μ-Meson Capture in Li^6 Leading to the Ground State of He^6

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We calculate the rate of the capture reaction of μ^- mesons in Li^6 leading to the He^6 ground state (Godfrey-type reaction), a process which is expected to give more accurate information on the μ-capture coupling constants than the capture in nuclei leading to all possible final states. Induced pseudoscalar coupling and Gell-Mann's conserved vector current are taken into account, and numerical results are given assuming a universal weak interaction. The Li^6 and He^6 wave functions are taken as shell model states with *LS* coupling and configuration mixing. It is found that the capture rate is sensitive to the p-shell radius, and for a determination of the latter, the Stanford electron scattering results for Li^6 have been analyzed taking into account the recoil motion of the α-particle core; however, the main portion of the radial integral in the theoretical capture rate can be read off the scattering data directly. The capture rate is found to be of the order of 0.4×10^3 sec^{-1}, its exact value still depending on some assumptions about the coupling.

I. INTRODUCTION

AFTER the establishment of nearly exact equality of the beta-decay and muon-decay coupling constants,[1] attempts are being made to determine the magnitude of the coupling responsible for muon capture. The capture rate in hydrogen being too slow for presently possible experiments as compared to the decay rate of the muon, absorption measurements have to be performed using complex nuclei. Most of these capture reactions will leave the final nucleus in many possible excited states, due to the large energy release when the muon is absorbed; the theory of this process, which may be carried through either making closure approximations[2] for the final states, or using explicit shell-model states,[3] can therefore have only an approximate character.[4] This is also seen by comparison of the existing experimental data[5,6] with the theoretical values, as done in reference 6, which shows deviations between theory and experiment (as well as between the two existing experiments) of 10 to 25% and more, for each of the nuclei used. Therefore, the approach taken by Godfrey,[7] namely to investigate a muon capture reaction which leads to the ground state of the final nucleus only, seemed to be more promising.[8] Godfrey measured the reaction rate of

$$\mu^- + C_6^{12} \rightarrow B_5^{12} + \nu, \tag{1}$$

and assured that the boron nucleus had been produced in a bound state by a simultaneous observation of the electrons from beta decay of the boron. His measurements were subsequently repeated by several experimental groups,[9–12] and Godfrey's theory has been refined by Fujii and Primakoff[13] (who also calculated the muon capture rate in Li_3^6 and He_2^3), and by Wolfenstein.[14] The experiments agree with each other within their rather large limits of error ($\gtrsim 10\%$) and do not contradict the assumption of a universal coupling also for muon capture, but it was shown by Wolfenstein[14] that even the theoretical capture rate carries a considerable uncertainty stemming from the not too well-known mixing of (jj-coupling) shell model configurations as well as from uncertain p-shell radii of carbon and boron. Moreover, the final boron nucleus possesses several bound excited states, and although most of the capture is presumed to lead to the ground state of B^{12}, there still exists considerable uncertainty on this point.[11,12]

It thus seems to be of interest to investigate the capture reaction

$$\mu^- + Li_3^6 \rightarrow He_2^6 + \nu \tag{2}$$

(leading to the ground state of He^6), which, although of smaller rate than the capture in C^{12}, nevertheless is quite accessible to experiment, and which has the significant advantage that He^6 does not possess any bound excited states.[15] Therefore, in a measurement of (2) with observation of a subsequent beta decay of He^6, one can be sure that the final nucleus had been produced in its ground state, which was not the case for reaction (1). A theory of the capture rate of (2) has already been given by Fujii and Primakoff,[13] and the present work

[1] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).
[2] H. Primakoff, Revs. Modern Phys. **31**, 802 (1959); H. A. Tolhoek, Nuclear Phys. **10**, 606 (1959).
[3] H. A. Tolhoek and J. R. Luyten, Nuclear Phys. **3**, 679 (1957).
[4] Cf. the remark of R. Peierls, *Proceedings of the 1958 Annual International Conference on High-Energy Physics at CERN*, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958), p. 249.
[5] Sens, Swanson, Telegdi, and Yovanovitch, Phys. Rev. **107**, 1464 (1957); J. C. Sens, Phys. Rev. **113**, 679 (1959).
[6] Astbury, Kemp, Lipman, Muirhead, Voss, Zangger, and Kirk, Proc. Phys. Soc. (London) **72**, 494 (1958).
[7] T. N. K. Godfrey, Ph.D. thesis, Princeton University, 1954 (unpublished); Phys. Rev. **92**, 512 (1953).
[8] This was discussed by Marshak, Telegdi, and Goldhaber, *Proceedings of the 1958 Annual International Conference on High-Energy Physics at CERN*, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958), p. 249.
[9] Fetkovich, Fields, and McIlwain, Bull. Am. Phys. Soc. **4**, 81 (1959).
[10] Love, Marder, Nadelhaft, Siegel, and Taylor, Bull. Am. Phys. Soc. **4**, 81 (1959).
[11] McGuire, Argo, Harrison, and Kruse, Bull. Am. Phys. Soc. **3**, 362 (1958).
[12] Burgman, Fischer, Leontic, Lundby, Meunier, Stroot, and Teja, Phys. Rev. Letters **1**, 469 (1958).
[13] A. Fujii and H. Primakoff, Nuovo cimento **12**, 327 (1959).
[14] L. Wolfenstein, Nuovo cimento **13**, 319 (1959).
[15] F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. **27**, 77 (1955).

was started with the intention to increase the accuracy of the calculation and to state the theoretical uncertainties precisely. We found, however, that our results were considerably smaller than those given in reference 13, due to their sensitivity to the p-shell radius of Li^6 (and He^6); this radius was obtained by us from a detailed analysis of the Stanford electron scattering data.[16,17]

In the following section, we formulate the weak interaction responsible for muon capture, including virtual pion effects which give rise to an induced pseudoscalar term as discussed by Goldberger and Treiman[18] and by Wolfenstein,[19] and possibly also to a "weak magnetic" term originating from a conserved vector current in the weak interaction, as suggested by Gell-Mann.[20] The muon capture rate will be given in terms of the nuclear matrix elements. In Sec. III, the Li^6 and He^6 ground states are specified—we adopt shell-model states with LS coupling for both of them—and the available information on the configuration mixing is discussed. In Sec. IV, we analyze the experimental data on electron scattering by Li^6 in order to obtain information on the radial distribution of the nucleons in the p shell; the analysis is made in the framework of the shell model, but takes into account the motion of the α-particle core around the center of mass of the Li^6 nucleus, which turns out to be of importance. Finally, in Sec. V, the matrix elements are evaluated, and the final results are discussed in Sec. VI.

