Electromagnetic Corrections to Isotopic Spin Conservation*

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If electromagnetic interactions are wholly responsible for all departures from isotopic spin invariance, then the strict conservation law $\Delta T=0$ may be replaced, to order e^2, by the rule $|\Delta T|\leqslant 2$. Consequences of this weaker restriction are discussed for elementary particle masses, scattering processes, and weak-interaction decay processes. The apparent absence in nature of particles with isotopic spins greater than one makes it difficult to find very practical experimental tests of this rule.

I. INTRODUCTION

SMALL violations of isotopic spin invariance in phenomena dominated by strong interactions are a familiar matter. They show up most clearly in the small mass differences within a single isotopic multiplet of elementary particles, and also in the observation of certain reactions, such as $O^{16}(d,\alpha)N^{14*}(T=1)$,[1] which would be forbidden if isotopic spin were strictly observed. The most widely held view seems to be that electromagnetism is the sole agency producing these deviations from strict isotopic spin invariance. Alternate possibilities have, however, been raised from time to time, most recently in an interesting series of papers by Pais.[2]

Electromagnetism is a very reasonable suspect. Indeed, whatever else also violates isotopic spin invariance, we expect the lowest-order electromagnetic process, emission, and reabsorption of a virtual photon, to give corrections of about the right order of magnitude, $\sim e^2$ (where we take $\hbar=c=1$ throughout). Beyond this, our inability to carry out really quantitative calculations involving strong interactions has so far prevented a direct test of these views.

What we remark in the present note is the following. To lowest order in e^2, electromagnetism does not completely destroy isotopic symmetry, i.e., the single photon picture described above has certain exact consequences which are independent of any detailed calculations involving the strong interactions. The reasoning is very simple. Emission or absorption of a virtual photon leads to change of isotopic spin by at most one unit. Therefore, to order e^2, strict isotopic spin conservation is replaced by $|\Delta T|\leqslant 2$. This represents a serious but not quite a total destruction of isotopic spin symmetry; in principle the surviving restriction is testable. We have explored the consequences of this weaker principle for particle masses, scattering, and decay processes. As it turns out, they are not impressive from a practical point of view, owing chiefly to the regrettable absence on the current scene of particle multiplets with $T>1$.

II. MASS DIFFERENCES

Consider a multiplet of particles with total isotopic spin T, and let the masses of the $2T+1$ charge states be denoted $M(T_3)$. For all known multiplets, these masses are equal to within a few Mev, reflecting the approximate conservation of isotopic spin. If the small variations in $M(T_3)$ arise from emission and reabsorption of a single virtual photon, then it follows from the $|\Delta T|\leqslant 2$ rule discussed above that the masses must lie on a parabola,

$$M(T_3)=\alpha T_3{}^2+\beta T_3+\gamma. \tag{1}$$

(This will be proven in greater detail below.) This relation, of course, has no significance for multiplets with $T\leqslant 1$, since any set of three or fewer points may always be joined by a parabola. Unfortunately there are no known elementary particles with $T\geq\frac{3}{2}$, so that we are unable at present to test Eq. (1) in particle physics. A similar relation also holds for light complex nucleii (to order Z^2e^2) but here verification of Eq. (1) would constitute a much weaker case for the purely electromagnetic origin of isotopic spin violations. Nucleii are such comparatively open structures that in any case one expects the main contributions to nuclear multiplet "tilt" to come from Coulomb forces and from the $n-p$ mass difference.[3] Still, it would be interesting to test Eq. (1) with nuclear quadruplets such as N^{17}, O^{17*}, F^{17*}, Ne^{17}, and B^{13}, C^{13*}, N^{13*}, O^{13}, the end members of which are stable against particle emission.[4]

Although we cannot verify Eq. (1) for the known particles, we can use their measured mass differences to obtain the parameters α, β for each multiplet. In the absence of conclusive field-theoretic calculations of

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[1] C. P. Browne, Phys. Rev. **104**, 1598 (1955).

[2] A. Pais, Phys. Rev. **112**, 624 (1958); **110**, 574 (1958); **110**, 1480 (1958).

[3] Formula (1) was derived and discussed for nuclear electrostatic energies by W. M. Macdonald, Phys. Rev. **98**, 60 (1955); **100**, 51 (1955); **101**, 271 (1956).

