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Effect of Simultaneous Doppler and Collision Broadening and of Hyperfine Structure on the Imprisonment of Resonance Radiation

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The transmission coefficient defined by Holstein for resonance radiation has been calculated when Doppler and collision broadening of the resonance line are present simultaneously. From this a simple formula is inferred for the imprisonment lifetime of the resonance radiation under this condition. The complication of the hyperfine structure is also taken into account and the results are found to give good agreement with experiments in mercury.

1. INTRODUCTION

THE imprisonment of resonance radiation plays an important role in many phenomena allied to gaseous discharges.[1–3] Its interpretation, however, is often difficult because of the presence of collision broadening in addition to the usual Doppler and natural broadening, and also because of the overlap of the hyperfine structure. The purpose of this paper is to investigate, in such cases, the imprisonment lifetime of resonance radiation as defined by Holstein.[4,5] The results are found to give excellent agreement with the measured lifetimes.[6]

2. TRANSMISSION COEFFICIENT OF A SINGLE LINE WITH COLLISION AND DOPPLER BROADENING PRESENT

The essential feature of this treatment is the calculation of the transmission coefficient, $\mathcal{T}$, for a single line in the presence of Doppler and collision broadening. When the transmission coefficient is determined we can, in principle, determine the imprisonment lifetime, T, through the proper mathematical manipulations. However, once the transmission coefficient is found, we forego further mathematics and infer the form of the imprisonment lifetime using the order of magnitude relation given by Holstein:

$$T/\tau=\kappa/\mathcal{T}(R). \tag{2.1}$$

τ is the natural lifetime of a resonance atom, κ is a constant of order unity, and R represents the radius of a cylindrical enclosure and one-half the width of a parallel plate enclosure. The complication of the hyperfine structure will then be taken into account in a reasonable manner.

The transmission coefficient, $\mathcal{T}(\rho)$, is defined as the probability of a resonance quantum traveling a distance ρ without being absorbed. Holstein gives the expression for $\mathcal{T}$ in the form

$$\mathcal{T}(\rho)=\int P(\nu)\exp[-k(\nu)\rho]d\nu, \tag{2.2}$$

where $P(\nu)$ is the frequency spectrum of the radiation emitted from a given volume element and $k(\nu)$ is the absorption coefficient as a function of frequency ν. For collision and Doppler broadening Holstein shows that P and k are proportional and, in fact,

$$k(\nu)=\lambda_0^2Ng_2P(\nu)/8\pi g_1\tau, \tag{2.3}$$

where λ_0 is the wavelength of the resonance line, N is the gas density and g_2, g_1 are the statistical weights of the excited and ground states, respectively. Equation (2.3) is not completely accurate in the case of natural broadening but for simplicity it can be adopted in that case also.

When Doppler, collision, and natural broadening are present at the same time, k is given by the approximate expression[7]

$$k(\nu)=k_0[\exp(-x^2)+(a/\pi^{\frac{1}{2}}x^2)], \tag{2.4}$$

where $k_0=\lambda_0^3Ng_2/8\pi^{\frac{3}{2}}g_1v_0\tau$ is the absorption coefficient for the center of the resonance line and

$$a=\left(\frac{1}{\tau}+\frac{1}{\tau_c}\right)\frac{\lambda_0}{4\pi v_0},$$

$$x=(\nu-\nu_0)c/\nu_0v_0,$$

$$v_0=(2k\theta/m)^{\frac{1}{2}}.$$

Here τ_c is an effective collision time for the resonance atom, c is the velocity of light, and v_0 is the average gas velocity at the absolute temperature, θ.

Formula (2.4) holds within 20% for values of $x>2$ and a up to 0.5. For $x>2$ and large values of a, (2.4) goes over to the correct form for collision broadening:

$$k(\nu)=k_0a/\pi^{\frac{1}{2}}x^2.$$

The integration we will perform will emphasize values of x larger than 2. Actual computation indicates that (2.4) is a reasonable form to use for the absorption coefficient throughout the whole range of a. In actual calculations in this paper, the largest value of a encountered was 0.011.

