

## Buildup of a Discharge in Argon

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Measurements were made of the rate of buildup of an electrical discharge in argon in the pressure range from 5 to 60 cm Hg. The results are interpreted on the basis of a secondary mechanism due to delayed photons. The photon delay times which fit the observed data are in the neighborhood of five microseconds over the range of pressures investigated. These photon delay times are compared with (a) calculated imprisonment times for resonance radiation, and (b) delay times for molecular radiation as observed by Colli. Considering the uncertainty in the calculations and the lack of knowledge about the energy distribution between the two main resonance lines, the imprisonment times are of the right magnitude to explain the observed buildup rates. The Colli-process delay times are in fair agreement with the data at the higher pressures; at the low pressures they are definitely too slow to explain the observed buildup rates.

### INTRODUCTION

FOR many gases the uniform field electrical breakdown near threshold is at present satisfactorily explained by a Townsend type of buildup.<sup>1</sup> The details of the secondary mechanism active in this Townsend buildup depend upon the particular gas and the pressure. In air at atmospheric pressure the secondary mechanism appears to be electron emission from the cathode by ultraviolet radiation created in the discharge.<sup>2</sup> Since the speed of propagation of the ultraviolet radiation is extremely high, the rate of buildup of the discharge is controlled by the electron velocity. In argon in the low-pressure range (less than several mm Hg), investigators have concluded that the secondary mechanism is due mostly to electron emission by positive ions.<sup>3,4</sup> Under those conditions the rate of buildup of the discharge becomes controlled by the positive-ion velocity and is, therefore, relatively slow.

Formative time-lag measurements by Kachikas and Fisher in argon in the pressure range from several cm Hg to atmospheric show that neither of the two secondary mechanisms described above control the discharge-buildup rate.<sup>5</sup> The formative time lag of breakdown is the time between the application of voltage to the gap and the establishment of a large gap current; it is thus inversely related to the rate of buildup of the discharge. The formative lags measured by Kachikas and Fisher were too short to be ascribable to a positive-ion secondary mechanism, but were too long to be due to a simple photon process. Kachikas and Fisher suggested that the data might be explained by a delayed-photon secondary mechanism. The buildup rate of the discharge would then be controlled by the delay time. Several possibilities for such a delayed-photon mechanism exist. Colli observed a delayed emission of ultraviolet light from a discharge

in argon.<sup>6</sup> This light is believed to be emitted by the radiative decay of a relatively long-lived excited molecular state which is formed by three-body collisions from metastable atoms created in the discharge. Another possibility for a delayed-photon mechanism is the imprisonment of resonance radiation. Since a large fraction of the radiation emitted by excited atoms created in the discharge is resonance radiation, the phenomenon of imprisonment may be expected to play an important role.

The present experiment covers much the same pressure range as the investigations of Kachikas and Fisher. The rate of buildup of the gap current was measured rather than the formative lag, since the former quantity is somewhat simpler to interpret. In addition, measurements of gap current during the final stages of buildup (just before gap breakdown) were made to investigate the mechanisms active during that phase of the discharge.

### THEORY

The theory used to analyze the results is essentially the simple exponential buildup theory due to Bartholomeyczuk.<sup>7</sup> Consider a plane parallel gap of spacing  $d$  (cathode at  $x=0$ , anode at  $x=d$ ). As a result of the applied field  $E=V/d$ , electrons have a drift velocity  $v_-$ , positive ions have a drift velocity  $v_+$ , and an electron multiplication coefficient  $\alpha$  exists giving the number of new electron-positive ion pairs created per electron per cm drift. The continuity equations for the electron and positive-ion currents are:

$$\frac{1}{v_-} \left( \frac{di_-}{dt} \right) = -\frac{di_-}{dx} + \alpha i_-, \quad (1)$$

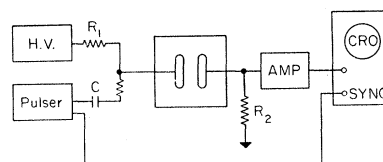


FIG. 1. Apparatus schematic.

