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Interference Effects in Neutral K-Particle Decay

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An analysis is made of interference effects between the K_1^0 and K_2^0 components of a neutral K beam for decay in the channel $\pi^++\pi^-+\pi^0$. The effects discussed, though expected to be small, may just be detectable. If so, they would serve as a test of the proposed $|\Delta T|=\frac{1}{2}$ rule.

THE $K_1^0-K_2^0$ complex, first discussed by Gell-Mann and Pais[1] and by Pais and Piccioni,[2] involves a number of curious and amusing interference effects, which have been widely discussed though not yet tested experimentally.[3] Our purpose here is to record the expectation of yet another such effect, concerning in particular the three-pion decay modes of neutral K particles. Aside from their intrinsic interest and novelty, all of these phenomena provide possible ways to determine the $K_1^0-K_2^0$ mass difference. In any case, they must be taken into account when one wants to study certain aspects of neutral K-particle decay.

If CP invariance (C=charge conjugation, P=space inversion) is valid, as we shall assume,[4] K_2^0 decay into two pions is forbidden, as is K_1^0 decay to three neutral pions. All other known decay modes of neutral K particles proceed both for K_1^0 and K_2^0. In a beam of particles which at the initial time were pure K^0, the relative amplitude of the K_1^0 and K_2^0 components varies with time. Their relative contributions to any fixed decay configuration varies accordingly, thus giving rise to time-dependent interference effects. This has already been discussed in connection with the decay modes $e^+(\mu^+)+\nu+\pi^-$ and $e^-(\mu^-)+\bar{\nu}+\pi^+$.[5] Here one of the theoretical issues currently at stake is the question whether the process $K^0\rightarrow e^-(\mu^-)+\bar{\nu}+\pi^+$, in contrast to $\bar{K}^0\rightarrow e^-(\mu^-)+\bar{\nu}+\pi^+$, is, in fact, allowed at all.[6] At large times one looks at a pure K_2^0 beam and cannot distinguish the contributions from the K^0 and $\bar{K}^0$ components. In fact, here the rates for $e^-(\mu^-)+\bar{\nu}+\pi^+$ and $e^+(\mu^+)+\nu+\pi^-$ must be identical. The question can be answered only close to the source, where the beam is still largely K^0; and here one encounters and can take advantage of time-dependent interference effects between the K_1^0 and K_2^0, or equivalently, the K^0 and $\bar{K}^0$ components of the beam.

A similar situation is to be expected for the three-pion decay modes of neutral K mesons, although in this case the interference effects will very likely prove to be small. As is well known, CP invariance implies that K_1^0 can go only to the $T=0$ and $T=2$ final states of isotopic spin, K_2^0 to the $T=1$ and $T=3$ states.[7] In connection with the much-discussed $|\Delta T|=\frac{1}{2}$ selection rule—but aside from this, just as a matter of intrinsic interest—one would like to establish the relative amplitudes for K decay into the various states of isotopic spin, both for neutral and charged K particles. For the charged K particles, the branching ratio $(\pi^++\pi^++\pi^-)/(\pi^++\pi^0+\pi^0)$ provides some information and, in fact, suggests that the symmetric $T=1$ state is the dominant one. The dominance of symmetric final states is, however, expected from centrifugal barrier considerations, quite aside from isotopic spin selection rules. Small admixtures of other states are apparently also present, but one cannot easily distinguish here the $T=2$ and the nonsymmetric $T=1$ states. For the neutral K particles, in particular for K_2^0, the branching ratio $(\pi^++\pi^-\pm\pi^0)/(\pi^0+\pi^0+\pi^0)$ will provide some information on the relative amplitudes for the final $T=1$ and $T=3$ states. To learn about the other final states, $T=0$ and $T=2$, one must study K_1^0 decay—in the presence of a K_2^0 component in the beam.

Since the K_1^0 and K_2^0 components decay to mutually orthogonal states, they of course cannot interfere, insofar as the total rate of three-pion production is concerned. But as already said, the dominant K_2^0 amplitude (that to the totally symmetric $T=1$ state) is expected to be much bigger than any of the K_1^0 amplitudes. In this case then, where squared amplitudes are being compared, there is presumably little hope of detecting the K_1^0 contributions to three-pion decay. If, however, one looks at a particular configuration of final pion momenta, the contributions from K_1^0 and K_2^0 can interfere. These effects, which would be time dependent, permit a comparison of amplitudes in contrast to intensities for K_1^0 and K_2^0 three-pion decay; and it is just conceivable that such effects could be detected.

We turn to this phenomenon now in more detail. Let

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[1] M. Gell-Mann and A. Pais, Phys. Rev. **97**, 1387 (1955).

[2] A. Pais and O. Piccioni, Phys. Rev. **100**, 1487 (1955).

[3] Other references are given in *Proceedings of 1958 Annual International Conference on High-Energy Physics at CERN, 1958* (CERN, Geneva, 1958), App. II, p. 346.

[4] S. Weinberg, Phys. Rev. **110**, 782 (1958).

[5] S. B. Treiman and R. G. Sachs, Phys. Rev. **103**, 1545 (1956).

[6] See for example, M. Gell-Mann and A. H. Rosenfeld, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Palo Alto, 1957), Vol. 7, p. 407.

[7] R. Gatto, Phys. Rev. **106**, 168 (1957); also, see G. Snow, Phys. Rev. **103**, 1111 (1956).

TABLE I. Amplitudes for K_1^0 and K_2^0 decay to three-pion states. In row 2 the symbols S, A, I denote states which are, respectively, totally symmetric, totally antisymmetric, or of intermediate symmetry.