[1] C. Kenty, J. Appl. Phys. **21**, 1309 (1950).
[2] P. J. Walsh, Phys. Rev. **107**, 338 (1957).
[3] A. V. Phelps, Phys. Rev. **110**, 1362 (1958).
[4] T. Holstein, Phys. Rev. **72**, 1212 (1947).
[5] T. Holstein, Phys. Rev. **83**, 1159 (1951).
[6] Alpert, McCoubrey, and Holstein, Phys. Rev. **76**, 1257 (1949).
[7] A. C. G. Mitchell and M. W. Zemansky, *Resonance Radiation and Excited Atoms* (The Macmillan Company, New York, 1934), pp. 321, 322, and 329.

The use of Eqs. (2.3) and (2.4) in (2.2) yields

$$\mathcal{T}(\rho)=\frac{1}{\pi^{\frac{1}{2}}}\int_{-\infty}^{\infty}\left(\exp(-x^2)+\frac{a}{\pi^{\frac{1}{2}}x^2}\right)\times\exp\left[-k_0\rho\left(\exp(-x^2)+\frac{a}{\pi^{\frac{1}{2}}x^2}\right)\right]dx. \quad (2.5)$$

The expressions for the transmission coefficient can be broken up into two integrations:

$$I_1=\frac{1}{\pi^{\frac{1}{2}}}\int_{-\infty}^{\infty}\exp(-x^2)\times\exp\left[-k_0\rho\left(\exp(-x^2)+\frac{a}{\pi^{\frac{1}{2}}x^2}\right)\right]dx, \quad (2.6)$$

$$I_2=\frac{a}{\pi}\int_{-\infty}^{\infty}\frac{1}{x^2}\exp\left[-k_0\rho\left(\exp(-x^2)+\frac{a}{\pi^{\frac{1}{2}}x^2}\right)\right]dx.$$

These integrals are evaluated by expanding in a Taylor series about $a=0$. The details are given in the Appendix. We give the results here:

$$\mathcal{T}(\rho)=\mathcal{T}_d\exp(-\pi\mathcal{T}_{cd}^2/4\mathcal{T}_c^2)+\mathcal{T}_cE_2(\pi^{\frac{1}{2}}\mathcal{T}_{cd}/2\mathcal{T}_c), \quad (2.7a)$$

where

$$\mathcal{T}_d=\frac{1}{k_0\rho(\pi\ln k_0\rho)^{\frac{1}{2}}},\quad \mathcal{T}_c=\frac{1}{\pi^{\frac{1}{2}}}\left(\frac{\pi^{\frac{1}{2}}a}{k_0\rho}\right)^{\frac{1}{2}},\quad \mathcal{T}_{cd}=\frac{2a}{\pi(\ln k_0\rho)^{\frac{1}{2}}}. \quad (2.7b)$$

$\mathcal{T}_d$ and $\mathcal{T}_c$ are the transmission coefficients in the case of pure Doppler broadening and collision boradening, respectively.[4] $\mathcal{T}_{cd}$ is the transmission coefficient under the condition of collision-type emission with Doppler-type absorption of the resonance line [ΔT_N in the notation of Holstein, reference 4, Eq. (5.14)]. E_2 signifies the error integral.[8]

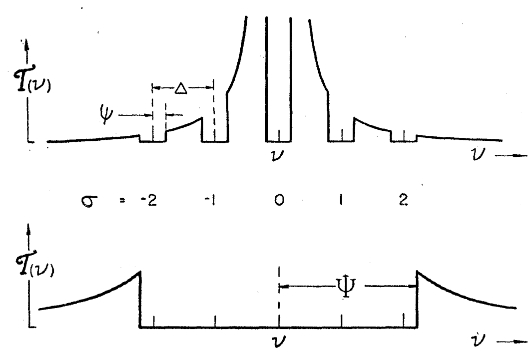

FIG. 1. Idealized transmission spectrum with the emission pictured as coming from the $\sigma=0$ hyperfine line. The upper curve displays the separation, Δ, and the absorption width, 2ψ of the hyperfine lines. The absorption widths are broad enough, in the lower curve, to produce overlapping with a composite width, Ψ. The actual emission is a sum over the transmitted emission from all the hyperfine lines.

The determination of the imprisonment lifetime, T, from the transmission coefficient given in (2.7a) is accomplished, in principle, by substituting the transmission coefficient correctly in the integral equation given by Holstein and then solving, for example, by a variational method. The net result is Eq. (2.1) where κ is determined exactly. Now for a cylindrical geometry κ has the following values,

$$\kappa_d=1.60,\quad \kappa_{cd}=1.00,\quad \kappa_c=1.115, \quad (2.8)$$

for the cases indicated by the subscripts. The differences among the κ's is much less than a factor of 2 and only 12% for the last two.