II. FORMULATION OF THE PROBLEM

We choose a rather general form for the muon capture Hamiltonian:

$$
\begin{aligned}
H=&C_S^*(\bar{u}_n u_p)\left(\bar{u}_\nu\frac{1-\gamma_5}{\sqrt{2}}u_\mu\right)\\
&+C_V^*(\bar{u}_n\gamma_\lambda u_p)\left(\bar{u}_\nu\frac{1-\gamma_5}{\sqrt{2}}\gamma_\lambda u_\mu\right)\\
&+C_M^*(\bar{u}_n\sigma_{\lambda\rho}u_p)\left(\bar{u}_\nu\frac{1-\gamma_5}{\sqrt{2}}\gamma_\lambda\frac{1}{2m}(\nu-\mu)_\rho u_\mu\right)\\
&+C_T^*\left(\bar{u}_n\frac{1}{\sqrt{2}}\sigma_{\lambda\rho}u_p\right)\left(\bar{u}_\nu\frac{1-\gamma_5}{\sqrt{2}}\frac{1}{\sqrt{2}}\sigma_{\lambda\rho}u_\mu\right)\\
&+C_A^*\left(\bar{u}_n\frac{1}{i}\gamma_\lambda\gamma_5 u_p\right)\left(\bar{u}_\nu\frac{1-\gamma_5}{\sqrt{2}}\frac{1}{i}\gamma_\lambda\gamma_5 u_\mu\right)\\
&+C_P^*(\bar{u}_n\gamma_5 u_p)\left(\bar{u}_\nu\frac{1-\gamma_5}{\sqrt{2}}\gamma_5 u_\mu\right),
\end{aligned}
\quad (3)
$$

with

$$\sigma_{\lambda\rho}=\frac{1}{2i}(\gamma_\lambda\gamma_\rho-\gamma_\rho\gamma_\lambda),$$

which is mostly taken from Lee and Yang,[21] but contains also the magnetic-moment term induced by virtual pions, as derived by Goldberger and Treiman.[18] As we shall consider no parity nonconserving effects, two-component theory with left-handed neutrinos is assumed. The Hamiltonian (3) describes the reaction

$$\mu^-+p\rightarrow n+\nu, \quad (4)$$

and the quantities ν_ρ, μ_ρ designate the four-momenta of the respective particles; m is the nucleon mass. (In the following, we express all energies in electron rest energies, all lengths in electron Compton wavelengths.) The matrix elements for muon capture will be given below using (3); for the numerical discussion, we shall, however, consider only the following special case: The interaction is invariant under time reversal ($C_i^*=C_i$); the interaction of bare particles is of the form postulated by Feynman and Gell-Mann[1] ($C_S=C_T=0$). In addition, various assumptions will be made as follows:

Assumption 1.—There is no induced pseudoscalar and weak magnetic term ($C_P=C_M=0$); in this case, dispersion relation techniques[18] suggest that with equality of the bare coupling constants, $C_V\cong C_V^\beta$ holds also for transitions between *dressed* single-nucleon states (C with superscript β stands for the beta-decay coupling constant, C without superscript for the one in muon capture).

Assumption 2.—There is no magnetic term, but an induced pseudoscalar, with the expected magnitude of coupling[18,19] ($C_P=\epsilon C_A$, $\epsilon\cong 8$; $C_M=0$; again $C_V\cong C_V^\beta$).

Assumption 3.—Both the induced pseudoscalar and the weak magnetic term are present (see Fujii and Primakoff[13] and Bernstein[22]), ($C_P=\epsilon C_A$, $C_M=cC_V$; $c=\mu_p-\mu_n$, where μ_p, μ_n are the anomalous proton and neutron magnetic moments). With the presence of the magnetic term, analogy with the isotopic vector part of the proton charge form factor[23] gives the relation[13]:

$$C_V\cong C_V^\beta(1-\tfrac{1}{6}q^2\langle r^2\rangle_p), \quad (5)$$

where q is the four-momentum transfer, and the mean square radius of the proton is determined from the Stanford experiments[24] to be $\langle r^2\rangle_p=(0.80\pm0.04)^2\times10^{-26}$ cm^2. This gives for $\xi\equiv C_V/C_V^\beta$ the value 0.9724, if a q appropriate to muon capture in Li^6 is used.

In assumptions 2 and 3, the relative sign of C_P and C_A has been taken as positive; this follows from arguments using perturbation theory[19] as well as dispersion relations,[18] although both signs are possible if C_P is de-

[16] G. R. Burleson and R. Hofstadter, Phys. Rev. **112**, 1282 (1958).
[17] See also Meyer-Berkhout, Ford, and Green, Ann. Phys. (to be published).
[18] M. L. Goldberger and S. B. Treiman, Phys. Rev. **111**, 355 (1958).
[19] L. Wolfenstein, Nuovo cimento **8**, 882 (1958).
[20] M. Gell-Mann, Phys. Rev. **111**, 362 (1958).
[21] T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956), Eq. (A.1).
[22] J. Bernstein, Phys. Rev. **115**, 694 (1959).
[23] J. Bernstein and M. Goldberger, Revs. Modern Phys. **30**, 465 (1958).
[24] Hofstadter, Bumiller, and Yearian, Revs. Modern Phys. **30**, 482 (1958).

termined phenomenologically from $\pi-\mu$ decay. The experimental evidence on this point from muon capture in carbon[2,3] can be considered as slightly in favor of the positive sign.

Although it was also suggested[18] that $C_A \cong C_A^\beta$ for transitions between single physical nucleons, we shall in all three assumptions set

$$C_A = RC_A^\beta, \tag{6}$$

to give some room to the possible presence of meson exchange effects[25] which could destroy exact equality of C_A and C_A^β in complex nuclei. R is at present unknown (see reference 2). For C_V, we shall however use the values for single physical nucleons, C_V^β or ξC_V^β as mentioned above.

Returning now to our general Hamiltonian (3), we approximate the neutron and proton wave functions u_n, u_p appearing in it by their nonrelativistic expressions, keeping powers of order zero and one in p/m, n/m, where p, n are the magnitudes of the proton and neutron three-momenta. Neglect of the neutron-proton mass difference does not give errors larger than 1%; also the small components of the muon K-shell wave function can be neglected without causing an appreciable error. Momentum conservation at the weak vertex, $\mathbf{p}-\mathbf{n}=\boldsymbol{\nu}$, then allows us to express all first order relativistic terms by ν/m and p/m, where ν stands for the magnitude of the neutrino three-momentum. The result is

$$H=\left(\phi_n^\dagger u_\nu^\dagger \frac{1+\gamma_5}{\sqrt{2}}\left[G_F^*+G_G^*\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}^N+G_P^*\frac{\boldsymbol{\nu}}{\nu}\cdot\boldsymbol{\sigma}^N+C_V^*\frac{\mathbf{p}}{m}\cdot\boldsymbol{\sigma}\right.\right.$$
$$\left.\left.+C_A^*\frac{\mathbf{p}}{m}\cdot\boldsymbol{\sigma}^N+C_T^*\left(\frac{i\mathbf{p}}{m}\times\boldsymbol{\sigma}^N\right)\cdot\boldsymbol{\sigma}\right]u_\mu\phi_p\right), \tag{7}$$

where ϕ_n and ϕ_p are the large components of u_n, u_p, and the new Fermi, Gamow-Teller and pseudoscalar type coupling constants are given by

$$G_F=C_S+\left(1+\frac{\nu}{2m}\right)C_V-\frac{\nu}{2m}C_T,$$
$$G_G=-\frac{\nu}{2m}(C_V+C_M)+\left(1+\frac{\nu}{2m}\right)C_T+C_A, \tag{8}$$
$$G_P=\frac{\nu}{2m}(-C_V-C_M+C_T-C_A+C_P).$$

The Pauli spin vector $\boldsymbol{\sigma}^N$ operates on the nucleons, the Dirac vector $\boldsymbol{\sigma}=i\gamma\gamma_4\gamma_5$ on the leptons.