[4] This point has been particularly emphasized by E. P. Wigner, Proceedings of the Robert A. Welch Foundation Conferences on Chemical Research, 1957 (unpublished), Vol. I, Chap. IV, p. 86.

electromagnetic mass shifts, it will perhaps be useful to perform an isotopic spin analysis, sorting out the way different virtual processes contribute to α and β.

To order e^2, elementary perturbation theory leads to the following expression for the masses $M_a(T_3)$ of particles of a multiplet a with unperturbed mass M_a:

$$M_a(T_3)=M_a+(2\pi)^3\langle a,T_3|H_2(0)|a,T_3\rangle - (2\pi)^6\sum_n \langle a,T_3|H_1(0)|n\rangle \times\langle n|H_1(0)|a_1T_3\rangle/E_n-M_a, \quad (2)$$

where the electromagnetic Hamiltonian density is

$$H(x)=H_1(x)+H_2(x),$$
$$H_1(x)=-j_\mu(x)A^\mu(x), \quad (3)$$
$$H_2(x)=-h_{\mu\nu}(x)A^\mu(x)A^\nu(x). \quad (4)$$

Here $j_\mu(x)$ is the first-order electromagnetic current, and $h_{\mu\nu}(x)$ is determined by the condition that to order e^2 the current is

$$J_\mu(x)=j_\mu(x)+2h_{\mu\nu}(x)A^\nu(x). \quad (5)$$

For example, charged spinless bosons interacting without derivative coupling contribute to j_μ and $h_{\mu\nu}$ the terms

$$j_\mu{}^{(\mathrm{bos})}=-ie(\phi\partial_\mu\phi^*-\phi^*\partial_\mu\phi), \quad (6)$$
$$h_{\mu\nu}{}^{(\mathrm{bos})}=-e^2g_{\mu\nu}\phi^*\phi. \quad (7)$$

(Strictly speaking, we should subtract from the expression for $M_a(T_3)$ the second order electromagnetic perturbation to the vacuum self-energy. Since we will only be concerned with mass differences this will be ignored.)

The sum in Eq. (2) runs over a complete set of intermediate states $|n\rangle$ each containing a photon of momentum $\mathbf{k}$ and a set of strongly interacting particles labelled by the symbol b. Then

$$M_a(T_3)=M_a-\langle a,T_3|h_\mu{}^\mu(0)|a,T_3\rangle\int\frac{d^3k}{2k}-(2\pi)^3\int\frac{d^3k}{2k}\sum_b\frac{\langle a,T_3|j_\mu(0)|b,T_3\rangle\langle b,T_3|j^\mu(0)|a,T_3\rangle\delta^3(\mathbf{p}_b+\mathbf{k})}{E_b+k-M_a}. \quad (8)$$

All field operators here obey the Heisenberg representation equations of motion determined by the strong interactions alone. One supposes, of course, that some sort of cutoff renders these integrals convergent.

Since the current j_μ is proportional to e it possesses the transformation property[5] of a mixed isoscalar $j_\mu{}^S$ and isovector $j_\mu{}^V$, so that by the Wigner-Eckart theorem,

$$\begin{aligned}\langle b,T_3|j_\mu|a,T_3\rangle&=\langle b\|j_\mu{}^S\|a\rangle+T_3\langle b\|j_\mu{}^V\|a\rangle, \quad T_a=T_b\\ &=[(T_a+1)^2-T_3{}^2]^{\frac{1}{2}}\langle b\|j_\mu{}^V\|a\rangle, \quad T_a=T_b+1 \quad (9)\\ &=[T_a{}^2-T_3{}^2]^{\frac{1}{2}}\langle b\|j_\mu{}^V\|a\rangle, \quad T_a=T_b-1\\ &=0, \quad |T_a-T_b|>1.\end{aligned}$$

On the other hand, $h_\mu{}^\mu$ is proportional to e^2, and thus transforms as a mixed isoscalar h^S, isovector h^V, and isotensor h^T, so that

$$\langle a,T_3|h_\mu{}^\mu|a,T_3\rangle=\langle a\|h^S\|a\rangle+T_3\langle a\|h^V\|a\rangle +[T_3{}^2-\tfrac{1}{3}T_a(T_a+1)]\langle a\|h^T\|a\rangle. \quad (10)$$