<sup>1</sup> For a review of the situation see, for instance, *Handbuch der Physik* (Springer-Verlag, Berlin, 1956), Vols. XXI and XXII.

<sup>2</sup> L. H. Fisher and B. Bederson, *Phys. Rev.* **81**, 109 (1951).

<sup>3</sup> H. L. von Gugelberg, *Helv. Phys. Acta* **20**, 307 (1947).

<sup>4</sup> J. A. Hornbeck, *Phys. Rev.* **83**, 374 (1951).

<sup>5</sup> G. Kachikas and L. H. Fisher, *Phys. Rev.* **95**, 892 (1954).

<sup>6</sup> L. Colli, *Phys. Rev.* **95**, 892 (1954).

<sup>7</sup> W. Bartholomeyczuk, *Z. Physik* **116**, 235 (1940).

$$\frac{1}{v_+} \left( \frac{di_+}{dt} \right) = \frac{di_+}{dx} + \alpha i_- \quad (2)$$

The secondary mechanism, which creates new electrons at the cathode, and thus allows the discharge to build up, is contained in the boundary conditions. The secondary mechanism is denoted by a coefficient  $\gamma$  which gives the number of secondary electrons emitted per incident ion or photon. (This  $\gamma$  includes the effect of back diffusion.) We will treat only the cases where the secondary mechanism is either due to positive ions,  $\gamma_i$ , or where it is due to a simple delayed-photon process,  $\gamma_p$ . For  $\gamma_i$  we have

$$i_+(x=d) = 0, \quad (3)$$

$$i_-(x=0) = \gamma_i i_+(x=0). \quad (4)$$

For a delayed-photon  $\gamma$ , we assume that the photons are emitted from an excited state having a mean lifetime  $\tau$  and that they proceed unhindered, a fraction  $g$  arriving at the cathode. The number of excited states created in the discharge is given by a coefficient  $\delta$  (similar to the ionization coefficient  $\alpha$ ) giving the number of excitations per electron per cm drift. This  $\delta$  includes not only excitations by direct impact, but also excitations due to decay from higher excited levels. We then have for the cathode boundary conditions, instead of (4),

$$i_-(x=0, t) = \gamma_p \delta g \int_0^d \int_{-\infty}^t e^{-(t-t')/\tau} i_-(x, t') dt' dx. \quad (5)$$

We assume solutions exponential in time of the form  $i(x, t) = j(x)e^{\lambda t}$ , where  $\lambda$  is the growth constant of the discharge. Solutions have been published which do not make this simple assumption, but instead consider the initial conditions under which the buildup is started.<sup>8</sup>

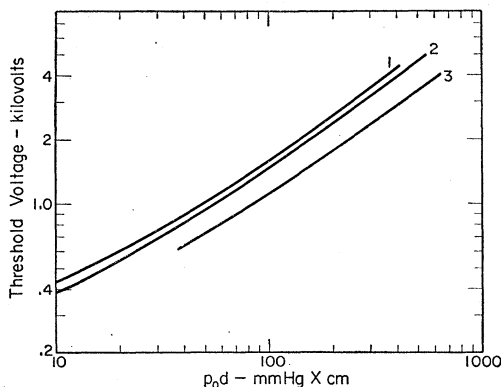


FIG. 2. Paschen curves for argon. Threshold breakdown voltage vs product of pressure and gap spacing. Pressures are reduced to 0°C. Curve 1—results of Penning and Addink; curve 2—this work; curve 3—results of Kachikas and Fisher.

<sup>8</sup> P. L. Auer, Phys. Rev. **111**, 671 (1958); **98**, 320 (1955); **101**, 1243 (1956). Dutton, Haydon, Jones, and Davidson, Brit. J. Appl. Phys. **4**, 170 (1953). P. M. Davidson, Phys. Rev. **99**, 1072 (1955); **103**, 897 (1956); H. W. Bandel, Phys. Rev. **95**, 1117 (1954).

