

have been uniformly divided by a speed of 8×10^5 cm/sec; the actual velocity range of the accepted 2S atoms is from 5.5 to 10×10^5 cm/sec.

The effectiveness of water in quenching the metastable 2S hydrogen atom is quite striking. Its large cross section may be related to the strong electric dipole moment of the water molecule.

V. RESULTS AND DISCUSSION

For residual gas pressures from 7 to 9×10^{-7} mm Hg, the average rate of single-quantum decay of the 2S hydrogen atom was found to be 930 sec^{-1} . Of this, 510 sec^{-1} is ascribable to collision quenching by the four most abundant gases present in the vacuum chamber. Thus, the final result of this measurement is that the apparent natural lifetime of the metastable hydrogen atom is 2.4 msec, with an estimated over-all probable error of 50%.

This figure can be regarded only as a lower limit on the true natural lifetime for two reasons. First, stray electric fields within the quenching region directed other than normal to the quench plates may have been present. Although a field of only 0.4 v/cm would completely account for the apparent natural decay, in view of the geometry of the quench region and the care

taken to avoid contact potentials and surface charging it is very doubtful that the observed decay arose solely from stray field quenching. Second, gas collision quenching corrections were made for only the four most prominent background gases; gas collision quenching by both trace gases and ground-state hydrogen atoms accompanying the metastable atoms may have contributed substantially to the decay rate.

From the measured lower limit of the metastable atom lifetime, it follows that an upper limit on the admixture (amplitudes squared) of the $2P_{\frac{1}{2}}$ state with the $2S_{\frac{1}{2}}$ state is 7×10^{-7} . By applying the result of this measurement in the formulas of Salpeter relating the lifetime of the H(2S) atom and the strength of an electronic electric dipole moment, it follows that the dipole moment strength cannot exceed $0.0045e\hbar/mc$.

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Mathematical Analysis of a Simple Model Related to the Stripping Reaction

ELLIOTT LIEB* AND HEINZ KOPPE†

Department of Physics, University of Illinois, Urbana, Illinois

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A scattering process in which an incoming bound particle can split up into its component parts when its total energy is above a threshold is of considerable interest physically (deuteron stripping, etc.) and mathematically (analytic properties of the S -matrix, etc.). For such complicated problems it is obviously convenient to have a simple, analytically soluble model for reference, but although many models have been suggested in the past none have proved analytically tractable. In this paper we propose and completely solve a one-dimensional model which, although it is not very physical, has all the desired characteristics. The problem is not mathematically trivial, however, and leads to a Wiener-Hopf integral equation.

WE have to begin by apologizing for the presumptuous title of the paper, our excuse being that a similar title has already been used for the same purpose.¹ The mathematical model treated in this paper has only one feature in common with the stripping reaction: It describes a scattering process in which an incoming complex particle might re-emerge as a complex particle or might be split into its components. Problems of this type lead to difficulties since they cannot be treated in the convenient interaction rep-

resentation. Therefore, there has been some desire for a simple mathematical model of such a reaction, and as such the problem has a minor history.

Heisenberg proposed an investigation of the Hamiltonian‡

$$-\frac{1}{2} \frac{\partial^2 \psi}{\partial u^2} - \frac{1}{2} \frac{\partial^2 \psi}{\partial v^2} - \{A\delta(u) + A\delta(v) + 2C\delta(u-v)\} \psi = E\psi.$$

This can be interpreted as describing two particles moving in one dimension and interacting with each other as well as with a fixed scattering potential at the origin. (They can be bound together with an

* Present address: Laboratory of Nuclear Studies, Cornell University, Ithaca, New York.

† Present address: Institut für theoretische Physik der Universität München, Munich, Germany.

¹ R. Jost, *J. Appl. Math. and Phys.* **6**, 316 (1955).

‡ Throughout we use units in which $\hbar = m = 1$.

energy, $-C^2$, or separately bound to the origin with an energy, $-A^2$.) This Hamiltonian looks very simple and symmetrical; however, it turned out to be quite unmanageable. Wildermuth² investigated it in two papers, but he could not give a solution in closed form.