Remembering now that Li^6 has spin 1, He^6 spin 0, the beta-decay transition probability contains only the Gamow-Teller term, and with V, A coupling is given by

$$w_\beta=\frac{1}{2\pi^3}|C_A^\beta|^2 f(Z,E_0)\sum_M|\mathbf{M}_G|^2, \tag{9}$$

where

$$f(Z,E_0)=\int_1^{E_0}F(Z,E)(E^2-1)^{\frac{1}{2}}E(E_0-E)^2dE, \tag{9a}$$

and

$$\mathbf{M}_G=(\Phi_{\mathrm{Li}}^\dagger\sum_i\tau_i^+\boldsymbol{\sigma}_i\Phi_{\mathrm{He}}). \tag{9b}$$

Here M is the magnetic quantum number of the Li^6 spin, E_0 the maximum total electron energy; $F(Z,E)$ is the Fermi function (given, e.g., by Feenberg and Trigg[26]), Φ_{Li} and Φ_{He} are the wave functions of Li^6 and He^6, and τ_i^+ transforms the ith neutron in He^6 into a proton.

The transition probability for muon capture is found to be

$$w_\mu=\frac{2}{(2\pi)^2a_\mu^3}\frac{\nu^2}{1+(\nu/m_{\mathrm{He}})}\frac{1}{2J+1}$$
$$\times\sum_{M,M_f}\left\{|G_F|^2|\mathfrak{M}_F|^2+|G_G|^2|\mathfrak{M}_G|^2\right.$$
$$-(2\,\mathrm{Re}G_GG_P^*-|G_P|^2)\left|\frac{\boldsymbol{\nu}}{\nu}\cdot\mathfrak{M}_G\right|^2$$
$$-2\,\mathrm{Re}\left(G_FC_V^*\frac{\boldsymbol{\nu}}{\nu}\cdot\mathfrak{M}_V\mathfrak{M}_F^\dagger\right)$$
$$-2\,\mathrm{Im}\left(G_FC_T^*\epsilon_{klm}\frac{\nu_k}{\nu}\mathfrak{M}_{Tlm}\mathfrak{M}_F^\dagger\right)$$
$$+2\,\mathrm{Im}\left[G_GC_V^*\frac{\boldsymbol{\nu}}{\nu}\cdot(\mathfrak{M}_V\times\mathfrak{M}_G^\dagger)\right]$$
$$-2\,\mathrm{Re}\left[(G_GC_A^*-G_PC_A^*)\mathfrak{M}_{Tkk}\frac{\boldsymbol{\nu}}{\nu}\cdot\mathfrak{M}_G^\dagger\right]$$
$$\left.+2\,\mathrm{Re}\left[G_GC_T^*\frac{\nu_k}{\nu}(\mathfrak{M}_{Tkl}-\mathfrak{M}_{Tlk})\mathfrak{M}_{Gl}^\dagger\right]\right\}, \tag{10}$$

where J is the spin of the initial nucleus, M and M_f the initial and final state magnetic quantum number. The matrix elements are

$$\mathfrak{M}_F=(\Phi_f^\dagger\sum_i\tau_i^-\exp(-i\boldsymbol{\nu}\cdot\mathbf{r}_i)\varphi(\mathbf{r}_i)\Phi_0),$$
$$\mathfrak{M}_G=(\Phi_f^\dagger\sum_i\tau_i^-\exp(-i\boldsymbol{\nu}\cdot\mathbf{r}_i)\varphi(\mathbf{r}_i)\boldsymbol{\sigma}^i\Phi_0),$$
$$\mathfrak{M}_V=\left(\Phi_f^\dagger\sum_i\tau_i^-\exp(-i\boldsymbol{\nu}\cdot\mathbf{r}_i)\varphi(\mathbf{r}_i)\frac{\mathbf{p}^i}{m}\Phi_0\right), \tag{10a}$$
$$\mathfrak{M}_{Tlm}=\left(\Phi_f^\dagger\sum_i\tau_i^-\exp(-i\boldsymbol{\nu}\cdot\mathbf{r}_i)\varphi(\mathbf{r}_i)\sigma_l^i\frac{p_m^i}{m}\Phi_0\right).$$

[25] J. S. Bell and R. J. Blin-Stoyle, Nuclear Phys. **6**, 87 (1958); Blin-Stoyle, Gupta, and Primakoff, Nuclear Phys. **11**, 444 (1959).

[26] E. Feenberg and G. Trigg, Revs. Modern Phys. **22**, 399 (1950).

Here, τ_i^- transforms the ith proton in the initial state Φ_0 into a neutron; $\mathbf{r}_i$ and $\mathbf{p}^i$ are position and momentum of the ith proton, and the K-shell muon space wave function is given by $(\pi a_\mu^3)^{-\frac{1}{2}}\varphi(\mathbf{r})$, $\varphi(\mathbf{r})=\exp(-r/a_\mu)$, where a_μ is the muon Bohr radius,

$$a_\mu=\frac{137.04}{Z}\frac{1}{\mu}\left(1+\frac{\mu}{m_{\mathrm{Li}}}\right), \tag{11}$$

with μ the muon mass, m_{Li} the mass of Li^6. In (10), twice appearing tensor indices are to be summed over, and ϵ_{klm} is $1(-1)$ for (klm) an even (odd) permutation of (123), zero otherwise. Momentum and energy conservation gives for ν:

$$\nu=m_{\mathrm{He}}\left\{\left[1+\frac{2}{m_{\mathrm{He}}}(\mu-\Delta M)\right]^{\frac{1}{2}}-1\right\}, \tag{12}$$
$$\nu=196.79=100.56 \text{ Mev},$$

using a muon rest mass $\mu=206.5$, a He$^6-$Li^6 atomic mass difference $\Delta M=6.95=3.55$ Mev[15] (Wu *et al.*[27] give the end point of the beta spectrum of He^6 at 3.50 ± 0.05 Mev; we shall consider the effect of this uncertainty on our results later), and a He^6 mass as calculated from the mass defect given by Ajzenberg and Lauritsen.[15] The muon binding energy was found to be negligible in (12).

