

Plausibility of a Nonlocal Optical Model*

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The implication of a simple two-kernel form of nonlocal potential is considered in the nuclear matter approximation. It is shown that this leads to a wave equation with a complex reduced mass. The parameters characterizing the real part of the optical potential are found to be in reasonable accord with expectations from two-body forces. The parameters associated with the imaginary part are handled only phenomenologically. A description is found which works quite well in the energy range 0 to 25 Mev. The results of this study compares favorably with the corresponding results of an investigation with a nonlocal model for finite nuclei.

I. INTRODUCTION

THE optical model¹ for nuclear reactions provides a good over-all description of scattering of nucleons by complex nuclei, if the effective scattering potential contains an absorptive (imaginary) term and is allowed to vary with the incident nucleon energy.

The momentum dependence of the effective nucleon-nucleus interaction potential is evident from the self-consistent treatment of the nuclear many-body problem by Brueckner *et al.*² This analysis shows that the effective interaction of a nucleon with a nucleus must, for finite nuclei, be nonlocal in both coordinate and momentum space, and is described by a potential matrix $(\mathbf{r}|V|\mathbf{r}')$ or $(\mathbf{k}|V|\mathbf{k}')$, rather than a local potential $V(\mathbf{r})$. The self-consistent determination of this potential matrix from two-body forces is an extremely involved problem and only preliminary results of calculations of this type have been reported as yet.³ For almost local potentials, however, a reasonable phenomenological form for the potential matrix may be taken as⁴

$$(\mathbf{r}|V|\mathbf{r}') = V[(\mathbf{r}+\mathbf{r}')/2]\delta_a(\mathbf{r}-\mathbf{r}'), \quad (1)$$

where $\delta_a(\mathbf{r})$ is a normalized approximation function of the delta function $\delta(\mathbf{r})$ with range parameter a . This range parameter, a , is a measure of the nonlocality in the potential, with $a=0$ corresponding to the local case.

A nonlocal interaction of the type discussed above leads to an integro-differential wave equation in coordinate space for the motion of a nucleon in a nucleus. By using form (1) for $(\mathbf{r}|V|\mathbf{r}')$ and expanding the interaction term in the resulting integro-differential equation about \mathbf{r} in a Taylor series, one obtains as a

first approximation, an equivalent Schrödinger wave equation containing, however, a spatially variable effective nucleon mass which appears in a fully symmetrized kinetic energy operator.⁴ An equation of this type has been used recently by various authors to study the approximate effect of a nonlocal nucleon-nucleus interaction on single-particle level schemes^{5,6} and nuclear binding energies.⁷

The case of an infinitely extended nucleus ($V=V_a$ = constant) can be treated exactly within the above framework, since the matrix (1) then has translational invariance in coordinate space, and hence is diagonal in momentum space. One finds

$$(\mathbf{k}|V|\mathbf{k}') = (2\pi)^3 V_a \tilde{\delta}_a[\frac{1}{2}(\mathbf{k}+\mathbf{k}')] \delta(\mathbf{k}-\mathbf{k}'), \quad (2)$$

in which $\tilde{\delta}_a(\mathbf{k})$ is the Fourier transform of $\delta_a(\mathbf{r})$ and $\hbar\mathbf{k}$ is the nucleon momentum.

Frahn⁸ has shown that the phenomenological form (2) for the nucleon-nucleus interaction can qualitatively account for the modified propagation of nucleons in nuclear matter and the energy dependence of the real part of the optical potential. One might hope to include absorptive effects in (2) by simply letting V_a be complex as is done in the local optical model. However, this procedure leads to an incorrect energy dependence of the imaginary part of the resulting optical potential as can be directly seen from the nonlocal continuity equation (Sec. II). In Sec. III, we discuss the inclusion of a repulsive ("hard core") contribution to (2) in the light of recent work on the related bound-state problem,^{9,10} and show that an improvement may be obtained. Finally in Sec. IV, we compare the potential parameters derived here with those inferred from a phenomenological study of finite nuclei using nonlocal optical potentials.

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¹ Feshbach, Porter, and Weisskopf, *Phys. Rev.* **96**, 448 (1954).