Danos³ considered the same Hamiltonian together with the boundary conditions $\psi=0$ for $u=0$ and $v=0$, (which is equivalent to $A=-\infty$). This means that a linear deuteron is smashed against a wall at the origin. Astonishingly, this has a very simple solution: ($x=u+v$; $y=u-v$; $E=k^2-C^2$)

$$\psi = e^{-C|y|-ikx} - e^{-C|x-ik|y|} - \frac{C-ik}{C+ik} \{ e^{-C|y|+ikx} - e^{-C|x+ik|y|} \}.$$

Asymptotically, this represents an incoming and an outgoing deuteron wave, both with unit amplitude. No fragments can occur, which is of course quite unexpected.

Therefore, Danos³ tried a modification of the problem by relaxing the boundary conditions to $\psi=0$ for $v=0$ only. This could be interpreted as a deuteron running against a potential wall which repels the proton and does not affect the neutron, so that at least at high energies there should be the possibility that the deuteron breaks up. But this modification destroys the symmetry of the original equation and leads again to considerable mathematical difficulties. Jost showed that the problem could be reduced to a difference-equation.¹ (In a previous paper⁴ he had shown that this difference-equation can be solved in principle.) The actual solution is not given, since its construction "might be manageable, though tedious." An essential point of the first paper of Jost is that he uses the Wiener-Hopf technique, which is obviously the appropriate tool for problems of this kind.

In this paper, we investigate a problem which is even more artificial, if somewhat similar to the Danos-Jost problem. We consider the equation

$$\frac{1}{2} \frac{\partial^2 \psi}{\partial u^2} - \frac{1}{2} \frac{\partial^2 \psi}{\partial v^2} - 2C\theta(u+v)\delta(u-v)\psi = E\psi, \quad (1)$$

$$\theta(x) = 1, \quad x \geq 0$$

$$= 0, \quad x < 0.$$

The interaction between the two particles vanishes if either of them has a negative coordinate. This could be interpreted as a situation in which at $u=0$ there is a potential wall which does not act on the neutron or proton, but excludes the mesons which are responsible for the interaction between the nucleons. Of course it is not necessary to enter into such speculations; $V(u,v) = -2C\theta(u+v)\delta(u-v)$ is simply a potential which has the property that outgoing waves can be eigenstates

² K. Wildermuth, Z. Physik **127**, 92 (1949).

³ J. H. D. Jensen (private communication mentioned Danos' work).

⁴ R. Jost, Comm. Math. Helv. **28**, 173 (1954).

of \mathcal{H}_0 or of \mathcal{H}_0+V . Consequently, if the incoming deuteron has negative total energy ($E=k^2-C^2$), i.e., below the stripping threshold, it can only be reflected from the origin as a deuteron. But if $E \geq 0$ the deuteron can either be reflected or break up into two free particles. Alternatively, Eq. (1) can be looked upon as the Hamiltonian of a particle rolling down a semi-infinite groove in two dimensions. If $E \geq 0$ the particle can "hop" out of the groove, but if $E < 0$ it cannot do so and must be reflected. It is interesting to note that classically the reflection probability is zero for $E > 0$ but that quantum mechanically it is not zero (of course it approaches zero as $E \rightarrow \infty$); see Fig. (1).

Thus, the whole interest in the problem lies in the fact that for $E=0$ new channels suddenly open up in the S -matrix. Since there is presently much interest in the analytic properties of S -matrices, this problem may be taken as an analytically soluble model. Alternatively, it may be used to try out approximation schemes. As an illustration the following point, although not unexpected, may be mentioned. If the Born approximation is tried on Eq. (1), one finds that for $E > 0$ a fairly good approximation is obtained for the probability of production of free particles, even for low energies in first Born approximation. However, to *any* order one finds that the probability of a reflected deuteron is zero. This is because a reflected deuteron must appear as a pole in the S -matrix, and one simply cannot obtain the pole by a power series expansion.

In the remainder of this paper we show how Eq. (1) can be solved in closed form using Wiener-Hopf techniques. The amplitudes for the various channels are quite complicated, but the probabilities turn out to be simple functions which we explicitly calculate.

