{9}$$

or

$$M_2'(\varphi)=2M_1'(\varphi)\tau-[M_1'(\varphi)]^2,$$

and therefore

$$\tau=M_1'(\varphi)\left\{1+\frac{1}{2}\frac{M_2'(\varphi)-[M_1'(\varphi)]^2}{[M_1'(\varphi)]^2}\right\}. \tag{10}$$

In a pure exponential decay ($\eta=1$), (9) would yield a consistency check; one should then get

$$M_2'(\varphi)=[M_1'(\varphi)]^2.$$

In the more general case $\eta\neq 1$ we evaluate $M_3'(\varphi)$ as a check. With our assumptions about φ one should get the identity

$$2M_1'(\varphi)M_3'(\varphi)=3[M_2'(\varphi)]^2+[M_1'(\varphi)]^4. \tag{11}$$

The degree to which (11) actually holds, is a measure of the accuracy and reliability of the measurement.

For the experimental evaluation of the higher moments the freedom from uncontrolled changes and drifts is even more important than in center-of-gravity shift measurements, and the fast-cycling procedure adopted in this experiment is essential.

There is another way of computing τ independently of η from the distribution functions which is less accurate than the moment analysis but more straightforward. This is based on the position of the maximum of the function $f(t)$ and constitutes a generalization of a theorem of Newton.[4] In the same manner as in reference 4 one can show that

$$1-\eta=\frac{f(t_M)-\bar{f}(t_M)}{-[\partial\bar{f}(t_M)/\partial t]\tau}, \tag{12}$$

where t_M is the point at which $f(t)$ attains its maximum. If we draw tangents to the curves $f(t)$, $\bar{f}(t)$ through the points $f(t_M)$, $\bar{f}(t_M)$, respectively, they will cut each other at a point O. The distance O–t_M we call Δ (see Fig. 2), and we get from (12)

$$1-\eta=\Delta/\tau.$$

One can therefore determine τ in principle without having recourse to higher moments, by the relation

$$\tau=M_1(f)-M_1(\bar{f})+\Delta.$$

It was also found expedient to compare directly two decaying radiations rather than to compare either of them with some prompt radiation. In the region where the pulse distributions from prompt radiations have dropped to zero, the decaying radiations exhibit a pure exponential decay $\exp(-t/\tau_1)$, $\exp(-t/\tau_2)$; and the

[4] T. D. Newton, Phys. Rev. **78**, 490 (1950).

ratio of two radiations varies as $\exp(-t/\tau')$, where

$$\frac{1}{\tau'}=\frac{1}{\tau_1}-\frac{1}{\tau_2}.$$

As the mean lives of the states investigated are all rather similar, τ' is rather long (of the order of 10^{-8} sec) and the measurement of τ' provides a very accurate comparison between the two mean lives.

RESULTS

The results of the measurements are presented in the table. The mean lives are given both relative to the mean life of Sm^{152} (these being the more accurate results) and in absolute value. The quoted errors were estimated from the statistical spread in repeated measurements and from comparison between the results of the different types of measurement described in the previous section. By applying the consistency relation (11), the mean life was evaluated from M_1, M_3 and compared with the values derived from M_1, M_2. The two values differed by 4–13%, the higher discrepancies being consistently associated with low-energy transitions and probably due to the increased background level (due to photomultiplier noise) encountered in this energy region.

The results of the measurements of reference 1 are also given in Table I for comparison. There appears to be a systematic difference between these values and the results of the present investigation, the values of reference 1 being consistently smaller.

The mean life of the first excited state of B^{10} was measured as a check; the excited state being produced in the reaction $B^{10}(p,p')B^{10}$. This reaction also yields some prompt high-energy radiation which could be corrected for directly by the methods described above. Earlier measurements of the mean life of this level are summarized by Holland *et al.*[5]

The prompt radiation was in this case found to comprise 0.03 of the total and the mean life determined from the third and first moments differed from the mean life determined from the second and first moments by 3%.

INTERPRETATION OF THE RESULTS

The mean lives of the 2^+ levels are related to the $B(E2)$ values for the transition $2^+\rightarrow 0^+$:

$$1/\tau=C_1(E_\gamma)B(E2)(1+\alpha), \tag{13}$$

where α is the conversion coefficient, and $C_1(E_\gamma)$ is a known function of the transition energy E_γ.

The conversion coefficients are not known at present with sufficient accuracy to allow a precise evaluation of the $B(E2)$ values from the measured mean lives. It is therefore envisaged to carry out an accurate determination of the yield of γ's per proton. This is given by

$$n_\gamma=C_2(E_\gamma,E_p,Z)\frac{B(E2)}{1+\alpha}, \tag{14}$$

where $C_2(E_\gamma,E_p,Z)$ is a function of the transition energy, the proton energy, and the charge of the target nucleus, and can be accurately evaluated. From (13) and (14) the $B(E2)$ values can be determined.

TABLE I. Mean lives of 2^+ states.

Nucleus	Mean life τ in units of 10^{-9} sec	$\tau/\tau_{Sm^{152}}$	Previous measurements; τ in units of 10^{-9} sec
Nd^{150}	2.22 ± 0.1	1.06 ± 0.02	
Sm^{152}	2.09 ± 0.08	1	2.02 ± 0.15[a]
Sm^{154}	3.95 ± 0.35	1.89 ± 0.15	
Gd^{154}	1.78 ± 0.15	0.85 ± 0.05	1.73 ± 0.15[a]
Gd^{156}	2.96 ± 0.15	$1.4\ \pm0.05$	
Gd^{158}	4.03 ± 0.35	1.93 ± 0.15	
Gd^{160}	3.64 ± 0.2	1.74 ± 0.09	
Dy^{172}	3.20 ± 0.2	1.53 ± 0.07	
Dy^{174}	3.49 ± 0.35	1.67 ± 0.15	
Er^{166}	2.61 ± 0.25	1.25 ± 0.09	2.46[a]
Er^{168}	2.65 ± 0.25	1.27 ± 0.09	
Hf^{178}	2.70 ± 0.15	1.29 ± 0.07	
Hf^{180}	2.38 ± 0.15	1.14 ± 0.07	2.02 ± 0.15[a]
W^{182}	2.23 ± 0.2	1.07 ± 0.09	1.84 ± 0.15[a]
W^{184}	1.92 ± 0.15	0.92 ± 0.07	
W^{186}	1.61 ± 0.1	0.77 ± 0.05	
B^{10}	0.94 ± 0.05		0.94 ± 0.06[b]

[a] From reference 1.
[b] From reference 5.

GYROMAGNETIC RATIOS

The gyromagnetic ratios of Nd^{150}, Sm^{152}, and Sm^{154} have been measured by a precession method.[6] The measured precession angles depend on the product τg. At the time of the experiment no direct measurements of the mean lives of the 2^+ states of Nd^{150} and Sm^{154} were available and the results quoted in reference 6 were based on estimates. The present measurements essentially vindicate the earlier estimates, so that no error in the determination of the g factors was committed in this respect.

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[5] Holland, Lynch, and Hanna, Phys. Rev. **112**, 903 (1958).

[6] G. Goldring and R. P. Scharenberg, Phys. Rev. **110**, 201 (1958).