² See for instance, H. A. Bethe, *Phys. Rev.* **103**, 1353 (1956).

³ Brueckner, Gammel, and Weitzner, *Phys. Rev.* **110**, 431 (1958).

⁴ W. E. Frahn and R. H. Lemmer, *Nuovo cimento* **5**, 1564 (1957).

⁵ Ross, Lawson, and Mark, *Phys. Rev.* **104**, 401 (1956).

⁶ W. E. Frahn and R. H. Lemmer, *Nuovo cimento* **6**, 1221 (1957).

⁷ A. E. S. Green, *Revs. Modern Phys.* **30**, 569 (1958); A. E. S. Green and P. Sood, *Phys. Rev.* **111**, 1147 (1958).

⁸ W. E. Frahn, *Nuovo cimento* **4**, 313 (1956).

⁹ Gomes, Walecka, and Weisskopf, *Ann. Phys. (N. Y.)* **3**, 241 (1958).

¹⁰ A. E. S. Green, *Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy, 1958* (United Nations, Geneva, 1958).

II. CONTINUITY EQUATION

The nonlocal wave equation for the motion of a nucleon in a nucleus may be written as

$$(\hbar^2/2M_0)\Delta\psi(\mathbf{r})+E\psi(\mathbf{r}) = \int U_a[\frac{1}{2}(\mathbf{r}+\mathbf{r}')] \delta_a(\mathbf{r}-\mathbf{r}')\psi(\mathbf{r}')d\mathbf{r}', \quad (3)$$

where $U_a = V_a + iW_a$, with V_a, W_a real, and we assume the form (1) for $(\mathbf{r}|\mathbf{V}|\mathbf{r}')$.

Since the reaction cross section is proportional to the number of particles removed from the incident beam per second, this is easily found to be given by

$$\int \text{div} \mathbf{S} d\mathbf{r} = (2/\hbar) \int W_a[\frac{1}{2}(\mathbf{r}+\mathbf{r}')] \times \delta_a(\mathbf{r}-\mathbf{r}')\rho(\mathbf{r},\mathbf{r}')d\mathbf{r}d\mathbf{r}', \quad (4)$$

where $\rho(\mathbf{r},\mathbf{r}') = \psi^*(\mathbf{r})\psi(\mathbf{r}')$ is the mixed nucleon density, and \mathbf{S} is the usual probability current density vector.

We confine the discussion to nucleons interacting in nuclear matter. Then W_a is a constant, and we can describe the nucleons by plane waves. The mixed density then becomes

$$\rho(\mathbf{r},\mathbf{r}') = \exp[-i\mathbf{k} \cdot (\mathbf{r}-\mathbf{r}')] \quad (5)$$

for a nucleon with momentum $\hbar\mathbf{k}$. The expression (5) essentially introduces the Fourier transform of $\delta_a(\mathbf{r}-\mathbf{r}')$ into (4). Taking a Gaussian representation for the function $\delta_a(\mathbf{r}-\mathbf{r}')$, the right-hand side of (4) becomes proportional to $W_a \exp(-k^2a^2/4) \equiv W_{\text{eff}}(k)$.

Clearly $W_{\text{eff}}(k)$, the effective absorptive potential, is a monotonically decreasing function of the nucleon momentum. This result is contrary to the findings based upon phenomenological fittings which indicate that the coefficient of the absorptive term is an increasing function of energy.¹¹ While it must be recognized that such studies are based upon actual fittings to finite nuclei, it is reasonable to suppose that the coefficient in front of the form function represents the well depth for absorption in nuclear matter. Since the result of the monotonic decreasing function follows so immediately from the continuity equation, one is greatly constrained as to how one can embody a theoretical modification which leads to an increasing absorptive part for scattering at low incident energies. A possible modification which may be made to the theory is to incorporate two nucleon-nuclear complex kernels. This seems reasonable in view of the work of Brueckner³ and others on nuclear structure in which the nucleon-nuclear

kernel contains an attractive part which arises from the attractive component of the two-body force and a repulsive part associated with the influence of the repulsive core when taken in conjunction with the exclusion principle. Assuming now that we do have two kernels, one associated with the attractive part and one with the repulsive part of the two-body force, we may write the single-particle wave equation as