CALCULATION

Transforming to $x=u+v$, $y=u-v$, Eq. (1) becomes

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + E \right) \psi = -2C\delta(y)\theta(x)\psi. \quad (2)$$

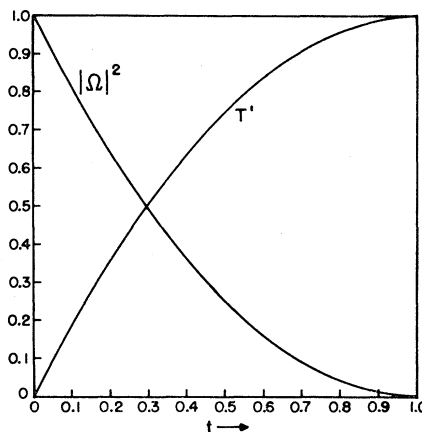


FIG. 1. $|\Omega|^2$ = Probability of outgoing deuteron. $T' = 1 - |\Omega|^2$ = Probability of stripping. $t = (k^2 - C^2)^{1/2} k^{-1} = E^{1/2} k^{-1}$ ($E > 0$).

The δ function has one bound state, with energy $-C^2$, which we shall call a deuteron. The wave function of the incoming deuteron (travelling along the x axis from $x=+\infty$ to $x=0$) with momentum $-k$ is

$$\psi_{in} = \exp(-C|y| - ikx); \quad E = k^2 - C^2. \quad (3)$$

Of course there can be no deuteron for $x < 0$ and therefore the problem is to find the outgoing wave which cancels ψ_{in} for $x < 0$. In order to separate incoming and outgoing waves it is convenient to suppose that E has a small positive imaginary part, $E = \kappa^2 + i2\epsilon\kappa$, and to take the limit $\epsilon \rightarrow 0$. Since C is real, k , as defined by Eq. (3), will also have a positive imaginary part:

$$k = K + i\epsilon\kappa K^{-1}; \quad K^2 - C^2 = \kappa^2 + O(\epsilon^2), \quad (4)$$

where K is the real part of k . Hence ψ_{in} diverges for large x whereas outgoing waves, including an outgoing deuteron, are square integrable.

We can rewrite Eq. (2) as a homogeneous integral equation:

$$\psi(x, y) = -2C \int_0^\infty dx' G(x, y | x', 0) \psi(x', 0), \quad (5)$$

where G is the Green's function of the left-hand side of (2) and is given by

$$G(x, y | x', y') = -(2\pi)^{-2} \int_{-\infty}^\infty \int_{-\infty}^\infty dpdq \frac{e^{ip(x-x') + iq(y-y')}}{p^2 + q^2 - E}. \quad (6)$$

Putting $y=0$ in Eq. (5) leads to the homogeneous Wiener-Hopf equation,

$$f(x) = 2C \int_0^\infty M(x-x') f(x') dx', \quad (7)$$

where $f(x) = \psi(x, 0)$ and $M(x-x') = -G(x, 0 | x', 0)$. This is an equation for the values of ψ along the positive x axis, from which one can calculate $\psi(x, y)$ with the help of (5).

Ordinarily, to solve the Schrödinger equation, (2), one would write $\psi = \psi_{in} + \text{outgoing wave}$. This is the prescription of formal scattering theory and would lead to an inhomogeneous version of (5) and consequently of (7). In principle there is no difficulty, but in fact an inhomogeneous Wiener-Hopf equation with kernel M is exceedingly intractable. On the other hand, all solutions of Eq. (5), if there are any, must have incoming parts since there is no solution to Eq. (2) which is purely outgoing. In view of the fact that Eq. (5) expresses ψ as an integral over an outgoing Green's function, it seems at first sight surprising that ψ can have an incoming part. Nevertheless, it is well known from Wiener-Hopf theory that precisely because M behaves as $\exp(-\epsilon|x-x'|)$ for large x' , ψ can behave as $\exp(+\epsilon'|x'|)$ and the integral in Eq. (5) will converge if $\epsilon' < \epsilon$. In other words, the convergence factor in G