These detailed solutions, however, are mainly of interest during the initial stages of the buildup; at later times these solutions tend to reduce to simple exponentials. Also, the experimentally observed gap currents showed an exponential increase in time as long as space charge effects were negligible.

Substituting these solutions, we obtain time-independent equations:

$$\frac{dj_-}{dx} = -\frac{\lambda}{v_-} j_- + \alpha j_-, \quad (6)$$

$$\frac{dj_+}{dx} = -\frac{\lambda}{v_+} j_+ - \alpha j_-, \quad (7)$$

$$j_+(d) = 0, \quad (8)$$

$$j_-(0) = \gamma_i j_+, \quad (9)$$

$$j_-(0) = \frac{\gamma_p \delta g}{1 + \lambda \tau} \int_0^d j_-(x) dx. \quad (10)$$

Equation (9) is the cathode boundary condition for a positive-ion  $\gamma$ , while (10) is for the case of a delayed-photon  $\gamma$ . Integrating (6) and (7) and satisfying the appropriate boundary conditions, we obtain for a positive-ion  $\gamma$ :

$$1 = \frac{\gamma_i \alpha}{\alpha - \lambda/v} [e^{(\alpha - \lambda/v)d} - 1], \quad (11)$$

where  $1/v = 1/v_+ + 1/v_-$ .

For a delayed-photon  $\gamma$ :

$$1 = \frac{\gamma_p \delta g}{(1 + \lambda \tau)(\alpha - \lambda/v_-)} [e^{(\alpha - \lambda/v_-)d} - 1]. \quad (12)$$

Setting  $\tau = 0$  gives the case of an undelayed-photon  $\gamma$ .

Equations (11) and (12) are implicit equations giving the rate of buildup  $\lambda$  in terms of the parameters of the discharge.

#### EXPERIMENTAL APPARATUS

The experimental apparatus consisted of the discharge chamber and associated vacuum system, of a voltage supply and a voltage step generator for initiating the discharge, and of an amplifier and oscilloscope for observing the gap current. A block diagram is shown in Fig. 1. A collimated photomultiplier was also used to observe the light emission in the gap, but its usefulness turned out to be limited because of insufficient sensitivity.

The discharge chamber had an internal diameter of six inches. It was made of stainless steel and was assembled with gold wire gaskets. The electrodes were introduced through glass to metal seals. The anode could be moved by means of a micrometer and bellows arrangement to change the gap spacing. The cathode was made from OFHC copper, the anode was nickel.

Both electrodes had an overall diameter of 3.5 in.; the faces were rounded to a radius of 12 in.; the edges had a radius of 0.6 in.

Both chamber and vacuum system could be baked at 350°C. The residual pressure after baking was  $3 \times 10^{-9}$  mm Hg with a rate of rise upon being shut off from the pump of  $1 \times 10^{-9}$  mm Hg/min.

The gas was Airco reagent-grade argon. It was admitted to the chamber through a metal system with packless valves and through a liquid nitrogen trap. The gas pressure was measured through the intermediary of a metal diaphragm comparison manometer.<sup>9</sup>

The discharge was initiated by suddenly applying the required voltage to the gap. Because of practical considerations only a fraction of the full gap voltage was switched by the step generator. The remainder of the gap voltage was continuously applied from the approach voltage supply. The step generator pulse rose to within two volts of its steady value in five microseconds and then showed a slow rise of two volts per kilovolt per 100 microseconds. The RC circuit coupling the step generator to the gap had  $R = 10^7$  ohms and  $C = 0.03$  mf. A resistor of 1170 ohms was in series with the coupling capacitor to reduce the energy released in the gap during breakdown.

The gap current was observed by its voltage drop across a resistor in series with the anode. This resistor ranged from 50 to 1000 ohms, the lower value being used to investigate the more rapid buildup preceding breakdown. The amplifier was of a nonoverloading type because of the sharp spike due to the step voltage coupling through the gap capacity. A differential amplifier together with a dummy gap capacity was also used, but did not give any significant improvement.