$$(\hbar^2/2M_0)\Delta\psi(\mathbf{r})+E\psi(\mathbf{r}) = \int U_a[\frac{1}{2}(\mathbf{r}+\mathbf{r}')] \delta_a(\mathbf{r}-\mathbf{r}')\psi(\mathbf{r}')d\mathbf{r}' + \int U_c[\frac{1}{2}(\mathbf{r}+\mathbf{r}')] \delta_c(\mathbf{r}-\mathbf{r}')\psi(\mathbf{r}')d\mathbf{r}', \quad (6)$$

where U_a and U_c are related to the attractive and repulsive parts of the nucleon-nucleon potential, respectively. In the case of nuclear matter, the natural division⁹ of the total energy into contributions from those two parts also seems to favor a formulation of the type (6).

Proceeding now to the continuity equation, we can determine the absorptive term in a similar fashion to the method used previously. The possibility of fitting the phenomenologically determined absorptive function now resolves about the relative magnitudes of U_a and U_c as well as the range parameters a and c associated with them.

III. EFFECTIVE LOCAL OPTICAL POTENTIALS

For the interaction of nucleons in infinitely extended nuclear matter, the Fourier transform of (6) leads directly to the dispersion relation⁸

$$E = (\hbar^2k^2/2M_0) + V_{\text{eff}}(k) + iW_{\text{eff}}(k), \quad (7)$$

where

$$V_{\text{eff}}(k) + iW_{\text{eff}}(k) = (V_a + iW_a) \exp(-k^2a^2/4) + (V_c + iW_c) \exp(-k^2c^2/4), \quad (8)$$

if Gaussian forms are used for δ_a and δ_c . $V_{\text{eff}}(k)$ and $W_{\text{eff}}(k)$ then represent the real and imaginary parts of the effective local potential felt by a nucleon.

In view of the fact that k^2 is complex, we write $k^2 = k_1^2 + ik_2^2$ and use the approximation

$$\exp(-ik_2^2a^2/4) \approx 1 - ik_2^2a^2/4. \quad (9)$$

Substituting (9) into (7) and (8), and making the reasonable assumption

$$|W_a k_2^2 a^2 / 4 V_a| \ll 1, \quad |W_c k_2^2 c^2 / 4 V_c| \ll 1,$$

it then follows that

$$V_{\text{eff}}(k) = V_a \exp(-k_1^2 a^2 / 4) + V_c \exp(-k_1^2 c^2 / 4), \quad (10)$$

$$W_{\text{eff}}(k) = \frac{W_a \exp(-k_1^2 a^2 / 4) + W_c \exp(-k_1^2 c^2 / 4)}{1 - (M_0 V_a a^2 / 2 \hbar^2) \exp(-k_1^2 a^2 / 4) - (M_0 V_c c^2 / 2 \hbar^2) \exp(-k_1^2 c^2 / 4)}, \quad (11)$$

$$E = (\hbar^2 / 2 M_0) k_1^2 + V_a \exp(-k_1^2 a^2 / 4) + V_c \exp(-k_1^2 c^2 / 4). \quad (12)$$

¹¹ See for instance, A. E. Glassgold, *Revs. Modern Phys.* **30**, 419 (1958).