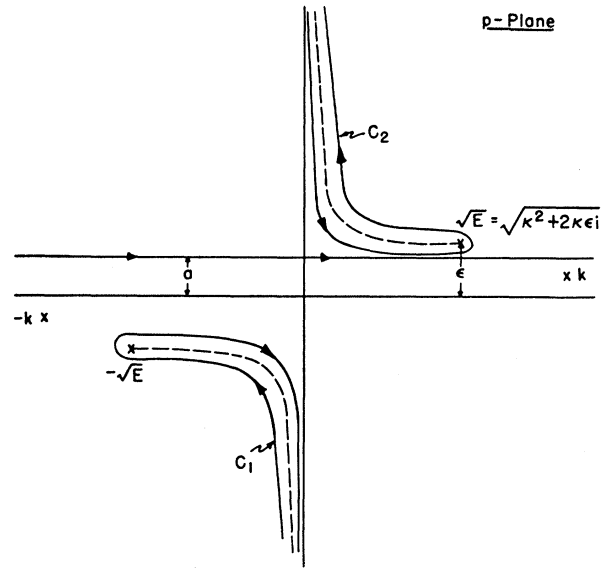


FIG. 2. Pertinent singularities and contours in the p -plane. Dashed lines are cuts.

allows ψ to diverge. It is in fact the case that Eq. (7), and hence Eq. (5), have a solution of the required form.

In the standard way⁵ we write $f(x) = f_+(x) + f_-(x)$; $F(p) = F_+(p) + F_-(p)$, where F_+ is the Fourier transform of f_+ , etc., and $f_+(x) \equiv 0$ for $x < 0$, $f_-(x) \equiv 0$ for $x \geq 0$:

$$F_+(p) = \int_{-\infty}^\infty dx e^{ipx} f_+(x) = \int_0^\infty e^{ipx} f(x) dx. \quad (8)$$

The Fourier transform of M [obtained from Eq. (6)] is

$$\tilde{M}(p) = \frac{1}{2} [p^2 - E]^{-\frac{1}{2}}. \quad (9)$$

Consequently,

$$-F_- = \frac{(p^2 - E)^{\frac{1}{2}} - C}{(p^2 - E)^{\frac{1}{2}}} F_+ = \frac{p^2 - k^2}{p^2 - E} \frac{(p^2 - E)^{\frac{1}{2}}}{(p^2 - E)^{\frac{1}{2}} + C} F_+. \quad (10)$$

A word about the square root in \tilde{M} : It has two cuts which must lie outside the strip $|I(p)| < \epsilon$ and are chosen so that $R[(p^2 - E)^{\frac{1}{2}}] \geq 0$ for all p . This choice is the most convenient; the cuts are shown in Fig. 2.

Since $(p^2 - E)^{\frac{1}{2}} \{ (p^2 - E)^{\frac{1}{2}} + C \}^{-1}$ is regular and free from zeros in the strip $|I(p)| < \epsilon$, we can write

$$\frac{(p^2 - E)^{\frac{1}{2}}}{(p^2 - E)^{\frac{1}{2}} + C} = \frac{\Gamma_+(p)}{\Gamma_-(p)} \equiv \Gamma(p), \quad (11)$$

where Γ_+ is regular and nonzero for $I(p) > -\epsilon$ and Γ_- is regular and nonzero for $I(p) < \epsilon$. We shall exhibit Γ_+ and Γ_- later, and it will be obvious that they are

⁵ For an exposition of Wiener-Hopf theory, see for example P. M. Morse and H. Feshbach, *Methods of Theoretical Physics* (McGraw-Hill Book Company, Inc., New York, 1953), p. 978; B. Noble, *Methods Based on the Wiener-Hopf Technique for the Solution of Partial Differential Equations* (Pergamon Press, New York, 1959).

asymptotically equal to one for large $|\mathcal{p}|$. Since F_+ and F_- go to zero for large $|\mathcal{p}|$, there is only one solution to Eq. (7)

$$F_+(\mathcal{p}) = A(\mathcal{p} + \sqrt{E})[\Gamma_+(\mathcal{p})(\mathcal{p}^2 - k^2)]^{-1}, \quad (12a)$$

$$F_-(\mathcal{p}) = -A[\Gamma_-(\mathcal{p})(\mathcal{p} - \sqrt{E})]^{-1}, \quad (12b)$$

where A is a constant. $f_+(x)$ is given by

$$f_+(x) = \frac{1}{2\pi} \int_{-\infty+ia}^{\infty+ia} F_+(\mathcal{p}) e^{-i\mathcal{p}x} d\mathcal{p}, \quad (13)$$

where $\epsilon\kappa K^{-1} < a < \epsilon$. It will be seen from (12a) and (13) that the pole in F_+ at $\mathcal{p} = k$ adds an outgoing part to $f_+(x)$.