III. NUCLEAR WAVE FUNCTIONS

Shell-model states with LS coupling will be assumed for both Li^6 and He^6 ground-state wave functions, as suggested by the "superallowed" character of the beta transition.[28,29] We shall always consider the α-particle core as not participating in either beta decay or muon capture,[30,31] considering the large binding energy of the α-particle and its lack of bound excited states, and write

$$\Phi_{\mathrm{Li,He}}=v_J(1,2)R_J(r_1)R_J(r_2)\phi_{JM}, \tag{13}$$

where $J=1$ for Li^6, 0 for He^6; 1, 2 designate the two $1p$ shell nucleons, v_J the isotopic spin wave function, R_J the radial wave function. In LS coupling:

$$\begin{aligned}\phi_{00}&=C_1\,{}^1S_0+C_2\,{}^3P_0,\\ \phi_{1M}&=C_3\,{}^3S_1+C_4\,{}^1P_1+C_5\,{}^3D_1,\end{aligned} \tag{14}$$

with C_i the mixing parameters, whose sum of absolute squares is normalized to 1 both for Li and He. Information about them is obtained from the Li^6 magnetic moment and quadrupole moment, the He^6 beta decay, and the positions of the lowest excited levels[15] of He^6 and Li^6. Analysis of these gives results not quite compatible with the Li^6 magnetic moment,[32] and we shall then disregard the latter, its information being considered unreliable due to unknown exchange and relativistic effects.[32]

The Li^6 quadrupole moment Q_6 is obtained[33,34] via the Li^7 quadrupole moment, whose value in the literature ranges between $Q_7=+3.5\times10^{-26}$ cm^2 [35] and -12×10^{-26} cm^2.[36] The shell model gives

$$Q_6=\left(\frac{1}{5}C_4^2-\frac{7}{50}C_5^2-\frac{4}{5\sqrt{5}}C_3C_5\right)\langle r^2\rangle, \tag{15}$$

where $\langle r^2\rangle^{\frac{1}{2}}$ is the p-shell radius, found in Sec. IV to be $\sim4.1\times10^{-13}$ cm. Taking account of the general uncertainty, we estimate

$$-0.03\leqslant Q_6/\langle r^2\rangle\leqslant 0.01. \tag{16}$$

The level structure of Li^6, He^6 has been analyzed by Pinkston and Brennan[32] and by Meshkov.[37] Extrapolating from their values to obtain Q_6 within the limits (16), we consider the following a reasonable set of configuration mixing parameters:

$$\begin{aligned}C_3&=0.988\pm0.004,\\ C_4&=0.147\pm0.025,\\ C_5&=0.055_{-0.065}^{+0.045};\end{aligned} \tag{17}$$

note that the mean values are normalized to 1. The large uncertainty in C_5 does not matter very much in the following, due to its small value. From the He^6 levels, we take according to Meshkov[37] (and extending the uncertainty somewhat beyond his limiting values):

$$\begin{aligned}C_1&=0.941\pm0.033,\\ C_2&=-0.339_{-0.080}^{+0.111}.\end{aligned} \tag{18}$$

Information from the helium beta decay will be considered in Sec. V.

IV. ANALYSIS OF THE ELECTRON SCATTERING DATA

The form factor of the Li^6 nuclear charge distribution was obtained from electron scattering experiments performed by Burleson and Hofstadter (reference 16, Table I; see also 17). Born approximation is justified for the analysis of these data, Li^6 being a sufficiently light nucleus. The results were analyzed by the same authors

[27] Wu, Rustad, Perez-Mendez, and Lidofsky, Phys. Rev. **87**, 1140 (1952).
[28] E. Feenberg, *Shell Theory of the Nucleus* (Princeton University Press, Princeton, 1955).
[29] H. A. Bethe and P. Morrison, *Elementary Nuclear Theory* (John Wiley and Sons, New York, 1956), second edition, p. 223.
[30] In the similar process of Li^6 photodisintegration, absorption of a quantum by the α core and the subsequent transfer of its excitation to the p-shell particles has indeed been considered by L. L. Foldy (unpublished), but the experimental results[31] do not give any conclusive evidence for the presence of this mechanism.
[31] T. A. Romanowsky and V. H. Voelker, Phys. Rev. **113**, 886 (1959).
[32] W. T. Pinkston and J. G. Brennan, Phys. Rev. **109**, 499 (1958).
[33] N. G. Cranna, Can. J. Phys. **31**, 1185 (1953).
[34] P. Kusch, Phys. Rev. **92**, 268 (1953).
[35] E. G. Harris and M. A. Melkanoff, Phys. Rev. **90**, 585 (1953).
[36] R. M. Sternheimer and H. M. Foley, Phys. Rev. **92**, 1460 (1953), who argue that their value should be more reliable than the one from reference 35.
[37] S. Meshkov (unpublished); see also S. Meshkov and C. W. Ufford, Phys. Rev. **101**, 734 (1956); S. Meshkov, Bull. Am. Phys. Soc. **4**, 255 (1959).

TABLE I. Experimental form factor of Li^6, after unfolding of the intrinsic proton charge distribution. The first three values were taken from Meyer-Berkhout *et al.*,[a] the rest from Burleson and Hofstadter.[b]

q (units 10^{13} cm^{-1})	$F(q)$	q	$F(q)$	q	$F(q)$
0.55	0.728	1.39	0.175	1.74	0.0830
0.78	0.537	1.47	0.153	1.80	0.0679
1.02	0.372	1.51	0.141	1.88	0.0561
1.22	0.242	1.56	0.124	1.97	0.0418
1.30	0.216	1.65	0.107	2.05	0.0381

[a] See reference 17.
[b] See reference 16.

using various static charge distributions; from the shell model point of view, the most interesting one of those is the "modified harmonic-well shell model," which assumes the charge distribution to consist of a sum of s- and p-shell Gaussian functions. Although this gives an over-all rms radius for the Li^6 of

$$\langle r^2\rangle_{\mathrm{Li}}^{\frac{1}{2}}=2.82\times10^{-13}\text{ cm},\qquad(19)$$

in agreement with results from other possible charge distributions, the rms s- and p-shell radii separately would come out as 3.24×10^{-13} cm and 1.69×10^{-13} cm, respectively. This clearly shows that the analysis is in need of refinement, as the p shell, being bound more weakly, is expected to have a larger radius than the s shell.