### EXPERIMENTAL RESULTS

Figure 2 shows a comparison of the observed threshold voltages with those obtained by Penning and

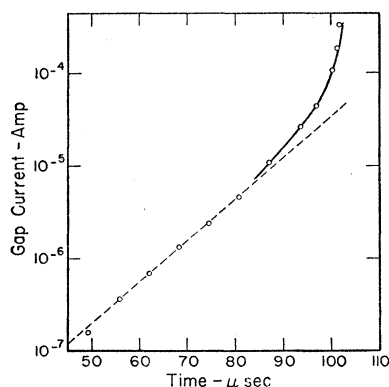


FIG. 3. Buildup of gap current for  $p = 181$  mm Hg and  $d = 0.99$  cm. Open circles are experimental points obtained from oscillogram. Solid curve shows result of numerical calculation of current buildup including the effects of space-charge field distortion.

<sup>9</sup> Alpert, Matland, and McCoubrey, Rev. Sci. Instr. 22, 370 (1951).

Addink<sup>10</sup> and Kachikas and Fisher.<sup>5</sup> The rather large difference between the threshold data of this experiment and that of Kachikas and Fisher is not understood. Because of this difference, no comparison of the buildup rates observed in this experiment with their formative time lags is attempted. Figure 3 shows a plot of gap current  $vs$  time after initiation obtained oscillographically. The current buildup is closely exponential except during the final stage where it becomes accelerated. This final increase in the rate of buildup will be discussed later.

The rate of buildup  $\lambda$  obtained from plots such as Fig. 3 is shown in Figs. 4–8 as a function of overvoltage above threshold (in volts) for several gap spacings and gas pressures ranging from 5 cm Hg to about 60 cm Hg.<sup>11</sup> Figure 9 shows a plot of observed  $\lambda$   $vs$  overvoltage together with calculated curves (a) for a positive-ion  $\gamma$ , (b) for an undelayed-photon  $\gamma$ , and (c) for a delayed-

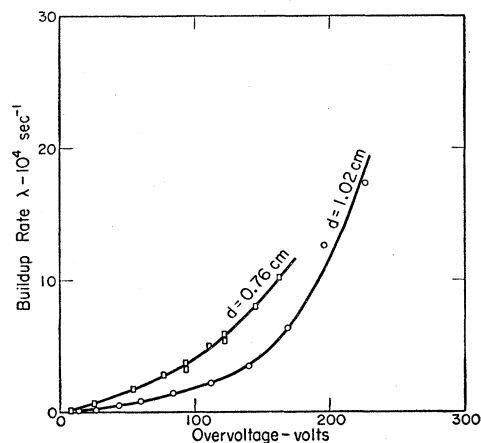


FIG. 4. Observed buildup rates  $vs$  overvoltage. Argon at 47.7 mm Hg. Gap spacings as shown.

photon  $\gamma$  with a delay time of 6 microseconds. Curve (1) was calculated from Eq. (11), curves (2) and (3) were calculated from Eq. (12). The various quantities entering into these equations were obtained as follows: positive-ion velocity  $v_+$  from Biondi and Chanin,<sup>12</sup> electron velocity  $v_-$  from extrapolated data of Nielsen,<sup>13</sup> multiplication coefficient  $\alpha$  from Druyvesteyn and Penning,<sup>14</sup> excitation coefficient  $\delta$  from Druyvesteyn and Penning.

Most of the data could be satisfactorily fitted by the simple delayed-photon model. Only at the lowest pressure, 4.77 cm Hg, was there evidence of an additional slower mechanism becoming active. The best fit value of  $\tau$  was obtained by rearranging Eq. (12) into the following form:

<sup>10</sup> F. M. Penning and C. C. J. Addinck, Physica 1, 1007 (1934).

<sup>11</sup> Gas pressures are reduced to 0°C.