If we choose A to be

$$A = 2ik\Gamma_+(k)[k + \sqrt{E}]^{-1}, \quad (14)$$

insert (6), (12a), and (13) into the right-hand side of Eq. (5), and integrate over q and x' , we find that

$$\begin{aligned} \psi(x,y) &= \frac{C}{2\pi} \int_{-\infty+ia}^{\infty+ia} d\mathcal{p} \exp\{-i\mathcal{p}x - (\mathcal{p}^2 - E)^{\frac{1}{2}}|y|\} \\ &\quad \times \frac{F_+(\mathcal{p})}{(\mathcal{p}^2 - E)^{\frac{1}{2}}} \quad (15a) \\ &= e^{-ikx - C|y|} + \Omega e^{ikx - C|y|} \end{aligned}$$

$$\begin{aligned} &+ \frac{C}{2\pi} \int_{C_1} d\mathcal{p} \exp\{-i\mathcal{p}x - (\mathcal{p}^2 - E)^{\frac{1}{2}}|y|\} \\ &\quad \times \frac{F_+(\mathcal{p})}{(\mathcal{p}^2 - E)^{\frac{1}{2}}}, \quad x > 0 \quad (15b) \end{aligned}$$

$$\begin{aligned} &= \frac{C}{2\pi} \int_{C_2} d\mathcal{p} \exp\{-i\mathcal{p}x - (\mathcal{p}^2 - E)^{\frac{1}{2}}|y|\} \\ &\quad \times \frac{F_+(\mathcal{p})}{(\mathcal{p}^2 - E)^{\frac{1}{2}}}, \quad x < 0. \quad (15c) \end{aligned}$$

The contours, C_1 and C_2 are shown in Fig. 2. A was chosen to make the coefficient of the incoming deuteron unity. $|\Omega|^2$ is then the probability of finding an outgoing deuteron. By evaluating the residue at $\mathcal{p} = -k$, Ω is found to be

$$\Omega = 2(k - \sqrt{E})[\Gamma_+(k)]^2(k + \sqrt{E})^{-1}. \quad (16)$$

Finally, we must evaluate the integrals around the contours C_1 and C_2 in (15a) and (15b). Firstly, it is clear that these integrals yield only outgoing waves (square integrable). Secondly, for very small ϵ the cuts and the contours will become L -shaped and it is clear that the only part of C_1 which is important is the line $-\kappa < \mathcal{p} < 0$ lying between the cut and the real axis. The rest of C_1 gives only damped waves. Similarly, the important part of C_2 is the line $0 < \mathcal{p} < \kappa$ below the cut. If we transform to two-dimensional polar coordinates and put

$$x = R \cos\theta, \quad y = R \sin\theta, \quad (17)$$

we are interested in evaluating these integrals for large R . We know that these integrals must be of the form

$$\begin{aligned} \lim_{\epsilon \rightarrow 0} \frac{C}{2\pi} \left\{ \left(\int_{C_1} \right) \theta(x) + \left(\int_{C_2} \right) [1 - \theta(x)] \right\} \\ = \chi(R, \theta) = R^{-\frac{1}{2}} \Phi(\theta) \exp(i\kappa R + i\pi/4), \quad (R \text{ large}) \quad (18) \end{aligned}$$

in the limit $\epsilon \rightarrow 0$. The easiest way to do these integrals is by the method of stationary phase; for both C_1 and C_2 the stationary phase point is $\mathcal{p} = -(E)^{\frac{1}{2}} \cos\theta$. There is one *caveat*. Although $F_+(\mathcal{p})$ will turn out to be zero for $\mathcal{p} = -\sqrt{E}$ [see Eq. (12a)] this is not so for $\mathcal{p} = +\sqrt{E}$. Consequently, the integrand in (15) will have a singularity at the end point $\mathcal{p} = +\sqrt{E}$. However, a *careful* analysis of the integrand and of the stationary phase procedure will show that the contribution from the end point is of higher order than the contribution from the stationary phase point. Hence we find that⁶

$$\Phi(\theta) = iC(2\pi\kappa)^{-\frac{1}{2}} F_+(-\kappa \cos\theta); \quad E_0 \equiv \kappa^2 > 0. \quad (19)$$