It seems that the most important effect to be taken into account would be the motion of the α-particle core around the center of mass of the Li^6 nucleus. Some other effects need however to be considered also. First of all, each of the protons in Li^6 has an intrinsic charge distribution whose shape can be taken as Gaussian,[24] with an rms radius $a=(0.80\pm0.04)\times10^{-13}$ cm. If this shape is preserved for a proton bound in Li^6, the observed charge distribution $\rho_{\mathrm{obs}}(r)$ is actually a folding of the proton intrinsic charge distribution $\rho_{\mathrm{prot}}(r)$ into the distribution of the proton's center of mass, $\rho(r)$:

$$\rho_{\mathrm{obs}}(r)=\int\rho(r')\rho_{\mathrm{prot}}(|\mathbf{r}-\mathbf{r}'|)d^3r'.$$

The corresponding relation for the form factors is

$$F(q)=\exp[\tfrac{1}{6}(qa)^2]F_{\mathrm{obs}}(q),\qquad(20)$$

and the experimental form factor after unfolding of the proton intrinsic spread is given in Table I. The error in these values, being $\sim5\%$ for $F_{\mathrm{obs}}(q)$, becomes $\sim7\%$ for $F(q)$ due to the uncertainty in a.

The α-particle core has an intrinsic spread which can also be described by a Gaussian; its rms radius after unfolding of the proton spread is given experimentally[38] by $a_\alpha=(1.40\pm0.11)\times10^{-13}$ cm from electron scattering, in agreement with a value of $(1.44\pm0.07)\times10^{-13}$ cm deduced from the He^4 photodisintegration.[39] As a result, the charge form factor is written as

$$F(q)=\tfrac{2}{3}\exp[-\tfrac{1}{6}(qa_\alpha)^2]F_r(q)+\tfrac{1}{3}F_p(q);\qquad(21)$$

the intrinsic α charge spread is separated here from the spread due to its recoil motion: $F_r(q)$ describes the motion of the α core center of mass around the Li^6 center of mass, and of course is closely related to the p-shell form factor $F_p(q)$ representing the motion of the (point) proton in the p shell.

In order to evaluate these remaining two functions, we adopt the shell model wave function (13) for the two outside particles. Their coordinates $\mathbf{r}_1$, $\mathbf{r}_2$ are then referred to the center of mass of the Li^6 nucleus, whereas a correct dynamical treatment would probably require taking the α particle as the origin. However, this is not expected to cause a great error,[30] and our treatment remains at least consistently within the framework of the shell model. In view of the values (17), we also consider the p-shell particles to be in the $^3S_1$ state only; this assumption is probably justified for the purpose of the present analysis, although we did not make an estimate of the contributions from the other configurations. The p-shell form factor is then easily shown to be

$$F_p(q)=\int R_1^2(r)j_0(qr)r^2dr,\qquad(21a)$$

where $j_l(x)$ is the spherical Bessel function of order l. The recoil motion of the core is determined by the motion of the outside proton and neutron; the coordinate of the core center is always given by

$$\mathbf{r}_c=-\tfrac{1}{4}(\mathbf{r}_1+\mathbf{r}_2),$$

with $\frac{1}{2}(\mathbf{r}_1+\mathbf{r}_2)$ the center-of-mass coordinate of the two p-shell particles. We can thus derive the following expression:

$$F_r(q)=F_p^2(q/4)+2G_p^2(q/4),\qquad(21b)$$

with

$$G_p(q)=\int R_1^2(r)j_2(qr)r^2dr.\qquad(21c)$$

The equations (21) now permit us to determine the p-shell radius, by fitting $F(q)$ of (21) to the experimental form factor of Table I. We tried for $R_1(r)$ an exponential function $re^{-\frac{1}{2}\alpha r}$, which gave no fit, but with a harmonic oscillator function

$$R_1=N_\alpha r\exp(-\tfrac{1}{2}\alpha^2r^2),\quad N_\alpha^2=8\alpha^5/3\pi^{\frac{1}{2}},\quad \alpha^2=\tfrac{5}{2}(1/b^2),\qquad(22)$$

we were able to fit the data fairly well, as demonstrated in Fig. 1. The p-shell radius thus obtained is

$$b\cong4.1\times10^{-13}\text{ cm}.\qquad(23)$$

This result may seem somewhat large, but it is supported by recent variational calculations on the Li^6

[38] R. W. McAllister and R. Hofstadter, Phys. Rev. **102**, 851 (1956); see also D. G. Ravenhall, Revs. Modern Phys. **30**, 430 (1958).

[39] M. L. Rustgi and J. S. Levinger, Phys. Rev. **106**, 530 (1957).

ground state,[40] and moreover it is consistent with the results (19): using the same methods which lead to (21b), we can derive the mean square radius of the recoil motion of the core center as

$$\langle r^2\rangle_r = \tfrac{1}{8}b^2, \tag{24a}$$

which gives for the mean square core radius (including its motion)

$$\langle r^2\rangle_c = \langle r^2\rangle_r + a_\alpha^2 = (2.02\times10^{-13})^2\ \text{cm}^2; \tag{24b}$$

the overall mean square radius of Li^6 then follows as

$$\langle r^2\rangle_{\text{Li}} = \tfrac{2}{3}\langle r^2\rangle_c + \tfrac{1}{3}b^2 = (2.88\times10^{-13})^2\ \text{cm}^2, \tag{24c}$$

in not too bad disagreement with the value (19) (even after the intrinsic proton radius was extracted from (19)). The adding of the squared radii in (24) is maybe not quite correct, as we are not always concerned with simple Gaussian distributions.

The slight disagreement of the curve in Fig. 1 with the experimental points need not worry us too much, as we shall use (22) with the radius (23) only for calculating small terms in the muon capture rate. It will be shown in the next section that the main part of the radial integral which determines the capture rate can be read off the experimental data directly.

V. EVALUATION OF THE CAPTURE RATE

The matrix elements in Eq. (10) for the muon capture rate have been evaluated using the shell model wave functions (13) for the initial Li^6 and final He^6 nuclear states; $\mathfrak{M}_F$ is then zero from the same arguments which lead to the Fermi selection rule in beta decay, as we have a transition from spin 1 to spin 0; also the beta decay of He^6 is a pure Gamow-Teller transition. The other results are

$$\Lambda_1 \equiv \sum_M |\mathfrak{M}_G|^2 = 6J^2K^2(J_0^2+2J_2^2D_1^2),$$

$$\Lambda_2 \equiv \sum_M \left|\frac{\mathbf{v}}{\nu}\cdot\mathfrak{M}_G\right|^2 = 2J^2K^2(J_0-2J_2D_1)^2,$$

$$\frac{\nu}{m}\Lambda_3 \equiv \sum_M 2\,\mathrm{Im}\frac{\mathbf{v}}{\nu}\cdot(\mathfrak{M}_V\times\mathfrak{M}_G^\dagger) = -12(\tfrac{2}{3})^{\frac{1}{2}}\frac{\nu}{m}J^2K^2J_1D_2(J_0+J_2D_1), \tag{25}$$

$$\frac{\nu}{m}\Lambda_4 \equiv \sum_M 2\,\mathrm{Re}\mathfrak{M}_{Tkk}\frac{\mathbf{v}}{\nu}\cdot\mathfrak{M}_G^\dagger = -4\frac{\nu}{m}J^2K^2(J_1D_3-J_1'D_4)(J_0-2J_2D_1);$$

[40] N. Austern (private communication). See also P. A. Wackman and N. Austern, Bull. Am. Phys. Soc. 4, 254 (1959).