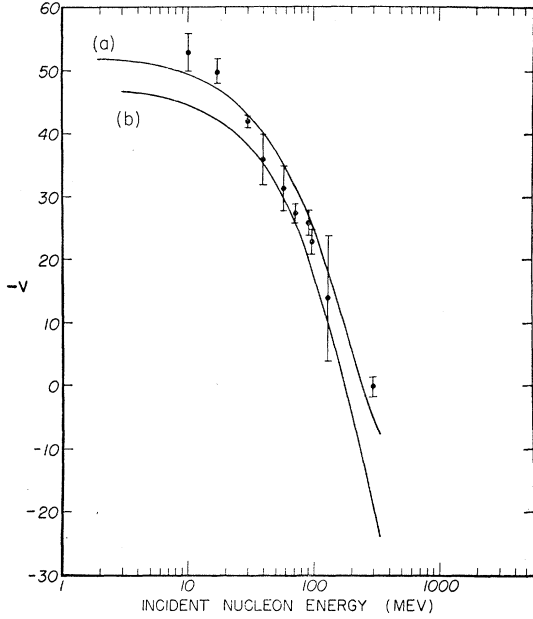


FIG. 1. The effective real optical model potential as a function of incident nucleon energy. The empirical points are those referred to in articles by Glassgold.^{11,15} Curve (a) is for the case: $V_a = -125$ Mev, $V_c = 43.4$ Mev, $a^2 = 0.48$ f², and $c^2 = 0.12$ f². Curve (b) is for the case: $V_a = -117$ Mev, $V_c = 43.4$ Mev, $a^2 = 0.48$ f² and $c^2 = 0$.

The method of Gomes, Walecka, and Weisskopf⁹ as extended by Green¹⁰ may be used to derive the parameters V_a and a from an assumed two-body interaction.¹² For example, choosing the Serber force with a repulsive core,

$$\begin{aligned} v(r) &= \infty, & r < r_c \\ &= -\frac{1}{2}(1 + \rho_m)V_0, & r_c < r < r_a \\ &= 0, & r > r_a \end{aligned} \quad (13)$$

then it follows from the Born approximation that the potential due to the attractive part of the two-body force exerted on a single particle of momentum $\hbar\mathbf{k}$ by 3Λ unequal particles is

$$V(k) = -3\{\Lambda V_0(r_a^3 - r_c^3)/2R^3 + (V_0/\pi)[g(r_a, k, k_f) - g(r_c, k, k_f)]\}, \quad (14)$$

where

$$g(r_a, k, k_f) = \frac{1}{2}\{\text{Si}[r_a(k + k_f)] + \text{Si}[r_a(k_f - k)] - 2 \sin k_f r_a \sin k r_a / k r_a\}. \quad (15)$$

where Si denotes the sine integral function.

The momentum dependence of the g -function gives rise to the nonlocality or the effective mass associated with the single-particle potential. One may characterize the momentum dependence by means of a parameter $\beta = (M_0/M^* - 1)$ in terms of the effective nucleon mass M^* . To evaluate β , one assumes that the function g is parabolic in k in the neighborhood of the Fermi surface.

¹² J. H. Van Vleck, Phys. Rev. 48, 367 (1935); W. E. Frahn, Nuovo cimento 5, 393 (1957).

Letting $R = r_0 A^{1/3}$ and $\Lambda = A/4$, we have

$$\beta = (3M_0 V_0 / \pi \hbar^2 k_f^2) [Y(r_a) - Y(r_c)], \quad (16)$$

where

$$Y(r_a) = \frac{1}{4} \sin 2k_f r_a + \frac{1}{2} k_f r_a - \sin^2 k_f r_a / k_f r_a. \quad (17)$$

If, for example, we assume the parameters⁹ $r_a = 2.3$ f, $V_0 = 28.34$ Mev, $r_c = 0.4$ f, which lead to a zero energy virtual singlet state of the deuteron, and take $k_f = 1.52/r_0$, one obtains nuclear stability near $r_0 = 1.2$ f with well parameters $V_a = -117$ Mev and $\beta = 0.535$. In terms of the nonlocality range a , it follows that $\beta = M_0 V(k_f) a^2 / 2\hbar^2$. Accordingly, these values of β and V_a corresponds to $a^2 = 0.48$ f².

If we assume that the effect of the two-body repulsive interaction leads to a local repulsive nucleon-nuclear force, then the methods of Huang and Yang¹³ yield $V_c = 43.4$ Mev and $c^2 = 0$ at the above core radius and nuclear radius constant.