<sup>12</sup> M. A. Biondi and L. M. Chanin, Phys. Rev. 94, 910 (1954).

<sup>13</sup> R. A. Nielsen, Phys. Rev. 50, 950 (1936).

<sup>14</sup> M. J. Druyvesteyn and F. M. Penning, Revs. Modern Phys. 12, 87 (1940).

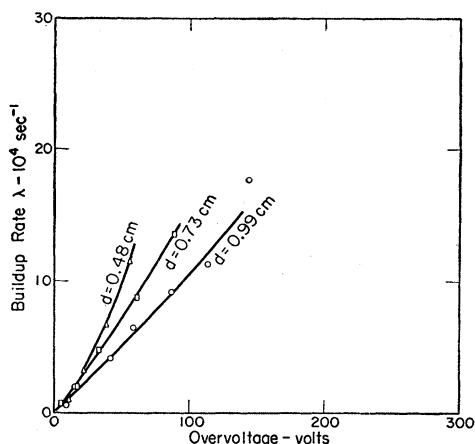


FIG. 5. Observed buildup rates vs overvoltage. Argon at 91 mm Hg. Gap spacings as shown.

$$-\lambda = \frac{1}{\tau} \left( \frac{\gamma_p}{\tau} \right) g \left[ \frac{\delta}{\alpha - \lambda/v_-} (e^{(\alpha - \lambda/v_-)d} - 1) \right]. \quad (13)$$

The quantity in square brackets was computed for each voltage and value of  $\lambda$ . If one then plots the observed value of  $\lambda$  against the quantity in square brackets, a straight line should result having intercepts  $1/\tau$  and  $1/g\gamma_p$ . Figure 10 shows such a plot for a gas pressure of 47.7 mm Hg and three gap spacings.

The values of the delay time  $\tau$  which give the best fit of the data are shown in Table I for the various gap spacings and gas pressures.

DISCUSSION

In the spectrum of atomic argon, only resonance photons have sufficient energy to emit electrons from the cathode. Because of imprisonment the only resonance photons likely to reach the cathode will be the 1048 Å and the 1063 Å lines. (Higher excited states will in general decay to one of the 4S states by emission of an intercombination photon.) Phelps has considered

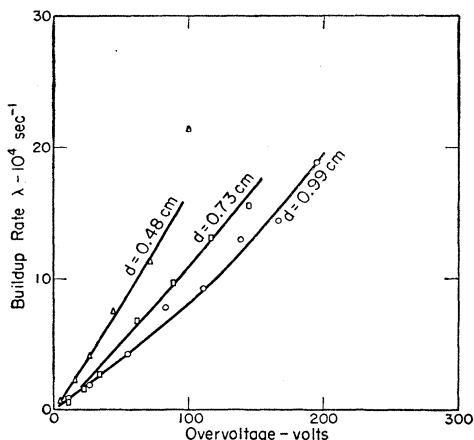


FIG. 6. Observed buildup rates vs overvoltage. Argon at 181 mm Hg. Gap spacings as shown.

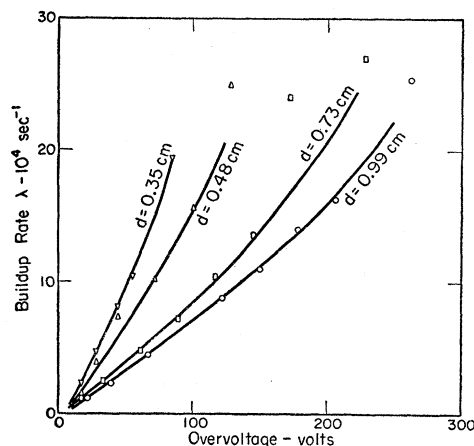


FIG. 7. Observed buildup rates vs overvoltage. Argon at 273 mm Hg. Gap spacings as shown.