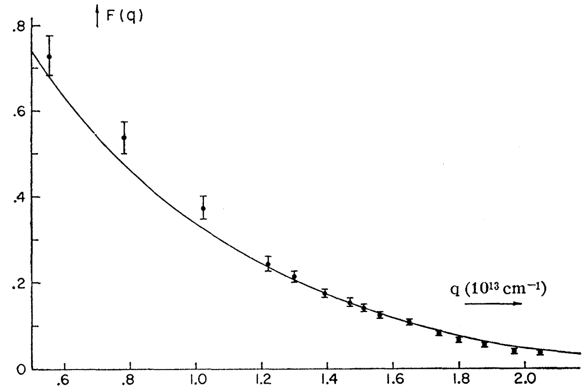

FIG. 1. Comparison of the Stanford experimental form factor of Li^6 for electron scattering (proton charge distribution unfolded) with a theoretical form factor using harmonic oscillator p-shell wave functions.

here the radial integrals are given by

$$\begin{aligned}
J_0 &= \int R_1(r)R_0(r)\varphi(r)j_0(\nu r)r^2dr,\\
J_1 &= \int R_1(r)R_0(r)\varphi(r)\frac{j_1(\nu r)}{\nu r}r^2dr,\\
J_2 &= \int R_1(r)R_0(r)\varphi(r)j_2(\nu r)r^2dr,\\
J_1' &= -\int r^2\left(\frac{d}{dr}\frac{R_1(r)}{r}\right)R_0(r)\varphi(r)\frac{j_1(\nu r)}{\nu r}r^2dr;
\end{aligned} \tag{25a}$$

further we have

$$\begin{aligned}
D_1 &= K^{-1}\left(\frac{1}{\sqrt{5}}C_1C_5+\frac{1}{2\sqrt{3}}C_2C_4+\frac{3}{2(10)^{\frac{1}{2}}}C_2C_5\right),\\
D_2 &= K^{-1}\left(C_1C_4-\frac{1}{\sqrt{3}}C_2C_3+\frac{1}{2}\frac{\sqrt{5}}{\sqrt{3}}C_2C_5\right),\\
D_3 &= K^{-1}\Big(C_1C_3+(5)^{\frac{1}{2}}C_1C_5-\sqrt{2}C_2C_3\\
&\qquad +\sqrt{3}C_2C_4+\frac{\sqrt{5}}{\sqrt{2}}C_2C_5\Big),\\
D_4 &= K^{-1}\left(C_1C_3+\frac{2}{\sqrt{5}}C_1C_5+\frac{3}{(10)^{\frac{1}{2}}}C_2C_5\right).
\end{aligned} \tag{25b}$$

The other quantities,

$$J = \int R_1(r)R_0(r)r^2dr \tag{26a}$$

and

$$K = C_1C_3 - \frac{1}{\sqrt{3}}C_2C_4 \tag{26b}$$

TABLE II. Values of ϵ, c, ξ under assumptions 1, 2, and 3.

	ϵ	c	ξ
1	0	0	1
2	8	0	1
3	8	3.7062	0.9724

appear already in the beta-decay matrix element, given by

$$\Lambda \equiv \sum_M |M_G|^2 = 6J^4K^2. \tag{26}$$

The results (25) have been obtained by forming irreducible tensors in the nuclear matrix elements (10a) and applying the Wigner-Eckart theorem; also certain identities for spherical harmonics[41] have been found useful.

For a numerical evaluation of the capture rate, we now make the assumptions listed after Eq. (4). The ratio of Gamow-Teller to Fermi coupling in beta decay,

$$X_\beta = |C_A{}^\beta / C_V{}^\beta|, \tag{27}$$

is according to the latest two experiments $X_\beta = 1.25 \pm 0.04$ and $X_\beta = 1.19 \pm 0.03$, with a negative relative sign.[42] We shall present our results for various values of X_β within these limits. For the ratio of He^6 beta decay to Li^6 muon capture half-life, we then find

$$\frac{1}{t_\mu}(ft)_\beta = C_0(AR^2+BR+C), \tag{28}$$

with

$$R = C_A/C_A{}^\beta \sim 1,$$

$$C_0 = \frac{1}{3}\frac{\pi}{a_\mu{}^3}\frac{\nu^2}{1+(\nu/m_{\mathrm{He}})} = 3.481\times 10^6,$$

and

$$\begin{aligned} \Lambda A &= \Lambda_1 - \epsilon\beta\Lambda_2 + \beta(\Lambda_2-\Lambda_4) \\ &\quad + \tfrac{1}{4}(\epsilon\beta)^2\Lambda_2 - \tfrac{1}{4}\epsilon\beta^2(\Lambda_2 - 2\Lambda_4), \\ X_\beta \Lambda B &= \xi[c\beta(\Lambda_1-\Lambda_2) \\ &\quad + \beta(\Lambda_1-\Lambda_2-\Lambda_3) + \tfrac{1}{4}(c+\epsilon)\beta^2\Lambda_2], \\ X_\beta{}^2\Lambda C &= \xi^2[\tfrac{1}{4}(c\beta)^2(\Lambda_1-\Lambda_2) + \tfrac{1}{2}c\beta^2(\Lambda_1 - \tfrac{1}{2}\Lambda_2 - \Lambda_3)], \end{aligned} \tag{29}$$

where $\beta = \nu/m = 0.1072$, and where according to the three assumptions taken in Sec. I, the quantities ϵ, c, and ξ are given by Table II. Equations (29) show the different relativistic orders of the terms (terms of order $\epsilon\beta^2$ and $c\beta^2$ were kept, although they are not the only ones of this order. Their contribution is at most 2%). The terms Λ_3 and Λ_4 were neglected by Fujii and Primakoff,[13] but $\beta\Lambda_3$, $\beta\Lambda_4$ are of the same size (~5%) as the other terms $\beta\Lambda_1$, $\beta\Lambda_2$ of this order.

Analysis of the beta-decay data shows that the overlap in J is not 100% complete. We here use the value $C_V{}^\beta \equiv G = (1.410\pm 0.009)\times 10^{-49}$ erg cm^3 (reference 42, p. 241), calculate $f=1030$ (using $\Delta M = 3.55$ Mev; $f=965$ for $\Delta M = 3.50$ Mev) including Coulomb corrections,[26] and take the He^6 half-life to be $t=0.82\pm 0.02$ sec, a weighted average over many experimental values.[43] Note that we will not insert this somewhat uncertain value in (28), but use it only for calculating the degree of overlap of J. Using (17) and (18), we obtain from (9) and (26) the values of J given in Table III, as a function of X_β.