Using these values, one might now find the predicted energy dependence of V_{eff} from the dispersion relation (12) by evaluating k_1 as a function of the nucleon energy E and inserting into (10). The result is shown in Fig. 1, curve (b). As is seen, the calculated values are, in general, larger than the empirical ones. The agreement is, however, improved if we assume that the core potential is also nonlocal, i.e., $c^2 \neq 0$. In modifying the previous expressions to embody this nonlocality, it is reasonable to hold $\beta' = M_0 [V_a(k_f) a^2 + V_c(k_f) c^2] / 2\hbar^2$ fixed at 0.535 and to vary V_a and c^2 . The best agreement with the empirical data is obtained by choosing $V_a = -125$ Mev and $c^2 = 0.12$ f² [Fig. 1(a)]. The former choice corresponds to a relatively small change in the well depth of the two-body force, and the latter to the type of nonlocality associated with the core term in the more detailed work of Brueckner.¹⁴

$W_{\text{eff}}(E)$ may now be obtained from (11) once the parameters W_a and W_c are chosen. Unfortunately, the derivation of these parameters from two-body forces constitutes an even more involved study than that associated with the real well parameters. Accordingly, only a limited phenomenological treatment of this term

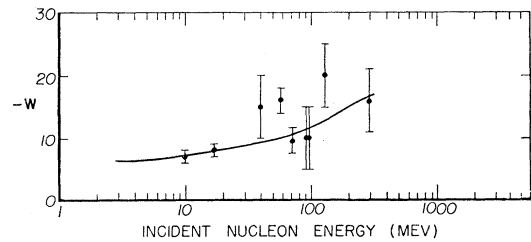


FIG. 2. The effective imaginary optical model potential as a function of incident nucleon energy. The empirical points are those referred to in articles by Glassgold.^{11,15} The curve is for the case: $W_a = 32$ Mev, $W_c = -37$ Mev, $a^2 = 0.48$ f² and $c^2 = 0$.

¹³ K. Huang and C. N. Yang, Phys. Rev. 105, 767 (1957).

¹⁴ K. A. Brueckner, Proceedings of the International Conference on the Optical Model, The Florida State University, Tallahassee, 1959, p. 145.

will be made. It should be clear from the form of $W_{\text{eff}}(k)$ that rather unusual measures are needed to obtain a function which rises with k . Accepting a and c from the derivation of the real potential, one is extremely restricted as to the choices of W_a and W_c which can accomplish such a behavior. The choice $W_a=32$ Mev and $W_c=-37$ Mev leads to the curve in Fig. 2. This gives a satisfactory description in the low-energy region (0 to 25 Mev). We feel, however, compelled to call attention to the implications of the negative sign associated with the core term which, if interpreted literally, would suggest that this component of the potential leads to emission rather than absorption. Since this is certainly rather nonphysical, we must conclude that either this representation of the imaginary term corresponds simply to a mathematical device, or else what is involved here is a type of interference effect as is also found in the case of Coulomb plus nuclear scattering.

It is seen from Figs. 1 and 2 that the real part of the effective potential inferred from a two-kernel form for the potential matrix is in fair agreement with that resulting from phenomenological analyses using local potentials. As to the imaginary part, although it gives a satisfactory description in the low-energy region (0 to 25 Mev), the over-all agreement is rather poor. The absence of a maximum around 70 Mev (as pointed out by many authors^{15,16}) and the nonzero contribution at the top of the Fermi sea indicate that nonlocality alone when embodied in the form assumed here cannot explain the energy dependence of the imaginary potential over a wide energy range.¹⁷

Therefore, in view of the above discussions, it would appear that a nonlocal optical model of the two-kernel type with constant parameters may be successfully used at low energies. However, to correlate experimental data over a wide energy range, an explicit variation of the nonlocal potential parameters seems necessary; alternatively, at the present stage of data fitting, one might simply use a local imaginary potential with a larger explicit variation of the potential parameters.

In our calculation, the nonlocality has been characterized by a Gaussian dependence. This assumption,

together with other simplifications, does not introduce any significant uncertainty in the behavior of the low-energy part of the curves. However, it does render the high-energy part somewhat doubtful, since the details here are expected to depend somewhat upon the shape of the nonlocality.