For $E_0 < 0$ the outgoing waves are, of course, all damped.

Equations (16) and (19) are the answers to the problem. However, since we are interested only in probabilities, *viz.*, $|\Omega|^2$ and $\sigma(\theta) = |\Phi(\theta)|^2$, we see from Eq. (12a) that it is necessary to know only $|\Gamma_+(\mathcal{p})|^2$, and that only for $-\kappa < \mathcal{p} < \kappa$ and $\mathcal{p} = K$. Hence the reason for doing the formalism first; although $\Gamma_+(\mathcal{p})$ will turn out to be a very complicated function, its modulus for the values of \mathcal{p} indicated above is expressible in terms of elementary functions.

In the usual way we write

$$\Gamma_+(\mathcal{p}) = \exp[\beta_+(\mathcal{p})], \quad (20)$$

where β_+ is analytic for $I(\mathcal{p}) > -\epsilon$ [which implies that Γ_+ is analytic and nonzero for $I(\mathcal{p}) > -\epsilon$] and is given by

$$\beta_+(\mathcal{p}) = \frac{1}{2\pi i} \int_D dt [\ln \Gamma(t)] (t - \mathcal{p})^{-1}. \quad (21)$$

The contour, D , is defined in the following way: if $I(\mathcal{p}) > 0$, D is the line $(-\infty, \infty)$; if $I(\mathcal{p}) < 0$, D is a deformation of the line $(-\infty, \infty)$ into the lower half plane which does not cross any of the singularities of Γ but which goes *under* the point $t = \mathcal{p}$.

Now, since we are interested only in $|\Gamma_+(\mathcal{p})|^2$ for the values of \mathcal{p} indicated above, we need calculate only $R[\beta_+(\mathcal{p})]$ for those values. It is this point that simplifies the calculation. $\beta_+(\mathcal{p})$ itself is a complicated function involving Eulerian Dilogarithms (Spence functions).

The evaluation of $R[\beta_+(\mathcal{p})]$ is a straightforward exercise involving a certain amount of juggling and integrations by parts. One must, however, take some care in passing to the limit $\epsilon \rightarrow 0$. The results are as

⁶ See A. Erdelyi, *Asymptotic Expansions* (Dover Publications, New York, 1956), p. 51.

follows for $E_0 = K^2 - C^2 > 0$:

$$p = K : \lim_{\epsilon \rightarrow 0} R[\beta_+(K)] = -\frac{1}{2} \ln \frac{2k}{C} - \frac{1}{4} \ln \left(\frac{k-\kappa}{k+\kappa} \right), \quad (22a)$$

$$p = \text{real}, p^2 < \kappa^2 = E_0 : \lim_{\epsilon \rightarrow 0} R[\beta_+(p)] = \frac{1}{2} \ln \left(\frac{\kappa+p}{k+p} \right). \quad (22b)$$

Inserting (22a) into Eq. (16), we can obtain the probability for an outgoing deuteron:

$$\lim_{\epsilon \rightarrow 0} |\Omega|^2 = \left| \frac{k-\kappa}{k+\kappa} \right| \left(\frac{C}{k} \right)^2, \quad E_0 > 0. \quad (23)$$

For $E_0 < 0$ Eq. (22a) must be modified. One finds easily that

$$\lim_{\epsilon \rightarrow 0} |\Omega|^2 = 1, \quad E_0 < 0, \quad (23a)$$

a result which is easily anticipated.