Inserting these results into Eq. (8), we obtain the expression for $M_a(T_3)$ given by Eq. (1), with parameters

$$\alpha=-\langle a\|h^T\|a\rangle\int\frac{d^3k}{2k}+(2\pi)^3\int\frac{d^3k}{2k}\sum_b\frac{\delta^3(\mathbf{p}_b+\mathbf{k})}{E_b+k-M_a}\times\{|\langle b\|j_0{}^V\|a\rangle|^2-|\langle b\|\mathbf{j}^V\|a\rangle|^2\}\times\{\delta_{T_b,T_a}-\delta_{T_b,T_a-1}-\delta_{T_b,T_a+1}\}, \quad (11)$$

$$\beta=-\langle a\|h^V\|a\rangle\int\frac{d^3k}{2k}+(2\pi)^3\int\frac{d^3k}{2k}\sum_b\frac{\delta^3(\mathbf{p}_b+\mathbf{k})}{E_b+k-M_a}\times 2\,\mathrm{Re}\{\langle b\|j_0{}^V\|a\rangle\langle b\|j_0{}^S\|a\rangle^* -\langle b\|\mathbf{j}^V\|a\rangle\cdot\langle b\|\mathbf{j}^S\|a\rangle^*\}\delta_{T_bT_a}, \quad (12)$$

$$\gamma=M_a-\{\langle a\|h^S\|a\rangle-\tfrac{1}{3}T_a(T_a+1)\langle a\|h^T\|a\rangle\}\times\int\frac{d^3k}{2k}+(2\pi)^3\int\frac{d^3k}{2k}\sum_b\frac{\delta(\mathbf{p}_b+\mathbf{k})}{E_b+k-M_a}\times\{|\langle b\|j_0{}^S\|a\rangle|^2-|\langle b\|\mathbf{j}^S\|a\rangle|^2\}\delta_{T_bT_a}. \quad (13)$$

We observe that the coefficient β receives contributions from intermediate states b with $T_b=T_a$ only. Moreover the pion pair contribution to h^V and hence to β, vanishes; since the pion field transforms as a pure isovector, the squared field in Eq. (7) transforms as a mixture of isoscalar and isotensor only. Thus the coefficients,[6]

$$\beta_N=m(p)-m(n)=-1.3 \text{ Mev},$$
$$\beta_K=m(K^\pm)-m(K^0)=-3.7\pm0.7 \text{ Mev},$$
$$\beta_\Sigma=\tfrac{1}{2}[m(\Sigma^+)-m(\Sigma^-)]=-3.2\pm0.2 \text{ Mev}$$

receive contributions only from intermediate states b with $T_b=\frac{1}{2}$, $\frac{1}{2}$, and 1, respectively, and from the K-meson pair contribution to $h_{\mu\nu}$ ($\beta_\pi=0$ by CPT invariance).

The coefficient α can be determined in isotopic

[5] This fact was used to derive a limitation on the magnetic moments of the Σ particles by Marshak, Okubo, and Sudarshan, Phys. Rev. **106**, 599 (1957).

[6] For K-meson, Rosenfeld, Solmitz, and Tripp, Phys. Rev. Letters **2**, 110 (1959); Crawford, Crest, Good, Stevenson, and Ticho, Phys. Rev. Letters **2**, 112 (1959). For other masses, see M. Gell-Mann and A. H. Rosenfeld, *Annual Review of Nuclear Science* (Annual Reviews Inc., Palo Alto, 1957), Vo. 7.

triplets, and takes the values[6]

$\alpha_\pi = m(\pi^\pm) - m(\pi^0) = 4.6$ Mev,

$\alpha_\Sigma = \frac{1}{2}(m(\Sigma^+)+m(\Sigma^-)) - m(\Sigma^0) = 3.0_{-1.2}^{+2.0}$ Mev,

From Eq. (11) we see that α receives contributions from intermediate states b with $T_b=0, 1, 2$ and from the pion-pair contribution to h^T. (The K-meson field has isospin $\frac{1}{2}$, so that its square in Eq. (7), transforms as a mixed isoscalar and isovector, and hence cannot contribute to h^T.) The algebraic sign of the first of these contributions is a product of three signs:

(i) A + (−) sign if the main contribution arises from states b with $T_b=1$ ($T_b=0$ or $T_b=2$).