Now the deviation of J from 1 will have to be taken into account in a calculation of J_i. We consider two possible reasons for $J\neq 1$: (a) He^6 being bound less tightly than Li^6, the radial p shell wave function R_0 has a larger radius than R_1; (b) R_0 and R_1 being equal, except for small r where they fail to overlap. This is assumed to involve only the wave function of the particle which undergoes the transition. Accordingly, the values of Table III have then to be identified with J^2 rather than J^4.

TABLE III. Overlap integral in the He^6 beta decay.

X_β	J^4	X_β	J^4
1.16	$0.99_{-0.11}{}^{+0.01}$	1.22	0.89 ± 0.10
		1.25	0.85 ± 0.10
1.19	$0.94_{-0.10}{}^{+0.06}$	1.29	0.80 ± 0.09

Both methods give very similar results; if we adopt Gaussian wave functions (22) with radius b from (23), the values of J_0/J thus calculated differ only by 2% (for $X_\beta = 1.16$) to 5% (for $X_\beta = 1.29$), method (a) giving the smaller values. In Table IV, we show the averages for J_0/J obtained in this way in the second column. For complete overlap ($J=1$), again using Gaussian functions, the radial integrals are calculated as $\bar{J}_0 = 0.441$, $\bar{J}_1 = 0.192$, $\bar{J}_2 = 0.134$, $\bar{J}_1{}' = 0.435$.

These values can be improved and made almost independent of any special shape of R_J as follows. Numerical analysis showed that a predominant part of (21) was contributed by the term $F_p{}^2(q/4)$ in (21b), for all the values of q considered. Now $F_p(\nu)$ is the same as J_0 in (25a), if in it we replace R_0 by R_1 and φ by 1. But the deviation of R_0 from R_1 and of φ from 1 gives only a small contribution to J_0 which can be calculated using our Gaussian R_1. Likewise, all the terms in $F(q)$ except the term $F_p{}^2(q/4)$ were calculated with Gaussian R_1 for

[41] H. A. Bethe and E. E. Salpeter, *Encyclopedia of Physics* (Academic Press, New York, 1957), Vol. 35, pp. 432, 435.

[42] M. Goldhaber, *Proceedings of the 1958 Annual International Conference High-Energy Physics, at CERN*, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958), p. 241 and p. 238 (footnote).

[43] We took from the authors quoted in reference 15 and in Strominger, Hollander, and Seaborg, Revs. Modern Phys. **30**, 585 (1958), and from two further papers: B. M. Rustad and S. L. Ruby, Phys. Rev. **97**, 991 (1955), and E. C. Campbell and P. H. Stelson, Oak Ridge National Laboratory ORNL-2076, 1956 (unpublished), p. 32.

$q=4\nu$; then $F_p(\nu)$ was determined by equating $F(4\nu)$ to the value $F_{\exp}(4\nu)=0.0390\pm0.0019$ interpolated from the experimental points of Table I, and identified with the principal part of J_0. This gave the values labeled $(J_0/J)_{\exp}$ in Table IV. The uncertainty quoted comes from the combined uncertainties of $C_V{}^\beta$, ft, $F_{\exp}(4\nu)$, and C_i in (17), (18). If ΔE were chosen 3.50 Mev[27] instead of 3.55 Mev, all J_0/J would shift closer to $\bar{J}_0$ by $\lesssim 5\%$ of their value (less for larger X_β; maximum shift for $X_\beta=1.22$).

For J_1, J_2, and J_1', no effect of incomplete overlap was calculated, as they appear only in small terms $\lesssim 10\%$. We nevertheless chose to reduce them by a fraction corresponding to the reduction of J_0 in going from the second to the third column of Table IV, and take for the complete overlap values: $\bar{J}_0=0.387$, $\bar{J}_1=0.170$, $\bar{J}_2=0.116$, $\bar{J}_1'=0.381$.

The difference between the "calculated" and the "fitted" values in Table IV is quite large, and reflects the fact that the points on the upper end of Fig. 1 lie off the theoretical curve by an amount of $\sim 10\%$. From the procedure outlined above, we consider $(J_0/J)_{\exp}$ as the correct values, to be used in our final results, although the derivation relies rather heavily on the correctness of (21b). The question of the trustworthiness of the results of Table IV for $X_\beta \gtrsim 1.22$, where J_0 deviated significantly from the complete overlap value $\bar{J}_0$, becomes less important by noticing that earlier sources, e.g., the work of Kofoed-Hansen on mirror nuclei,[44] have always favored smaller values of X_β, and the more recent Russian values on the neutron decay[42] go in the same direction.

TABLE IV. The radial integral J_0.

X_β	$(J_0/J)_{\mathrm{Av}}$	$(J_0/J)_{\exp}$
1.16	0.428	$0.374_{-0.041}^{+0.013}$
1.19	0.410	$0.356_{-0.034}^{+0.030}$
1.22	0.398	$0.344_{-0.029}^{+0.033}$
1.25	0.389	$0.335_{-0.027}^{+0.030}$
1.29	0.377	$0.323_{-0.023}^{+0.029}$

With all this information now gathered, we finally obtain the values for A, B, and C under our three assumptions, and for $t_\mu{}^{-1}(ft)_\beta$ if we set $R=1$, as listed in Table V.

VI. DISCUSSION

In the derivation of the values in Table V, large uncertainties have been circumvented by plotting the results for various X_β, by taking $(ft)_\beta$ out of the capture rate and by leaving R undetermined. These three quantities may become better known in the future. Nevertheless, the values obtained still carry large uncertainties, up to 20% for some cases; the accuracy to which they may serve for determining the coupling constant is thus not too high. It seems however the best one can do at present.

TABLE V. Muon capture rate in Li^6.

X_β	A	B	C	$(10^3 t_\mu)^{-1}(ft)_\beta$
		Assumption 1		
1.16	$0.141_{-0.029}^{+0.010}$	0.012	0	533_{-101}^{+35}
1.19	$0.128_{-0.023}^{+0.023}$	0.011	0	483_{-80}^{+80}
1.22	$0.119_{-0.019}^{+0.024}$	0.010	0	449_{-66}^{+84}
1.25	$0.113_{-0.017}^{+0.021}$	0.009	0	424_{-59}^{+73}
1.29	$0.105_{-0.014}^{+0.020}$	0.008	0	393_{-49}^{+70}
		Assumption 2		
1.16	$0.110_{-0.021}^{+0.009}$	0.013	0	431_{-73}^{+31}
1.19	$0.100_{-0.018}^{+0.018}$	0.011	0	386_{-63}^{+63}
1.22	$0.093_{-0.015}^{+0.018}$	0.010	0	359_{-52}^{+63}
1.25	$0.088_{-0.013}^{+0.016}$	0.010	0	340_{-45}^{+56}
1.29	$0.082_{-0.011}^{+0.015}$	0.009	0	316_{-38}^{+52}
		Assumption 3		
1.16	$0.110_{-0.021}^{+0.009}$	$0.044_{-0.006}^{+0.002}$	0.005	556_{-95}^{+39}
1.19	$0.100_{-0.018}^{+0.018}$	$0.039_{-0.005}^{+0.005}$	0.004	498_{-81}^{+81}
1.22	$0.093_{-0.015}^{+0.018}$	$0.036_{-0.004}^{+0.005}$	0.004	462_{-66}^{+81}
1.25	$0.088_{-0.013}^{+0.016}$	$0.033_{-0.003}^{+0.004}$	0.003	432_{-57}^{+71}
1.29	$0.082_{-0.011}^{+0.015}$	$0.030_{-0.003}^{+0.004}$	0.003	399_{-48}^{+66}