TABLE I. Observed and calculated photon delay times in microseconds.

p mm Hg	Gap spacing, cm					Calc. delay colli process	
	0.48	0.63	0.73	0.99	1.52	1/11p <sup>2</sup>	Excited A <sub>2</sub> decay
47.7		5.0	6.0	6.7	40	3.3	
91	5.2	5.2	7.0	11	3.3		
181	4.4	6.0	6.7	2.8	3.3		
273	3.8	5.5	5.5	1.2	3.3		
548	3.0	3.0	3.0	0.3	3.3		
Calculated imprisonment times for resonance radiation							
1067 Å	9.0	10.0	11.0	13.0	16.0		
1048 Å	2.2	2.5	2.7	3.2	4.0		

the effect of imprisonment of resonance radiation on the buildup of a discharge.<sup>15</sup> He has shown that the effect of imprisonment upon the rate of buildup can be described closely by a relation of the form of Eq. (12),

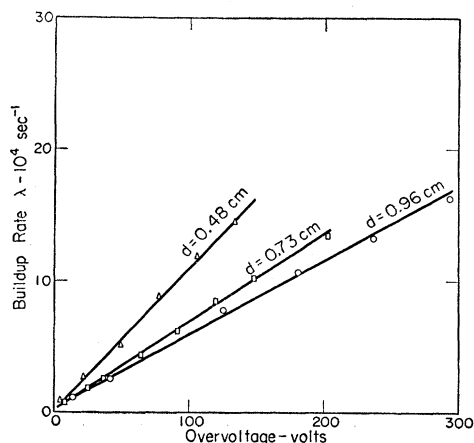


FIG. 8. Observed buildup rates vs overvoltage. Argon at 548 mm Hg. Gap spacings as shown.

<sup>15</sup> A. V. Phelps, Bull. Am. Phys. Soc. Ser. II, 4, 115 (1959); also Phys. Rev. (to be published).

where  $\tau$  is now an effective imprisonment time. Under conditions where the imprisonment is controlled by dipolar pressure broadening, the imprisonment time is independent of gas pressure and varies as the square root of the gap spacing.<sup>15</sup> Fursov, Organov, and Striganov<sup>16</sup> estimate the transition probability of the 1048 Å line to be  $10^9 \text{ sec}^{-1}$ . Calculations by Knox<sup>17</sup> give values for the transition probability of  $4 \times 10^8 \text{ sec}^{-1}$  for the 1048 Å line and  $1 \times 10^8 \text{ sec}^{-1}$  for the 1067 Å line. Using the values of Knox with the analysis of Phelps, one obtains the imprisonment times listed in Table I. Considering the uncertainties in the calculations and the lack of knowledge about the spectral energy distribution between the 1048 Å and the 1067 Å lines, the values of imprisonment times shown are consistent with the observed data.

The mechanism proposed by Colli<sup>6</sup> is another means by which radiation of sufficient energy to emit electrons can reach the cathode. This is a two-step mechanism:

- (a)  $A(\text{metastable}) + 2 A(\text{atom})$   
 $= A_2(\text{excited}) + A(\text{atom});$
- (b)  $A_2(\text{excited}) = 2 A(\text{atom}) + \text{photon}.$

The extra atom is necessary in the first process to take up the excess energy and stabilize the molecule; the rate was determined by Colli to be: rate =  $11p^2$ , where  $p$  is the gas pressure in mm Hg. Phelps and Molnar<sup>18</sup> and Futch and Grant<sup>19</sup> found a  $p^2$  term in the destruction rate of the  $^3P_2$  metastable state which they attributed to this process; the rates they obtained were, respectively,  $12p^2$  and  $16.3p^2$ . The value of Colli is used since it was obtained from the observed delayed light

