"Nucleon-Structural" Corrections to First Forbidden Unique Beta Transitions

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The coupling of leptons to a conserved isovector current is shown to result in a small change in the shape of the electron momentum spectrum in forbidden unique beta transitions. The experimental advantages and disadvantages of studying such nucleon-structural effects in forbidden unique transitions rather than in allowed transitions, as suggested by Gell-Mann, are discussed. To facilitate the computations, the beta-decay interaction Hamiltonian is written in a form in which the terms leading to the "nucleon-structural" corrections associated with the coupling of the leptons to a conserved isovector current and the terms leading to effects of comparable order of magnitude are simply identified.

INTRODUCTION

To explain the remarkable numerical agreement between the vector coupling constants in nuclear beta decay and in muon beta decay, Feynman and Gell-Mann[1] have hypothesized that the lepton pair emitted in beta decay is coupled primitively to the same conserved isovector current as the photon emitted in an electromagnetic interaction. If the weak-coupling Hamiltonian does indeed have the conjectured form, then the associated "nucleon-structural" effects can be exactly taken into account, since the known values of the nucleon magnetic moments and the results of the Hofstadter experiments[2] on electron-nucleon scattering provide the necessary structural data.

Gell-Mann[3,4] has shown that the coupling of the lepton field to a conserved isovector current yields a small but possibly observable distortion in the shape of the electron momentum spectrum in allowed beta decay obeying Gamow-Teller selection rules. A more detailed study of the effect of a lepton-conserved isovector current coupling on the shape of allowed beta transition momentum spectra and on beta-gamma and beta-alpha directional correlations has been given by Bernstein and Lewis.[5] Boehm, Soergel, and Stech[6] have also suggested the study of beta-gamma directional correlation to detect the lepton-conserved isovector current coupling and have sought the effect experimentally. Furthermore, Fujii and Primakoff[7] have demonstrated that the transition rate for μ capture is pronouncedly affected by the weak coupling of the leptons to the conserved isovector current implied by the Feynman–Gell-Mann hypothesis, the relatively large effect in μ capture being a consequence of the large momentum transfer between nucleon and lepton field, the latter acting as a structural probe.

Another effect of the conservation of isovector current in weak interactions, which is explored here, is a small change in the shape of the electron momentum spectrum in forbidden unique transitions. The experimental detection of such nucleon-structural corrections to forbidden unique transitions is facilitated by their long lifetime and flat spectral shape, relative to allowed transitions of comparable energy release. To compute the shape corrections, an "anomalous magnetic moment" term is added to the usual V-A beta decay interaction Hamiltonian, and the resulting interaction is expressed in terms of appropriate even Dirac operators describing the lowest order terms plus higher order terms involving, in general, odd Dirac operators. All leading corrections to the ordinary beta transition matrix elements are then identified. A similar procedure has been applied to the study of nucleon-structural effects in muon capture by Fujii and Primakoff.[7] Detailed considerations are limited to first forbidden unique transitions, of which many examples are known.[8]

I. LOW-MOMENTUM TRANSFER INTERACTION

The current of nucleons and of particles strongly interacting with nucleons, e.g., pions, which is coupled to the electromagnetic field is, to first order in α, the fine structure constant

$$j_\mu(x)=ie\bar{\psi}_N(x)\gamma_\mu\left(\frac{1+\tau_3}{2}\right)\psi_N(x)-ie\phi^\dagger(x)t_3\frac{\partial}{\partial x_\mu}\phi(x),$$

$$\psi_N(x)\equiv\text{nucleon field amplitude}=\begin{pmatrix}\psi_p\\ \psi_n\end{pmatrix}, \qquad \text{(I-1)}$$

$$\phi(x)\equiv\text{pion field amplitude}=\begin{bmatrix}\phi_1\\ \phi_2\\ \phi_3\end{bmatrix},$$

$$\tau_3\equiv\begin{pmatrix}1&0\\0&-1\end{pmatrix}\qquad t_3\equiv\begin{bmatrix}0&-i&0\\ i&0&0\\ 0&0&0\end{bmatrix}.$$

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[1] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).
[2] Hofstadter, Bumiller, and Yearian, Revs. Modern Phys. **30**, 482 (1958).
[3] M. Gell-Mann, Phys. Rev. **111**, 362 (1958).
[4] M. Gell-Mann, *1958 Annual International Conference on High-Energy Physics at CERN*, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958), p. 256.
[5] J. Bernstein and R. Lewis, Phys. Rev. **112**, 232 (1958).
[6] Boehm, Soergel, and Stech, Phys. Rev. Letters **1**, 2 (1958).
[7] A. Fujii and H. Primakoff, Nuovo cimento **12**, 327 (1957).
[8] E. Feenberg, *Shell Theory of the Nucleus* (Princeton University Press, Princeton, New Jersey, 1955).