As mentioned, we shall not attempt an exact computation of the decay time. We merely infer the form of T from an inspection of (2.7a). Noting that $4/\pi\approx(1.115)^2$, the simplest equation for T, as indicated by (2.7a), is

$$\frac{1}{T}=\frac{\exp(-T_c^2/T_{cd}^2)}{T_d}+\frac{E_2(T_c/T_{cd})}{T_c}. \quad (2.9)$$

Note that this equation reduces to the form for pure Doppler broadening when $a=0$, and to the form for pure collision broadening when a is very large. It also has the correct limit for $T_{cd}\gg T_c$, T_d. Because Eq. (2.9) has these correct limiting values and the proper functional dependence as indicated by Eq. (2.7a), we expect Eq. (2.9) to be quite accurate in predicting imprisonment lifetimes.

3. INFLUENCE OF OVERLAPPING OF HYPERFINE STRUCTURE ON IMPRISONMENT LIFETIME

Our expression for the decay time has been derived from considerations of a single resonance line. Actually, the resonance line may be multiple, as for example in mercury, where the resonance line has a hyperfine structure consisting of 5 lines of (very roughly) equal amplitude and separation. We will investigate the effect of this hyperfine structure on the decay times T_c, T_d, T_{cd} and determine the changes necessary to make Eq. (2.9) applicable in the presence of hyperfine structure.

We first note two important facts concerning the Eqs. (2.5), (2.6) and (2.7a) for the transmission coefficient. First, $\mathcal{T}(\rho)$ can be broken down to the sum of I_1 and I_2 as already mentioned. This means that the effect of the hyperfine structure overlap on I_1 and I_2 are separated. Second, the expression for I_2,

$$I_2=\mathcal{T}_cE_2(\pi^{\frac{1}{2}}\mathcal{T}_{cd}/2\mathcal{T}_c)$$

essentially separates $\mathcal{T}_{cd}$ and $\mathcal{T}_c$. This is so because I_2 is given quite closely by the smaller of the two transmission coefficients. If one coefficient is twice the other,

[8] E. Jahnke and F. Emde, *Tables of Functions* (Dover Publications, New York, 1945), fourth edition, pp. 23 ff.

then I_2 is equal to the smaller one within 4%. Even when $\mathcal{T}_{cd}=\mathcal{T}_c$, the error in choosing I_2 as equal to either is only 27%.

These two facts strongly suggest that all we need do to take the hyperfine structure into account in (2.9) is to consider separately the hyperfine structure effect on $T_d \exp(T_c^2/T_{cd}^2)$ and on T_{cd}, T_c where these last two quantities appear in the second term on the right in Eq. (2.9). This corresponds to separating I_1 from I_2 and, in I_2, to separating $\mathcal{T}_{cd}$ from $\mathcal{T}_c$. This is the procedure we adopt and it will greatly reduce the effort involved in computing the effect of the overlapping of the hyperfine structure.

4. COMPUTATION OF THE HYPERFINE STRUCTURE EFFECT FOR MERCURY

The computation of the hyperfine structure effect in the manner suggested above will be done for the $^3P_1$ resonance state in mercury. We assume, in this case, a model for the hyperfine structure consisting of five lines of equal amplitude and equal separation, $\delta=11.7$ mA (see reference 7, Fig. 13).

The formula for the transmission in the presence of hyperfine structure is given by Holstein in the general form

$$\mathcal{T}'(\rho)=\sum_n \int P_n(\nu)\exp[-\sum_\sigma k_\sigma(\nu)\rho]d\nu, \tag{4.1}$$

where the P_n, k_σ are the spectral emission and absorption coefficients of the individual components and $\sum\int P_n(\nu)d\nu=1$. This last equation expresses the normalization of the emission coefficients. The relation between k and P is still given by (2.3). We use primes throughout to denote the presence of hyperfine structure.