IV. COMPARISON WITH RESULTS FOR A FINITE NUCLEUS

In a study of Wyatt, Wills, and Green,¹⁸ an attempt has been made to find a phenomenological description of the nucleon-nuclear interaction for finite nuclei which could handle without explicit variations of the parameters, both bound and scattering phenomena. On the basis of their attempts to fit angular distributions and total cross sections as well as mass separation energies, they have arrived at parameters which might now be compared with the results of this current study. Let us assume a two-kernel nonlocal optical model potential of the type given by (6). In the case of almost local potentials given by (1), it can be easily shown that at low energies, the wave equation may be reduced to a form which is independent of the shape of the nonlocality.⁴ This equation may be written as

$$-\left(\hbar^2/8\right)\{\Delta[1/M^*(\mathbf{r})]+\nabla[2/M^*(\mathbf{r})]\nabla \\ +[1/M^*(\mathbf{r})]\Delta\}\psi(\mathbf{r})+[(V_a+V_c)\xi(\mathbf{r}) \\ +i(W_a+W_c)\eta(\mathbf{r})]\psi(\mathbf{r})=E\psi(\mathbf{r}), \quad (18)$$

where

$$M^*(\mathbf{r})=M_0/\{1-(M_0/2\hbar^2)[(V_a a^2+V_c c^2)\xi(\mathbf{r}) \\ +i(W_a a^2+W_c c^2)\eta(\mathbf{r})]\} \quad (19)$$

is the effective mass and $\xi(\mathbf{r})$, $\eta(\mathbf{r})$ are the form functions for V_a , V_c , W_a , W_c , respectively. It should be clear that insofar as this characterization of the problem is concerned, the combinations of parameters which are essential to the determination of the cross sections are those listed in Table I. In this table, the combinations deduced from the parameters of the preceding Sec. are given in the second column. In the third column are given the parameters deduced directly by fitting data for finite nuclei.¹⁹ The limits shown are based upon the range of parameters which lead to good fits. In comparing the imaginary part, a correction was made for the differences in form factors used in the two studies.

The agreement between the two sets of combinations is gratifying in view of the considerable difference in viewpoint used in arriving at these numbers. It is also gratifying that the agreement occurs at a nonlocality corresponding to a reduced mass at the center of the nucleus of the magnitude which has been used successfully in fitting total energies and densities of nuclei in

TABLE I. Parameter combinations.

	This study	W.W.G. ^a
V_a+V_c	-81 Mev	-70±3 Mev
W_a+W_c	-10 Mev	-14±6 Mev
$V_a a^2+V_c c^2$	-54 Mev f^2	-47±3 Mev f^2
$W_a a^2+W_c c^2$	+22 Mev f^2	+32±7 Mev f^2

^a See reference 19.

¹⁵ A. E. Glassgold and P. J. Kellogg, Phys. Rev. **109**, 1291 (1958).

¹⁶ A. M. Lane and C. F. Wandel, Phys. Rev. **98**, 1524 (1955); W. B. Riesenfeld and K. M. Watson, Phys. Rev. **102** (1956); E. Clementel and C. Villi, Nuovo cimento **10**, 176 (1955).

¹⁷ H. Feshbach, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Palo Alto, 1958), Vol. 8, p. 49.

¹⁸ A. E. S. Green, Proceedings of the International Conference on the Optical Model, The Florida State University, Tallahassee, 1959, p. 44.

¹⁹ Wyatt, Wills, and Green (to be published).

various nuclear structure calculations. One might therefore expect that a good mass surface would result from a nuclear potential characterized by the parameters arrived at here.⁷ In conclusion, we believe this study in conjunction with the work of Wyatt, Wills, and Green suggests that the simple nonlocal nucleon-

nuclear potentials when pursued in the Frahn-Lemmer approximation can serve to describe the average behavior of nucleons with nuclei in the range of energies from minus 70 Mev to plus 25 Mev which corresponds roughly to the entire range of concern of classical nuclear physics.

Mass Assignments and Some Decay Characteristics of Gd¹⁴⁵, Eu¹⁴⁵, Gd¹⁴⁶, and Eu¹⁴⁶†

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The nuclides Gd¹⁴⁵, Eu¹⁴⁵, Gd¹⁴⁶, and Eu¹⁴⁶ were prepared by the interactions of 20- to 40-Mev helium ions with Sm¹⁴⁴, and found to decay with half-lives of 25±2 minutes, 5.6±0.3 days, 46±2 days, and 4.4±0.1 days, respectively. The mass number assignments were made on the basis of excitation functions, and chemical evidence of parent-daughter relationships, with special reference to the previously known nuclide Eu¹⁴⁵. The most prominent gamma rays appearing in the decay of each of these four nuclides are as follows: in Gd¹⁴⁵ decay, at 0.80, 1.03, and 1.75-Mev; in Eu¹⁴⁵ decay, at 0.53, 0.64, and 0.89 Mev; in Gd¹⁴⁶ decay, at 0.114 and 0.153 Mev; and in Eu¹⁴⁶ decay, at 0.63 and 0.74 Mev. There is also a strong *K* x-ray line in each spectrum. In addition, Gd¹⁴⁵ was found to emit positrons with an end-point energy of about 2.4±0.2 Mev.