(ii) A + (−) sign if the main contribution arises from timelike (spacelike) photons, interacting with $J_0(\mathbf{j})$.

(iii) A + (−) sign if the main contribution arises from intermediate states with $E_b+k>M_a$ ($<M_a$). In multiplets stable against single γ-decay, such as the pion triplet, only $E_b+k>M_a$ is possible. For the Σ triplet, E_b+k can be as small as $m(\Lambda^0)$.

As for the pion-pair contribution, we can again insert a sum over states $|b,T_3\rangle$, obtaining

$$-\langle a\|h^T\|a\rangle = e^2\,\Sigma_c |\langle c\|\phi_\pi\|a\rangle|^2 \times(\delta_{T_a,T_c}-\delta_{T_a,T_c+1}-\delta_{T_a,T_c-1}), \quad (14)$$

which contributes a positive (negative) term to α if the main contribution comes from states c with $T_c=1$ ($T_c=0$ or 2).

All in all, it seems rather more reasonable than not that in fact α is positive for the known cases. These considerations do not provide any hint, however, regarding the sign of β.

III. SCATTERING AND DECAY PROCESSES

We turn now to the effects of electromagnetism in strong scattering processes. Let us consider a process $a\to b$, where a and b are sets of strongly interacting particles, with total isotopic spin T_a and T_b, respectively. Since isotopic spin is not precisely conserved, it is of course not necessarily true that $T_a=T_b$ (but T_3 is conserved—we are neglecting weak interactions). In the most general case, the S matrix then has the form

$$S=S_0^0+S_1^0+S_2^0+S_3^0+\cdots, \quad (15)$$

where S_T^0 transforms under isotopic spin rotations like the $T_3=0$ component of a spherical tensor of rank T; by the Wigner-Eckart theorem,

$$\langle b,T_3'|S_T^0|a,T_3\rangle = \delta_{T_3'T_3}C_{TT_a}(T_bT_3;0T_3)\langle b\|S_T\|a\rangle, \quad (16)$$

where C_{TT_a} is the usual Clebsch-Gordan coefficient.[7] In particular, the matrix element of S_T^0 vanishes unless

$$|T_a-T_b|\leqslant T\leqslant T_a+T_b.$$

The dominant term in Eq. (15) is of course S_0^0. As for the remaining terms, their relative importance depends on the mechanism that produces violations of isotopic spin conservation. If it is electromagnetism, then to order e^2 only the S_1^0 and S_2^0 terms can contribute, so that violation of isotopic spin conservation cannot occur in an entirely arbitrary way. Indeed, using the orthogonality properties of the Clebsch-Gordan coefficients, we see from (15) that

$$\sum_{T_3} C_{TT_a}(T_bT_3;0T_3)\langle b,T_3|S|a,T_3\rangle=0. \quad \text{for } T>2. \quad (17)$$

If $T_a+T_b\leqslant 2$, Eq. (17) tells us nothing (except that 0=0). If $T_a+T_b>2$, (17) is a set of T_a+T_b-2 relations among the scattering amplitudes. (A more detailed proof may easily be constructed using perturbation theory as in Sec. II.)

We mention first an entirely academic example. For pion-nucleon scattering, Eq. (17) provides a restriction on the isotopic spin $\frac{3}{2}$ amplitudes

$$A_{\frac{3}{2}}(T_3)=\langle\pi N,\,T=\tfrac{3}{2},\,T_3|S|\pi N,\,T=\tfrac{3}{2},\,T_3\rangle:$$
$$-A_{\frac{3}{2}}(\tfrac{3}{2})+3A_{\frac{3}{2}}(\tfrac{1}{2})-3A_{\frac{3}{2}}(-\tfrac{1}{2})+A_{\frac{3}{2}}(-\tfrac{3}{2})=0. \quad (18)$$