For five lines of equal strength, we have $\int P_n(\nu)d\nu=\frac{1}{5}$. *The $\frac{1}{5}$ becomes a normalizing constant which should now appear in front of P and k in all our preceding equations.* To convert our equations to the form including the hyperfine structure we must include the sums indicated in (4.1), replace P, k by $P/5$, $k/5$, and x by $x-n\Delta$, where n is an integer and $\Delta=\delta c/\nu_0 v_0$. Then

$$\begin{aligned}\mathcal{T}_{cd}'&=\frac{a}{5\pi}\sum_n\int_{-\infty}^{\infty}\frac{1}{(x-n\Delta)^2}\\&\qquad\times\exp\left\{-\sum_\sigma\frac{k_0\rho}{5}\exp[-(x-\sigma\Delta)^2]\right\}dx,\\ \mathcal{T}_c'&=\frac{a}{5\pi}\sum_n\int_{-\infty}^{\infty}\frac{1}{(x-n\Delta)^2}\\&\qquad\times\exp\left[-\sum_\sigma\frac{k_0\rho}{5}\frac{a}{\pi^{\frac{1}{2}}}\frac{1}{(x-\sigma\Delta)^2}\right]dx.\end{aligned} \tag{4.2}$$

The integrations are performed by noting that the exponential terms are very close to unity except in the region bounded by $x_{cd}=\sigma\Delta\pm\psi_{cd}$, $x_c=\sigma\Delta\pm\psi_c$ where the exponential term drops sharply to a low value. Here we have approximately

$$\psi_{cd}=\ln(k_0\rho/5),\quad \psi_c=(4k_0\rho a/5\pi^{\frac{3}{2}})^{\frac{1}{2}},\quad \psi_d=\ln^{\frac{1}{2}}(k_0\rho/5). \tag{4.3}$$

ψ_d is noted for future reference. In essence, the exponential term gives each hyperfine component a spectral width of 2ψ. Within this spectral range, centered on each hyperfine component, the resonance absorption is complete. For $\psi>\Delta/2$, the width of the lines overlap and the hyperfine structure disappears leaving a single line of total width, Ψ, where

$$\begin{aligned}\frac{k_0\rho}{5}\sum_{\sigma=-2}^{\sigma=+2}\exp[-(\Psi_{cd}-\sigma\Delta)^2]&=1,\\ \frac{k_0\rho}{5}\sum_{\sigma=-2}^{\sigma=+2}\frac{1}{(\Psi_c-\sigma\Delta)^2}&=\frac{4}{\pi}.\end{aligned} \tag{4.4}$$

These remarks are illustrated schematically in Fig. 1 where the transmission is illustrated for the radiation from the $\sigma=0$ hyperfine line for ψ less than $\frac{1}{2}\Delta$ and ψ greater than $\frac{1}{2}\Delta$.

Following the ideas outlined above, we have:

$$\begin{aligned}&\int_{-\infty}^{\infty}\frac{\exp[-\sum ak_0\rho/5\pi^{\frac{1}{2}}(x-\sigma\Delta)^2]dx}{(x-n\Delta)^2}\\&=\left[\int_{-\infty}^{-(2\Delta+\psi_c)}+\int_{-(2\Delta-\psi_c)}^{-(\Delta+\psi_c)}+\cdots+\int_{2\Delta+\psi_c}^{\infty}\frac{dx}{(x-n\Delta)^2}\right],\quad \psi_c<\tfrac{1}{2}\Delta\\&=\left[\int_{-\infty}^{-(2\Delta+\Psi_c)}+\int_{2\Delta+\Psi_c}^{\infty}\frac{dx}{(x-n\Delta)^2}\right],\quad \psi_c>\tfrac{1}{2}\Delta.\end{aligned} \tag{4.5}$$

After performing the integrations and manipulating the fractions, the result is

$$\begin{aligned}\frac{T_c}{T_c'}&=\frac{1}{5}\sum_{n=-2}^{2}\sum_{\sigma=-2}^{2}\left[1-(\sigma-n)^2\left(\frac{\Delta}{\psi_c}\right)^2\right]^{-1},\quad \psi_c<\tfrac{1}{2}\Delta\\&=\frac{1}{5}\left(\frac{\psi_c}{\Psi_c}\right)\sum_{n=-2}^{2}\left[1-n^2\left(\frac{\Delta}{\Psi_c}\right)^2\right]^{-1},\quad \psi_c>\tfrac{1}{2}\Delta,\end{aligned} \tag{4.6}$$

where ψ_c is given by (4.3) and Ψ_c is given by (4.4) in the form

$$\left(\frac{\psi_c}{\Psi_c}\right)^2\sum_{\sigma=-2}^{2}\left[1-\sigma\left(\frac{\Delta}{\Psi_c}\right)\right]^{-2}=1, \tag{4.7}$$

with

$$\frac{\Delta}{\psi_c}=\left(\frac{\Psi_c}{\psi_c}\right)\frac{\Delta}{\Psi_c}.$$