INTRODUCTION

RECENT reports by several workers¹⁻⁴ suggest the existence of a gadolinium isotope with mass number less than 147, and having a half-life in the range between 25 and 70 days. The mass number of this nuclide has been estimated variously to be 145 and 146.

This paper reports work in which both Gd¹⁴⁵ and Gd¹⁴⁶ were prepared and partially characterized under conditions in which the assignment of mass number was relatively unambiguous. In particular, the nuclides in question were prepared by the interaction of 20-Mev to 40-Mev helium ions with Sm¹⁴⁴, the mass assignments being made on the basis of the excitation functions for their production.

The observed half-lives were found to be 25 minutes and 46 days, respectively, for Gd¹⁴⁵ and Gd¹⁴⁶, suggesting that Gd¹⁴⁶ had been responsible for the observations mentioned above.

† Research performed under the auspices of the U. S. Atomic Energy Commission.

¹ J. R. Grover, thesis, University of California Report UCRL-3932, September, 1957 (unpublished).

² Shirley, Smith, and Rasmussen, *Nuclear Phys.* 4, 395 (1957).

³ Gorodinskii, Murin, Pokrovskii, Preobrazhenskii, and Titov, *Doklady Akad. Nauk S.S.S.R.* 112, 405 (1957) [translation: *Soviet Phys. Doklady* 2, 39 (1957)].

⁴ Gorodinskii, Murin, Pokrovskii, and Preobrazhenskii, The Program and Abstracts of the Eighth Annual Conference for Nuclear Spectroscopy, Leningrad, January 27 to February 3, 1958 (unpublished), p. 22; Gorodinskii, Murin, and Pokrovskii, *Izvest. Akad. Nauk S.S.S.R. Ser. Fiz.* 22, 811 (1958) [translation: *Bull. Acad. Sciences U.S.S.R.* 22, 805 (1958)].

EXPERIMENTAL

Target Foils

By means of a Zapon painting technique described elsewhere,⁵ reasonably uniform deposits of samarium oxide were formed on very pure (99.99%) 0.001-inch aluminum foil. The thickness of the samarium oxide layer in typical target foils was about 400 micrograms per cm². A simple backscattering type of beta gauge was used to assure sufficient target uniformity in those experiments in which uniformity was necessary. The samarium oxide enriched in Sm¹⁴⁴ was obtained from Oak Ridge National Laboratory and had the following isotopic composition (expressed in atom percent): Sm¹⁴⁴, 58.9%; Sm¹⁴⁷, 13.5%; Sm¹⁴⁸, 5.3%; Sm¹⁴⁹, 3.2%; Sm¹⁵⁰, 1.4%; Sm¹⁵², 3.8%; Sm¹⁵⁴, 14.0%. Some foils were also prepared using natural samarium oxide (Sm¹⁴⁴, 3.1%).

Irradiations

Bombardments were carried out in the 40-Mev external helium ion beam of the Brookhaven 60-inch cyclotron. The bombarding energy was adjusted by means of aluminum absorber foils, employing for this purpose the range-energy relation of Aron *et al.*⁶ The full energy of helium ions incident on the target stack was taken to be 40.5 Mev, on the basis of approximate range measurements.

⁵ Dodson, Graves, Helmholtz, Hufford, Potter, and Povelites, *Miscellaneous Physical and Chemical Techniques of the Los Alamos Project*, edited by A. C. Graves and D. K. Froman (McGraw-Hill Book Company, Inc., New York, 1952), p. 1.

⁶ Aron, Hoffman, and Williams, U. S. Atomic Energy Commission Report AECU-663, May, 1951 (unpublished).