In terms of individual processes:

$$\begin{aligned}&-A(\pi^++p\to\pi^++p)+2A(\pi^0+p\to\pi^0+p)\\&+\sqrt{2}A(\pi^0+p\to\pi^++n)+\sqrt{2}A(\pi^++n\to\pi^0+p)\\&+A(\pi^++n\to\pi^++n)-A(\pi^-+p\to\pi^-+p)\\&-\sqrt{2}A(\pi^0+n\to\pi^-+p)-\sqrt{2}A(\pi^-+p\to\pi^0+n)\\&-2A(\pi^0+n\to\pi^0+n)+A(\pi^-+n\to\pi^-+n)=0. \quad (19)\end{aligned}$$

The relations (17), of which (19) is an example, require some further comment. They connect amplitudes for different scattering processes, involving particles with slightly different masses. Thus, if $A(\pi^++p\to\pi^++p)$ in (19) refers to particles with momenta $\mathbf{k}$, $\mathbf{p}$, $\mathbf{k}'$ and $\mathbf{p}'$, respectively, what momenta are implied for $A(\pi^-+p\to\pi^0+n)$? The two sets of momenta cannot be precisely identical, owing to the slight mass differences within multiplets. It is clear, however, what must be done. In general, we deal with a set of processes

$$a_1,t_1+a_2,t_2+\cdots+a_m,t_m\to n_1,t_1'+b_2,t_2'+\cdots+b_n,t_n',$$

where $a_1\cdots b_n$ denote particle multiplets; and $t_1,\cdots,t_n'$ are the values of T_3 for the individual particles. We compare processes where $t_1,\cdots,t_n'$ run over all possible values consistent with T_3 conservation. To ensure the validity of Eqs. (13), the correct prescription is this: The momenta of all of the particles are to be chosen as smooth functions of the various masses $M_a(t)$, $M_b(t')$ subject to energy-momentum conservation. For example, fix the energies of all but two of the particles and the directions of motion of all but one. The remaining direction and energies are then fixed by energy-momentum conservation in each case. Now as we compare one process with another, the variations of the

[7] For notation, see J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952), Appendix A.

dependent variables are equivalent to what we would obtain by including mass shift terms in the perturbation Hamiltonian. In effect, we acquire a correction term δS_{mass} of the same order as S_1^0 and S_2^0; but from Eq. (1) it is clear that to lowest order δS_{mass} also transforms like a mixed isoscalar, isovector, and isotensor. Hence it can be absorbed in S_1^0 and S_2^0, so that Eq. (17) remains valid. By the same argument, we can also ignore variations in phase volume in comparing one process with another, except of course near thresholds.

Let us then return to the restrictions on isotopic spin violation implied by Eq. (17). These restrictions come into play for reactions in which the final and initial total isotopic spins can sum to a value greater than two. It is not difficult to list many examples; but in most cases a test of Eqs. (17) would require hopelessly difficult experiments, such as the study of the process $\pi^0+n\rightarrow\pi^0+n$ in the example of Eq. (19). Even when this problem can be avoided, as it is in a number of examples, one encounters a second practical difficulty: Eqs. (17) relate scattering amplitudes, whereas one actually measures cross sections. In the general case, therefore, a precision phase-shift analysis would be required to determine the amplitudes; and for practical purposes this is out of the question.

A simple example of a process for which both these difficulties are avoided is the reaction $\text{He}^4+\text{He}^4\rightarrow 3\pi+\text{He}^4+\text{He}^4$ or alternatively, the decay of a $T=0$ nucleon-antinucleon state (even parity singlet, or odd parity triplet) into three pions. Suppose we denote a particular three-pion channel by the momenta $\mathbf{k}_1$, $\mathbf{k}_2$ and $\mathbf{k}_3$; and let $A(+,-,0)$ be the amplitude for producing π^+ with momentum $\mathbf{k}_1$, π^- with momentum $\mathbf{k}_2$ and π^0 with momentum $\mathbf{k}_3$. The other six amplitudes $A(-+0)$, $A(0+-)$, $A(0-+)$, $A(+0-)$, $A(-0+)$, and $A(0,0,0)$ are defined similarly. The cross sections σ are proportional to $|A|^2$. Now if charge independence were exactly satisfied, only the $T=0$ three pion state would be produced. There is just one such state and it is totally antisymmetric under permutations of the pions. We therefore write