In the same manner we find

$$\frac{T_{cd}}{T_{cd}'}=\frac{1}{5}\sum_{n=-2}^{2}\sum_{\sigma=-2}^{2}\left[1-(\sigma-n)^2\left(\frac{\Delta}{\psi_{cd}}\right)^2\right]^{-1},\ \psi_{cd}<\tfrac{1}{2}\Delta$$
$$=\frac{1}{5}\left(\frac{\psi_{cd}}{\Psi_{cd}}\right)\sum_{n=-2}^{2}\left[1-n^2\left(\frac{\Delta}{\Psi_{cd}}\right)^2\right]^{-1},\quad \psi_{cd}>\tfrac{1}{2}\Delta, \tag{4.8}$$

where Ψ given by (4.4). However, Δ is large, of the order of 7 to 8, so that in (4.4) the only term in the sum which is important is $\exp[-(\Psi_{cd}-2\Delta)^2]$. Since $\Psi_{cd}>2\Delta$, all other terms are negligible in value. Then, using (4.3) in (4.4),

$$\psi_{cd}/\Psi_{cd}=1-(2\Delta/\psi_{cd}). \tag{4.9}$$

The computation is carried out quite easily. In the region of $\psi_c<\frac{1}{2}\Delta$, values of Δ/ψ_c are assumed and T_c/T_c' computed from (4.6). The result for T_{cd}/T_{cd}' is the same in this region for numerical values of Δ/ψ_{cd} equal to Δ/ψ_c. In the region $\psi_c>\frac{1}{2}\Delta$, values of Δ/Ψ_c are assumed, ψ_c/Ψ_c calculated from (4.7), and Δ/ψ_c computed. Then (4.6) yields T_c/T_c' as a function of Δ/Ψ_c and hence of Δ/ψ_c. An analogous solution yields T_{cd}/T_{cd}' as a function of Δ/ψ_{cd}. Figure 2 gives the curves constructed.

In the vicinity of Δ/ψ_{cd}, $\Delta/\psi_c=2$, the mathematical assumptions embodied in (4.5) will break down and, in addition, the model assumed will not be accurate enough to give correct results. The net result we believe will be that Eqs. (4.6) and (4.8) will give values which are too low. We have consequently joined the curves for T_c/T_c', T_{cd}/T_{cd}' smoothly, but somewhat arbitrarily, in this region to give values of the ratio of the T's higher than indicated by our equations.

The computation of the effect of the hyperfine structure on I_1' can be carried out in a similar way with some modification. The essential features of the overlap of hyperfine structure on I_1' as found by such a computation are these. There is no change in I_1' for values of ψ_d within a percent or so of $\frac{1}{2}\Delta$. For $\psi_d>\frac{1}{2}\Delta$, I_1' falls sharply, depending on ψ_d and the actual value of k_0R.

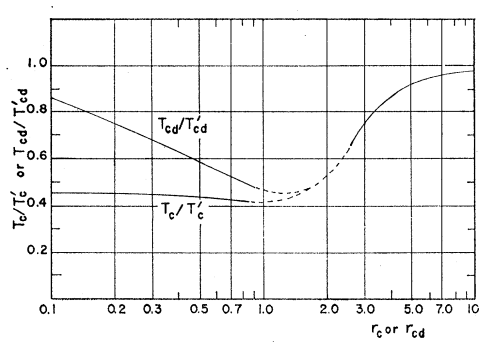

FIG. 2. Ratio of the imprisonment lifetime with overlap to imprisonment time without overlap, T/T', as a function of ratio of hyperfine separation to line width, $r=\Delta/\psi$

Fortunately, in the case of mercury, the term I_1' becomes negligible before this occurs. We can thus assume without any error that I_1' is unchanged.

The final equation for T is then

$$\frac{1}{T}=\frac{\exp(-T_c^2/T_{cd}^2)}{T_d}+\frac{E_2(T_c'/T_{cd}')}{T_c'}. \tag{4.10}$$

T_{cd}', T_c' are given in terms of T_{cd}, T_c and Δ/ψ_{cd}, Δ/ψ_c by Fig. 2. T_c, T_{cd}, T_d are given by Eqs. (2.1), (2.7b), and (2.8) with $k_0/5$ in place of k_0.