The pion field can be eliminated from explicit consideration if the interaction between the physical nucleon field and the electromagnetic field involves a momentum transfer small compared to $m_\pi c$. The current in the limiting case of small momentum transfer takes a form which describes the nucleon-structural effects and which is essentially experimentally justified:

$$j_\mu(x)=ie\left\{\bar{\psi}_N(x)\gamma_\mu\left(\frac{1+\tau_3}{2}\right)\psi_N(x)+i\bar{\psi}_N(x)\sigma_{\mu\nu}\times\left[\frac{(\kappa_p+\kappa_n)}{2e}+\frac{(\kappa_p-\kappa_n)}{2e}\tau_3\right]\psi_N(x)\frac{\partial}{\partial x_\nu}\right\}, \quad \text{(I-2)}$$

$$\begin{aligned}\kappa_p&\equiv\text{proton ``anomalous'' magnetic moment}=1.79e/2m_p,\\ \kappa_n&\equiv\text{neutron ``anomalous'' magnetic moment}=-1.91e/2m_p.\end{aligned} \quad \text{(I-2)}$$

The current (I-1) and (I-2) can be decomposed into an isoscalar part and an isovector part, each of which is separately conserved:

$$j_\mu(x)=j_\mu^{(S)}(x)+[\mathbf{j}_\mu^{(V)}(x)]_3,$$

$$j_\mu^{(S)}(x)=ie\left\{\bar{\psi}_N(x)\frac{\gamma_\mu}{2}\psi_N(x)+i\bar{\psi}_N(x)\sigma_{\mu\nu}\frac{(\kappa_p+\kappa_n)}{2e}\psi_N(x)\frac{\partial}{\partial x_\nu}\right\},$$

$$\mathbf{j}_\mu^{(V)}(x)=ie\left\{\bar{\psi}_N(x)\gamma_\mu\frac{\boldsymbol{\tau}}{2}\psi_N(x)+i\bar{\psi}_N(x)\sigma_{\mu\nu}\frac{(\kappa_p-\kappa_n)}{2e}\boldsymbol{\tau}\psi_N(x)\frac{\partial}{\partial x_\nu}\right\}, \quad \text{(I-3)}$$

$$\partial j_\mu^{(S)}(x)/\partial x_\mu=0;\quad \partial\mathbf{j}_\mu^{(V)}(x)/\partial x_\mu=0.$$

The conservation of total isocurrent insures the conservation of electric charge; the conservation of isoscalar current insures the conservation of nucleon charge or number; the conservation of isovector current insures the conservation of "weak" charge if the lepton field in weak interactions is coupled to the relevant component of the conserved isovector current.

The Feynman–Gell-Mann hypothesis now states that the lepton field vector covariant

$$A_\mu(x)\equiv\bar{\psi}_e(x)\gamma_\mu(1+\gamma_5)\psi_\nu(x)$$

is indeed coupled to the (+) component of the conserved isovector current of nucleons, pions, $\cdots$ in just the same manner as the electromagnetic field vector amplitude $A_\mu(x)$ is coupled to the 3-component of the same conserved isovector current, the only difference being in the value of the effective coupling constant. Accordingly, the effective interaction Hamiltonian density, generating the low-momentum transfer weak nucleon-lepton interaction, is

$$\mathcal{H}^{(V)}(x)=C_V\left\{\bar{\psi}_N(x)\gamma_\mu\tau_+\psi_N(x)+i\left(\frac{\kappa_p-\kappa_n}{e}\right)\bar{\psi}_N(x)\sigma_{\mu\nu}\tau_+\psi_N(x)\frac{\partial}{\partial x_\nu}\right\}A_\mu(x)+\text{H.a.}, \quad \text{(I-4)}$$

$$\tau_+\equiv\frac{\tau_1+i\tau_2}{2};\quad \sigma_{\mu\nu}\equiv\frac{\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu}{2i};\quad \sigma_{lm}\equiv\epsilon_{lmn}\sigma_n;\quad (l,m=1,2,3),$$

where "H.a." represents the Hermitian adjoint.

In addition, the lepton field axial vector covariant, $A_\mu'(x)\equiv\bar{\psi}_e(x)i^{-1}\gamma_\mu\gamma_5(1+\gamma_5)\psi_\nu(x)=i^{-1}A_\mu(x)$, is coupled to an axial vector nucleon current which is not conserved[9]:

$$\begin{aligned}\mathcal{H}^{(A)}(x)&=C_A\bar{\psi}_N(x)i^{-1}\gamma_\mu\gamma_5\tau_+\psi_N(x)A_\mu'(x)\\&=-C_A\bar{\psi}_N(x)\gamma_\mu\gamma_5\tau_+\psi_N(x)A_\mu(x).\end{aligned} \quad \text{(I-5)}$$

The "induced" pseudoscalar interaction arising from the $\pi\rightarrow e+\nu$ decay of a pion in the virtual pion cloud surrounding the bare nucleon is completely negligible according to recent estimates [$C_P\cong(8/207)C_A$].[10] Thus $\mathcal{H}(x)=\mathcal{H}^{(V)}(x)+\mathcal{H}^{(A)}(x)$ should describe the effective interaction responsible for nuclear beta decay. It is, at present, quite definitely established that the first term of $\mathcal{H}^{(V)}(x)$ and of $\mathcal{H}^{(A)}(x)$ are both present in nuclear beta decay, with C_V and C_A given by

$$\begin{aligned}C_V&=(1.00\pm0.01)\times10^{-49}\text{ erg-cm}^3,\\ C_A&=-(1.19\pm0.03)C_V.\end{aligned} \quad \text{(I-6)}$$

However, the presence of the second, "anomalous magnetic moment," term in $\mathcal{H}^{(V)}(x)$ has not yet been demonstrated experimentally in nucleon beta decay and it is to this term that we now direct our attention.