Similarly, inserting (22b) into Eq. (12a) we find that

$$\lim_{\epsilon \rightarrow 0} |F_+(p)|^2 = 2k(p+k) [(k+\kappa)(k+p)(k-p)^2]^{-1}, \quad (24)$$

$$E_0 > 0, \quad p = \text{real}, \quad p^2 < \kappa^2.$$

It is convenient to introduce the dimensionless variable

$$t = \kappa k^{-1}, \quad E_0 > 0, \quad (25)$$

from which it follows that t goes from 0 to 1 as E_0 goes from 0 to ∞ . Inserting Eq. (24) into Eq. (19),

$$\sigma(\theta) = |\Phi(\theta)|^2 = \frac{(1-t)t}{\pi\kappa} \frac{1-\cos\theta}{1-t^2 \cos^2\theta} \frac{1}{1+t \cos\theta} \quad (26)$$

$$|\Omega|^2 = (1-t)^2, \quad E_0 > 0. \quad (27)$$

In order to check (26) and (27) we can investigate the conservation of current condition in the xy plane. The current of ψ_{in} is simply $kC^{-1} = (1-t^2)^{-\frac{1}{2}}$; that for a plane wave of total energy E_0 is simply κ . Hence, the condition of conservation of current becomes

$$(1-|\Omega|^2)(1-t^2)^{-\frac{1}{2}} = \kappa \int_0^{2\pi} \sigma(\theta) d\theta. \quad (28)$$

Equations (26) and (27) satisfy Eq. (28).

The cross section for the stripping reaction is $\sigma(\theta)$ as given by (26). Here it should be remembered that the original Hamiltonian admits of two different interpretations: (a) as describing two particles in one dimension as stated above; (b) as describing one particle in two

dimensions. As it stands (26) is obviously adapted to the second interpretation, where θ is the direction of scattering. In the first interpretation θ must be connected with the distribution of energy between the stripping fragments. In order to work out this connection it is convenient to revert to Eq. (18). In the neighborhood of a point R_0 with coordinates (17), the exponential in (18) can be approximated by a plane wave $\exp\{i\kappa(x \cos\theta + y \sin\theta)\}$. If we reintroduce the original coordinates u and v , this goes over into

$$\exp\{i\kappa[(\cos\theta + \sin\theta)u + (\cos\theta - \sin\theta)v]\}. \quad (29)$$

Let us call the particle with the coordinate u the proton and the particle with the coordinate v the neutron. Then their respective momenta are given by

$$p = \kappa(\cos\theta + \sin\theta), \quad (30)$$

$$p' = \kappa(\cos\theta - \sin\theta).$$

If we square both equations and add, we obtain $\frac{1}{2}(p^2 + p'^2) = \kappa^2 = E$; $p_{\pm}' = \pm(2E - p^2)^{\frac{1}{2}}$. Solving for $\cos\theta$, we get $\cos\theta = (p + p')/2\kappa$ or

$$\cos(\theta_{\pm}) = \frac{1}{2\kappa} [p \pm (2E - p^2)^{\frac{1}{2}}]. \quad (31)$$

There are two solutions corresponding to the positive or negative sign of p' .

Finally, we obtain for the probabilities $W_+(p)$, $W_-(p)$ that the proton emerges with momentum p , and the neutron is scattered *backwards*, or correspondingly *forwards*:

$$W_+(p) = \sigma(\theta_+) |d\theta_+/dp| = \sigma(\theta_+) (2E - p^2)^{-\frac{1}{2}}, \quad (32)$$

$$W_-(p) = \sigma(\theta_-) |d\theta_-/dp| = \sigma(\theta_-) (2E - p^2)^{-\frac{1}{2}}.$$

Since $\sigma(\theta)$ increases with decreasing $\cos\theta$, and $\cos\theta_- \leq \cos\theta_+$, W_- is always larger than W_+ , i.e., for a given proton momentum, p , the neutron always prefers to be scattered forward. From (26) it can be seen that $W_+ = 0$ for $\cos\theta = 1$, i.e., if both particles are scattered backwards with equal momenta. Since in this case they must remain close to each other, they would form a deuteron and it was to be expected that the stripping cross section is zero. σ is largest for $\cos\theta = -1$, which means that there is always a preference to forward scattering of both particles with equal momenta. This effect becomes more and more pronounced for high energies.

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