5. COMPARISON WITH EXPERIMENT

The foregoing analysis was applied to the imprisonment lifetime of the 2537 A radiation from the $^3P_1$ resonance state of mercury. Fowler[9] has fitted the experimental data[6] to a curve combining simple diffusion of the resonance state with loss by molecular formation at large densities. However, the diffusion coefficient derived from this fitting is 80 times larger than the value expected theoretically and 7 times the experimental upper limit for this coefficient.[10]

Our analysis is based on Eq. (4.10) and Fig. 2. The following relations were used for mercury:

$$\frac{\lambda_0}{\tau}\frac{1}{4\pi v_0}=\frac{0.0208}{\theta^{\frac{1}{2}}},\quad \frac{k_0}{5}=\frac{2.23\times10^{-12}}{\theta^{\frac{1}{2}}}N(\text{cm}^{-1}),$$

$$\tau=1.08\times10^{-7}(\text{sec}).$$

Holstein[11] gives $\tau/\tau_c=0.26\times10^{-15}N\alpha$, where α is a coefficient, less than unity, by which τ/τ_c is reduced due to the hyperfine structure. The actual calculations do not depend too strongly on the value of α, and α was adjusted somewhat to give the best fit. The value of α found was 0.7 which is consistent with the rough estimate of one-half given by Holstein.

The theoretical and experimental curves are given in Fig. 3. The agreement is within $\pm4\%$ except for the last three points which are in the region where the band fluorescence of mercury complicates the experiments and the loss by molecular formation, as suggested by Fowler, may become important. The good agreement should be regarded with some suspicion since the theory on which our calculations have been used does not warrant this type of agreement; perhaps $\pm10\%$ is a more reasonable expectation. In any case, the agreement does recommend Eq. (4.10) and Fig. 2 to predict accurate imprisonment lifetimes in the presence of collision and Doppler broadening and hyperfine structure overlap.

[9] R. G. Fowler, *Handbuch der Physik* (Springer-Verlag, Berlin, 1956), Vol. 22, p. 226.

[10] A. V. Phelps, Third International Conference on Ionization Phenomena in Gases, Venice, Italy, 1957 (unpublished).

[11] See reference 4, equation following (5.17). An extra factor of π should appear in the numerator.

APPENDIX

The evaluation of I_1 and I_2 is carried out by expanding them in a Taylor series about $a=0$. First consider I_1:

$$I_1=I_1(0)+\left.\frac{\partial I_1}{\partial a}\right|_{a=0} a+\frac{1}{2!}\left.\frac{\partial^2 I_1}{\partial a^2}\right|_{a=0} a^2+\cdots. \quad (1)$$

Now

$$\left.\frac{\partial^n I_1}{\partial a^n}\right|_{a=0}=\frac{(-1)^n}{\pi^{\frac{1}{2}}}\left(\frac{k_0\rho}{\pi^{\frac{1}{2}}}\right)^n\int_{-\infty}^{\infty}\frac{\exp(-x^2)}{x^{2n+1}}\times\exp[-k_0\rho\exp(-x^2)]dx. \quad (2)$$

The evaluation of this latter integral follows the method given by Holstein.[4] Set

$$y=k_0\rho\exp(-x^2).$$

$$\left.\frac{\partial^n I_1}{\partial a^n}\right|_{a=0}=\frac{(-1)^n}{\pi^{\frac{1}{2}}}\left(\frac{k_0\rho}{\pi^{\frac{1}{2}}}\right)^n\int_0^{k_0\rho}\frac{e^{-y}dy}{k_0\rho(\ln k_0\rho-\ln y)^{n+\frac{1}{2}}}. \quad (3)$$

The main contribution to the integrand comes from the vicinity of $y=1$. In the denominator we then set $y=1$ and extend the upper limit of the integral to infinity, since $k_0\rho$ is always large in the application of Holstein's theory. Then

$$\left.\frac{\partial^n I_1}{\partial a^n}\right|_{a=0}=\frac{(-1)^n}{\pi^{\frac{1}{2}}}\left(\frac{k_0\rho}{\pi^{\frac{1}{2}}}\right)^n\frac{1}{k_0\rho[\ln(k_0\rho)]^{n+\frac{1}{2}}}. \quad (4)$$

The approximation used to evaluate (3) should be more accurate as n increases. The accuracy for $n=1$ was determined by expanding the integrand $e^{-y}(\ln k_0\rho-c\ln y)^{-\frac{3}{2}}$ in a Taylor series about $c=0$, evaluating the subsequent integrals and then setting $c=1$ in the series. For $8<k_0\rho<\infty$ the approximate solution (4) at $n=1$ is accurate within $\pm 3\%$.