The first feature we observe is the reduction of the capture rate by a factor $\gtrsim 4$ compared to the value given by Fujii and Primakoff.[13] We ascertained that this came mostly from their choice of $b=2.40\times10^{-13}$ cm, whereas the correct p-shell radius (23) obtained from the analysis of the Stanford data should be used. This point was noticed by Primakoff, the resulting uncertainty was, however, underestimated.

Further sources of error in our results may be mentioned. The value $\epsilon=8$ in the induced pseudoscalar is not known too accurately; it was evaluated[19] using the experimental π-μ decay rate and Chew's pion-nucleon coupling constant $f^2\sim0.08$, both quantities carrying certain errors, and off-energy shell effects and contributions of three and more pions to the induced pseudoscalar were not considered quantitatively in reference 19. We completely disregarded possible core excitation effects,[30] and the evaluation of J_0 in connection with incomplete overlap in J may be open to criticism. Finally, the applicability of the shell model may be questioned.

As far as effects of the induced pseudoscalar and Gell-Mann's weak magnetic term are concerned, they may not be recognizable with very great certainty from our results. Comparing assumptions 1 and 3, we see that they almost cancel each other if present simultaneously. Our large limits of error permit at $X_\beta=1.22$, e.g., that a measured value for $(10^3 t_\mu)^{-1}(ft)_\beta$ of ~400 fits all three assumptions. However, a difference of a factor four in the capture rate should immediately be detectable experimentally and may thus confirm our choice of the p-shell radius.

To give an idea of the actual capture rate, we shall state the most likely values of the muon capture

[44] See, e.g., C. S. Wu, in *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, 1955).

probability for $X_\beta=1.19$; they are:

$$w_\mu=0.396\times10^8 \text{ sec} \quad \text{(assumption 1)}$$
$$0.317\times10^8 \text{ sec} \quad \text{(assumption 2)}$$
$$0.409\times10^8 \text{ sec} \quad \text{(assumption 3)},$$

where $(ft)_\beta=845$ sec is used according to Sec. V.

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Determination of the Pion-Nucleon Coupling Constant from *n-p* Scattering Angular Distribution*†

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By the use of a method recently proposed by Chew, the pion-nucleon coupling constant is determined from differential cross sections for neutron-proton scattering. Data at 90 and 400 Mev were used. Details of the extrapolation procedure are discussed and the statistical methods used in interpreting the results are explained. The resulting value of the coupling constant is between 0.06 and 0.07, depending on the range and energy of the data included in the analysis. The discrepancy between this value and the usually quoted 0.08 should not be taken seriously, however, because several nonstatistical uncertainties could not be taken into account. The origin of these uncertainties is discussed.

I. INTRODUCTION

IN a recent paper Chew[1,2] suggested a method of determining the pion-nucleon coupling constant from differential cross sections for nucleon-nucleon scattering. The method is based on the conjectured occurrence of poles in the nucleon-nucleon scattering amplitude at certain unphysical values of the scattering angle. If p_1 and q_1 are the initial four-momenta of the nucleons, and p_2 and q_2 the final four-momenta, the momentum transfer is defined as $t=-(p_2-p_1)^2$ and the crossed momentum transfer as $\bar{t}=-(p_2-q_1)^2$. We use a metric such that $p^2=\mathbf{p}^2-E^2$, and our units are $\hbar=c=1$. There is then a pole at $t=\mu^2$ and another at $\bar{t}=\mu^2$, where μ is the pion mass. In terms of the barycentric scattering angle θ and the barycentric three-momentum p, the first of these poles[3] corresponds to $\cos\theta=+(1+\mu^2/2p^2)$, and the second to $\cos\theta=-(1+\mu^2/2p^2)$. If in the case of neutron-proton scattering one associates p_1 and p_2 with the proton, and q_1 and q_2 with the neutron, then in terms of Feynman diagrams the first pole gives the contribution of the exchange of a single neutral pion (forward scattering) whereas the second pole gives the contribution of the exchange of a single charged pion (charge-exchange scattering). In addition to the poles, one conjectures also the existence of branch points, corresponding to higher-order processes, when t or $\bar{t}$ becomes $(2\mu)^2$, $(3\mu)^2$, etc. In terms of $\cos\theta$ these branch points occur at

$$\cos\theta=\pm(1+4\mu^2/2p^2), \quad \pm(1+9\mu^2/2p^2), \quad \cdots. \tag{1}$$

They are thus considerably farther from the ends of the physical region than are the poles. One may therefore hope that the poles will tend to dominate the physical region, especially near the ends.[1] The residues of the poles are known exactly and are proportional to g^2, the pion-nucleon coupling constant. In fact the pole terms are formally identical with the two second-order one-pion exchange diagrams in perturbation theory. It must be pointed out, however, that we are not merely doing perturbation theory. In the first place the coupling constant and masses that are involved refer to actual physical particles and not to bare particles as would be the case in perturbation theory. Secondly, once the existence and position of the poles are accepted, their contribution to the scattering amplitude can be calculated without any reference whatever to perturbation theory, for instance by the method used by Goldberger, Nambu, and Oehme.[4]

Chew's suggestion then entails determining the resi-

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† A preliminary account of this work was given in Bull. Am. Phys. Soc. Ser. II, **3**, 404 (1958).

[1] Geoffrey F. Chew, Phys. Rev. **112**, 1380 (1958).

[2] Geoffrey F. Chew, *Proceedings of the 1958 Annual International Conference on High-Energy Physics at CERN* (CERN, Geneva, 1958), p. 96.

[3] It might be instructive to point out that this pole also occurs in the classical Born approximation which is simply the Fourier transform of $g^2r^{-1}\exp(-\mu r)$. For a massless carrier of the field, the corresponding pole is *at* the edge of the physical region, which is the reason why the differential cross section for Coulomb scattering at 0° is infinite.

[4] Goldberger, Nambu, and Oehme, Ann. Phys. **2**, 226 (1957). See especially pp. 243 to 245.