$$\begin{aligned} A(+-0)&=A_0+\delta A(+-0),\\ A(-+0)&=-A_0+\delta A(-+0),\\ A(+0-)&=-A_0+\delta A(+0-),\\ A(-0+)&=A_0+\delta A(-0+),\\ A(0+-)&=A_0+\delta A(0+-),\\ A(0-+)&=-A_0+\delta A(0-+),\end{aligned} \tag{20}$$

where the δA are of order e^2 relative to A_0. To order e^2 then, we have

$$\begin{aligned}\sigma_a(+-0)&\equiv\sigma(+-0)-\sigma(-+0)+\sigma(-0+)\\ &\quad-\sigma(+0-)+\sigma(0+-)-\sigma(0-+)\\ &=2\,\text{Re}\{A_0^*[\delta A(+-0)+\delta A(-+0)\\ &\quad+\delta A(+0-)+\delta A(-0+)+\delta A(0+-)\\ &\quad+\delta A(0-+)]\}.\end{aligned} \tag{21}$$

But our selection rule $|\Delta T|\leqslant 2$ tells us that the $T=3$ amplitude,

$$\begin{aligned}A_3=2A(0,0,0)&+A(+-0)+A(-+0)+A(+0-)\\ &+A(-0+)+A(0+-)+A(0-+),\end{aligned} \tag{22}$$

is zero, at least to order e^2. Thus

$$\sigma_a(+-0)=-4\,\text{Re}[A_0^*A(000)], \tag{23}$$

and hence

$$\sigma_a^2(+-0)\geq 16|A_0|^2|A(000)|^2. \tag{24}$$

Finally, we have $\sigma(000)=|A(000)|^2$ which is of order e^4; and to zero order

$$\begin{aligned}\sigma_s(+-0)&\equiv\sigma(+-0)+\sigma(-+0)+\sigma(-0+)\\ &\quad+\sigma(+0-)+\sigma(0+-)+\sigma(0-+)\\ &=6|A_0|^2.\end{aligned} \tag{25}$$

Altogether then, we find to lowest order,

$$|\sigma_a(+-0)|\geq[(8/3)\sigma_s(+-0)\sigma(000)]^{\frac{1}{2}} \tag{26}$$

both sides of this inequality being of order e^2.

With particles of isotopic spin greater than one available, it would be possible to find more practical tests than the above example of the hypothesis that isotopic spin violations are solely due to electromagnetism; and unlike the above example they would involve equalities rather than inequalities.

Finally, we comment on the question of isotopic symmetries for the weak reactions. It has been suggested by a number of authors that weak nonleptonic processes obey a selection rule $|\Delta T|=\frac{1}{2}$ to a more or less good approximation. The extreme view is that this rule is violated only by electromagnetic effects. From the observed rate for the decay $K^+\rightarrow\pi^++\pi^0$ it appears that the correction terms are rather too large to be explained by electromagnetic effects alone. In any event, if electromagnetism is indeed the sole agency which destroys the $|\Delta T|=\frac{1}{2}$ selection rule, it follows that to lowest order in e^2, $|\Delta T|\geq\frac{7}{2}$ is still forbidden. This is an incredibly weak restriction, but in principle at least it would be testable in a reaction like $K\rightarrow 3\pi$. In general $|\Delta T|=\frac{7}{2}$ could contribute here. The absence of a $|\Delta T|=\frac{7}{2}$ component would imply

$$A_{K^+}(3)=(\tfrac{2}{3})^{\frac{1}{2}}A_{K_2^0}(3), \tag{27}$$

where $A_{K^+}(3)$ and $A_{K_2^0}(3)$ are the amplitudes for K^+ and K_2^0 decay, respectively, into three-pion states with $T=3$. This relation could in principle be tested,[8] but only in principle.

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[8] The time-dependent interference between the decay modes $K_1^0\rightarrow\pi^++\pi^-+\pi^0$ and $K_2^0\rightarrow\pi^++\pi^-+\pi^0$ could possibly serve to measure the interference between the $T=0$ and $T=3$ states. See S. B. Treiman and S. Weinberg, Phys. Rev. **116**, 239 (1959).