T	0	1			2		3
Symmetry character	A	S	I	I	I	I	S
K_1^0	a_0				$a_2^{(1)}$	$a_2^{(2)}$	
K_2^0		a_1^s	$a_1^{(2)}$	$a_1^{(3)}$			a_3

TABLE II. The coefficients A_1 and A_2 in Eq. (1), for various distributions of charges in the three-pion system; we define $a^s = a_1^s + a_3$.

Charge distribution	A_1	A_2
$+-0$	$\sqrt{2}a_0 + 2a_2^{(1)}$	$\sqrt{2}a^s - 2a_1^{(2)}$
$-+0$	$-\sqrt{2}a_0 - 2a_2^{(1)}$	$\sqrt{2}a^s - 2a_1^{(2)}$
$+0-$	$-\sqrt{2}a_0 + a_2^{(1)} + \sqrt{3}a_2^{(2)}$	$\sqrt{2}a^s + a_1^{(2)} + \sqrt{3}a_1^{(3)}$
$-0+$	$\sqrt{2}a_0 - a_2^{(1)} - \sqrt{3}a_2^{(2)}$	$\sqrt{2}a^s + a_1^{(2)} + \sqrt{3}a_1^{(3)}$
$0+-$	$\sqrt{2}a_0 - a_2^{(1)} + \sqrt{3}a_2^{(2)}$	$\sqrt{2}a^s - a_1^{(2)} + \sqrt{3}a_1^{(3)}$
$0-+$	$-\sqrt{2}a_0 + a_2^{(1)} - \sqrt{3}a_2^{(2)}$	$\sqrt{2}a^s - a_1^{(2)} + \sqrt{3}a_1^{(3)}$

us denote by (1,2,3) a particular configuration of final pion momenta; i.e., particle "one" has given momentum $\mathbf{p}_1$, etc. For the given configuration, we want to compare the various possible distributions of charges: (π^+,π^-,π^0), (π^-,π^+,π^0), etc. We do not consider $3\pi^0$ decay, which is forbidden for K_1^0. The momentum configuration being fixed, the decay amplitudes for the various distributions of charge depend in an obvious way on the K_1^0 and K_2^0 amplitudes in the beam and on the intrinsic K_1^0 and K_2^0 amplitudes for decay into the various isotopic spin states of the three pion system. For any particular distribution of the charges, the general structure of the decay amplitude is

$$\text{Amplitude} = A_1 \exp(-\tfrac{1}{2}\lambda_1 t - i\omega_1 t) + A_2 \exp(-\tfrac{1}{2}\lambda_2 t - i\omega_2 t); \quad (1)$$

and the decay rate is

$$\text{Rate} = |A_1|^2 e^{-\lambda_1 t} + |A_2|^2 e^{-\lambda_2 t} + 2|A_1||A_2| \times \cos(\Delta\omega t + \varphi) \exp[-\tfrac{1}{2}(\lambda_1 + \lambda_2)t], \quad (2)$$

where φ is the phase between A_1 and A_2. Here λ_1 and λ_2 are the inverse lieftimes for K_1^0 and K_2^0 decay, respectively; and $\Delta\omega$, for low K-particle energies, is the $K_1^0 - K_2^0$ mass difference. Our expectation is that the second term in Eq. (2) is always the dominant one; that, at small time the third term may just be detectable in comparison with it; and that the first term is always negligible.

It is a straightforward matter to express the amplitude A_1 as a linear combination of the amplitudes for K_1^0 decay into the two $T=2$ and one $T=0$ final three-pion states. Likewise, one can express A_2 in terms of the amplitudes for K_2^0 decay into the three $T=1$ and one $T=3$ states. The basic amplitudes are defined in Table I. We expect that a_1^s, the amplitude for the symmetric $T=1$ state, is the dominant one. In Table II, we express A_1 and A_2 in terms of these basic amplitudes for the various distributions of charge.

From a practical point of view, what we have been saying is that the three-pion decay spectrum for a beam of particles, initially pure K^0, should change with time. If this effect proves to be detectable, one could analyze the situation by means of Eq. (2), distinguishing the various distributions of charge for given momentum configuration. In practice, one would of course not analyze separately for each sharp momentum configuration but rather in terms of ranges of momenta. One notices from Table II that the amplitudes A_2 are symmetric under the interchange $\pi^+ \rightleftarrows \pi^-$; the amplitudes A_1, antisymmetric. This is as expected and, in fact, would form the simplest basis for detecting the presence of K_1^0 contributions to three-pion decay. That is, the simplest first approach would be to group events into two classes, according to whether the π^+ energy is greater or smaller than the π^- energy. At large times, the rates for these two classes should be identical (if CP invariance is valid). At small times, any departure would indicate the presence of K_1^0 contributions. To distinguish the $T=0$ and $T=2$ amplitudes a more detailed analysis of the data would be required. Notice that according to the $|\Delta T| = \frac{1}{2}$ rule, the $T=2$ amplitudes vanish. In this case one has the simple result

$$A_1(+-0) = A_1(-0+) = A_1(0+-) = -A_1(-+0) = -A_1(+0-) = -A_1(0-+).$$

In Eq. (2), the interference term—which one could experimentally isolate—measures $A_1A_2 \equiv I$. According to our expectation, we can approximate this to lowest order in small quantities by $I \approx \sqrt{2}A_1 a^s$ (see Table II). Since the amplitude a^s is totally symmetric, we see that the vanishing of the $T=2$ amplitudes would imply

$$I(+-0) = I(-0+) = I(0+-).$$

Any departure from these equalities would represent contributions from the $T=2$ states and hence, violations of the $|\Delta T| = \frac{1}{2}$ rule.