Inserting the values of $\partial^n I_1/\partial a^n$ into the Taylor series (1), we have

$$I_1=\frac{1}{k_0\rho(\pi\ln k_0\rho)^{\frac{1}{2}}}\left[1-\frac{ak_0\rho}{\pi^{\frac{1}{2}}\ln(k_0\rho)}+\frac{1}{2!}\left(\frac{ak_0\rho}{\pi^{\frac{1}{2}}\ln(k_0\rho)}\right)^2-\cdots\right]$$

$$=\mathcal{T}_d\exp(-\pi\mathcal{T}_{cd}^2/4\mathcal{T}_c^2). \quad (5)$$

In evaluating I_2 by a Taylor series similar to (1),

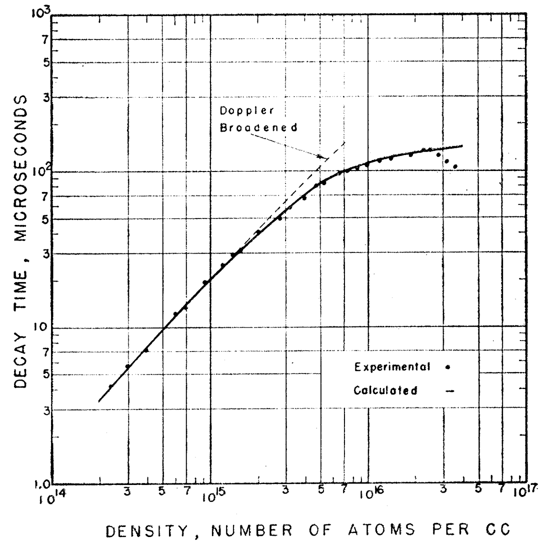

FIG. 3. Decay or imprisonment lifetime for resonance radiation in mercury vapor *vs* density of mercury atoms. The dotted line gives the calculated curve for the case of Doppler broadening with no overlap of the hyperfine structure as taken from reference 6. The heavy line includes the effects of the overlapping and of collision broadening.

we have

$$\left.\frac{\partial^n I_2}{\partial a^n}\right|_{a=0}=\frac{(-1)^n a}{\pi}\times\int_{-\infty}^{\infty}\left(\frac{k_0\rho}{\pi^{\frac{1}{2}}}\right)^n\frac{\exp[-k_0\rho\exp(-x^2)]}{x^{2(n+1)}}dx. \quad (6)$$

For large $k_0\rho$ the term $\exp[-k_0\rho\exp(-x^2)]$ has a very small value for x values up to $k_0\rho\exp(-x^2)\approx 1$ at which value of x the term $\exp[-k_0\rho\exp(-x^2)]$ rises sharply to unity. We then replace this term by a step function which is zero for $-[\ln(k_0\rho)]^{\frac{1}{2}}<x<[\ln(k_0\rho)]^{\frac{1}{2}}$ and unity for all other values of x. Then

$$\left.\frac{\partial^n I_2}{\partial a^n}\right|_{a=0}=\frac{(-1)^n a}{\pi}\left(\frac{k_0\rho}{\pi^{\frac{1}{2}}}\right)^n 2\int_{[\ln(k_0\rho)]^{\frac{1}{2}}}^{\infty}\frac{dx}{x^{2(n+1)}}$$

$$=\frac{2}{2n+1}\frac{(-1)^n a}{\pi}\left(\frac{k_0\rho}{\pi^{\frac{1}{2}}}\right)^n\Big/[\ln(k_0\rho)]^{n+\frac{1}{2}}, \quad (7)$$

and

$$I_2=\frac{2a}{\pi[\ln(k_0\rho)]^{\frac{1}{2}}}\left[1-\frac{1}{3}\left(\frac{ak_0\rho}{\pi^{\frac{1}{2}}\ln(k_0\rho)}\right)+\frac{1}{5}\frac{1}{2!}\left(\frac{ak_0\rho}{\pi^{\frac{1}{2}}\ln(k_0\rho)}\right)^2-\cdots\right]$$

$$=\mathcal{T}_c E_2(\pi^{\frac{1}{2}}\mathcal{T}_{cd}/2\mathcal{T}_c).$$