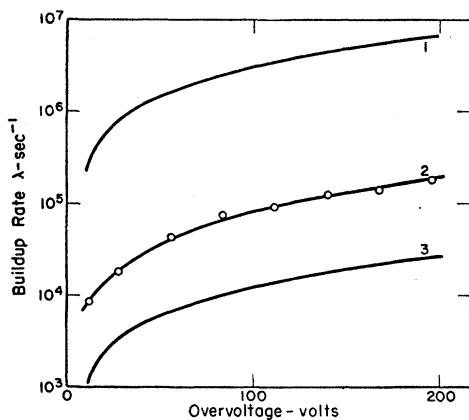


FIG. 9. Discharge buildup rate vs overvoltage for  $p=181$  mm Hg and  $d=0.99$  cm. Points are observed values. Curve 1 is computed for an undelayed-photon secondary mechanism. Curve 2 is computed for a delayed-photon secondary mechanism having a delay time of 6 microseconds. Curve 3 is computed for a positive-ion secondary mechanism.

<sup>16</sup> Fursov, Organov, and Striganov, Doklady Akad. Nauk, S.S.S.R. **101**, 453 (1955).

<sup>17</sup> R. S. Knox, Phys. Rev. **110**, 375 (1958).

<sup>18</sup> A. V. Phelps and J. P. Molnar, Phys. Rev. **89**, 1202 (1953).

<sup>19</sup> A. J. Futch and F. A. Grant, Phys. Rev. **104**, 356 (1956).

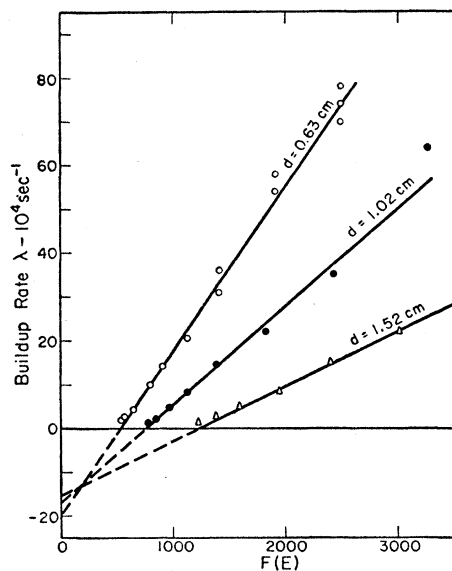


FIG. 10. Observed buildup rate vs  $F(E)$  for  $p=47.7$  mm Hg; where  $F(E) = [\delta/(\alpha - \lambda/v_-)] [\exp(\alpha - \lambda/v_-)d - 1]$ . Photon delay time  $\tau$  is the reciprocal of the vertical intercept.

emission. The second process is the decay of the excited molecular state with emission of a photon of approximately 1250 Å; the lifetime is pressure independent and is approximately 3.3 microseconds.

At high pressures the three-body collision process occurs rapidly enough so that the delay in the light emission is due to the lifetime of the molecular state—3.3 microseconds. This is in fair agreement with the observed data at higher pressures, though there is a suggestion of a dependence on the gap spacing which would not be explainable by this mechanism.

At the lower pressures the formation of the molecule by three-body collisions becomes the slower and thus the controlling process.<sup>20</sup> For  $p_0=47.7$  mm Hg, the delay time  $\tau=1/11p^2=40$  microseconds. As mentioned, at that pressure the observed buildup rate appears to be controlled by two different secondary mechanisms. The dominant one has a time constant,  $\tau$ , of 6 microseconds and contributes about 80% of the threshold secondary yield,  $\gamma$ , for a spacing of 1 cm. The slower one has an estimated time constant  $\tau$  of the order of 20 microseconds. It is not possible to decide whether this slow secondary mechanism is due to the Colli process or to a positive-ion  $\gamma$  since both these processes have comparable speeds at that pressure. It appears fairly certain, however, that the dominant secondary process, having a  $\tau$  of 6 microseconds, is not due to the delayed-photon mechanism of Colli.

### The Later Stages of the Buildup

At gap currents larger than  $10^{-5}$  ampere the observed discharge buildup is no longer exponential, but becomes

<sup>20</sup> In the intermediate region where both times are comparable an equation somewhat more complex than Eq. (12) will hold.

accelerated. This has been generally attributed to space charge distortion of the gap field leading to a greater electron multiplication. As a check on this hypothesis we computed the discharge buildup by numerically integrating the charge carrier equations, taking into account the distortion of the gap field by the accumulated space charge. The numerical computation was initiated from a gap current of  $5 \times 10^{-6}$  amp. At this value the current buildup is still exponential, and the charge distribution at that point was taken accordingly. The secondary mechanism assumed was a delayed-photon mechanism using the observed value of photon delay.

Figure 3 shows the results of this computation compared with the actual gap current buildup. Since the

computation depends upon the current density, while the total gap current is observed, the effective discharge area remains as an open parameter. The computed curve shown is for an effective discharge area of  $16 \text{ cm}^2$  which agrees quite well with the electrode diameter.

It thus appears that the acceleration of the discharge buildup up to currents of the order of  $10^{-3}$  amp can be explained by the space charge distortion of the gap field.

#### ACKNOWLEDGMENT

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## Transport Phenomena in Slightly Ionized Gases: Low Electric Fields\*

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Chapman and Cowling have obtained the velocity distribution function of electrons in a slightly ionized gas when an electric field  $E$  is applied by assuming elastic collisions and validity of Lorentzian approximation. The author has expanded this distribution function, neglecting terms involving fourth and higher powers of field and used this expansion to calculate various transport properties, when a magnetic field is also applied. To simplify mathematics the relaxation time has been taken as a power function of electron velocity. Some approximations have also been discussed. It is seen that at low electric fields the transport properties can be expressed as linear functions of  $E^2$ .

### INTRODUCTION

IN discussions of transport phenomena a slightly ionized gas is usually considered as a Lorentzian gas consisting of a large number of neutral molecules and a small number of electrons. This treatment neglects the part played by ions in transport phenomena, which is justifiable on account of their heavy mass and small number. Electron-electron collisions are also neglected because of their small number.

Numerous theoretical investigations (for partial listing see Loeb<sup>1</sup>) of the velocity distribution of electrons in the presence of an electric field have been carried out. Most of these treatments assume the collisions between electrons and molecules to be elastic; the remaining ones are at the least very difficult to handle mathematically in discussing transport properties. However, detailed analysis of transport phenomena has been made only by assuming a Maxwellian distribution of electron velocities in the case of vanishingly small electric fields.

In this communication the author has expanded the distribution function, obtained by Chapman and

Cowling,<sup>2</sup> neglecting terms involving fourth and higher powers of electric field. This expansion has been used to investigate various transport phenomena in the presence of a magnetic field. The analysis is valid in a good range, where useful data can be obtained. Results have been given in the case when the relaxation time  $\tau \propto x^n$ ,  $x$  being the dimensionless electron velocity.

### VELOCITY DISTRIBUTION FUNCTION AT LOW ELECTRIC FIELDS

Chapman and Cowling<sup>2</sup> have derived the distribution function of electrons in the presence of an electric field, which may be written as

$$f(\mathbf{c}_2) = f_0(c_2) + c_x f_1(c_2), \quad (1A)$$

where

$$f_0(c_2) = A \exp\left(-\int \frac{m_2 c_2 dc_2}{kT + m_1 F_2^2 v^2 / 3c_2^2}\right), \quad (1B)$$

$$f_1(c_2) = (F_2/c_2^2)(\partial f_0/\partial c_2), \quad (1C)$$

$\mathbf{c}_2$  is the electron velocity,  $c_x$  is the  $x$  component of  $c_2$ ,  $m_2$  and  $m_1$  are the electronic and atomic masses,

\* Work supported by Armour Research Foundation.

<sup>1</sup> L. B. Loeb, *Basic Processes of Gaseous Electronics* (University of California Press, Berkeley, 1955), p. 270.

<sup>2</sup> S. Chapman and T. G. Cowling, *The Mathematical Theory of Nonuniform Gases* (University Press, Cambridge, England, 1953), p. 350.