II. NUCLEAR MATRIX ELEMENT

For the sake of definiteness, consider a beta-decay transition from a nuclear state $|N\rangle$ to a nuclear state $|N'\rangle$ with the emission of a negatron and an antineutrino. The relevant matrix element governing the transition rate is

$$\begin{aligned}\left\langle N'e\bar{\nu}\left|\int\mathcal{H}(x)d^3x\right|N\right\rangle&=\int\langle N'|C_V\bar{\psi}_N(x)\gamma_\mu\tau_+\psi_N(x)\\&\quad-C_A\bar{\psi}_N(x)\gamma_\mu\gamma_5\tau_+\psi_N(x)|N\rangle\langle e\bar{\nu}|A_\mu(x)|0\rangle d^3x\\&\quad+\int\left\langle N'\left|iC_V\left(\frac{\kappa_p-\kappa_n}{2e}\right)\bar{\psi}_N(x)\sigma_{\mu\nu}\tau_+\psi_N(x)\right|N\right\rangle\\&\quad\times\frac{\partial}{\partial x_\nu}\langle e\bar{\nu}|A_\mu(x)|0\rangle d^3x.\end{aligned} \quad \text{(II-1)}$$

[9] M. Goldberger and S. Treiman, Phys. Rev. **111**, 354 (1958).
[10] M. Goldberger and S. Treiman, Phys. Rev. **110**, 1478 (1958).

In the expression (II-1), we understand the time integration to have been carried out yielding the energy conservation condition so that $\psi_N(x)$, $A_\mu(x)$, and $\partial A_\mu(x)/\partial x_\nu$ are all to be evaluated at $x_4=0$, a prescription which will be henceforth implied.

The lepton matrix elements can be immediately evaluated:

$$\begin{aligned}\langle e\bar{\nu}|A_\mu(x)|0\rangle&=J_\mu e^{-iK\cdot x},\\ \langle e\bar{\nu}|\partial A_\mu(x)/\partial x_\nu|0\rangle&=-iK_\nu J_\mu e^{-iK\cdot x},\\ J_\mu&=\bar{u}_e\gamma_\mu(1+\gamma_5)u_{\bar{\nu}},\\ K_\nu&=(p_e+p_{\bar{\nu}})_\nu.\end{aligned}\tag{II-2}$$

Further, for the first forbidden unique transitions, $|N\rangle$ and $|N'\rangle$ differ by two units of total angular momentum and have opposite parities: $\Delta J=2$; "yes." As a result, only irreducible tensor operators of rank two or higher which induce a parity change, D, yield nonvanishing matrix elements connecting $|N\rangle$ to $|N'\rangle$.

To select such appropriate operators D, we, in effect, expand $\mathcal{H}(x)$ in powers of the nucleon velocity/c by the use of the equations of motion of quantized field amplitudes ψ_p, ψ_n in the interaction picture

$$\begin{aligned}m_N\psi_N(x)&=-\gamma_\mu\frac{\partial}{\partial x_\mu}\psi_N(x),\\ m_N\bar{\psi}_N(x)&=\frac{\partial\bar{\psi}_N}{\partial x_\mu}(x)\gamma_\mu.\end{aligned}\tag{II-3}$$

We also recall that $\mathcal{H}(x)$ occurs in an integral over all space [Eq. (II-1)] so that an integration by parts of terms in $\mathcal{H}(x)$ containing $(\partial/\partial x_m)$ $(m=1, 2, 3)$ is always permitted.

We begin by applying the identities (II-3) to the first term in $\mathcal{H}^{(V)}(x)$ of (I-4):

$$\begin{aligned}&\bar{\psi}_p(x)\gamma_\mu\psi_n(x)A_\mu(x)\\ &=\frac{1}{m_p+m_n}\left[\frac{\partial\bar{\psi}_p(x)}{\partial x_\nu}\gamma_\nu\gamma_\mu\psi_n(x)-\bar{\psi}_p(x)\gamma_\mu\gamma_\nu\frac{\partial}{\partial x_\nu}\psi_n(x)\right]A_\mu(x)\\ &=\frac{1}{m_p+m_n}\left[\frac{\partial\bar{\psi}_p(x)}{\partial x_4}\psi_n(x)-\bar{\psi}_p(x)\frac{\partial}{\partial x_4}\psi_n(x)\right]A_4(x)\\ &+\frac{1}{m_p+m_n}\left[\frac{\partial\bar{\psi}_p(x)}{\partial x_l}\gamma_l\gamma_m\psi_n(x)-\bar{\psi}_p(x)\gamma_m\gamma_l\frac{\partial}{\partial x_l}\psi_n(x)\right]A_m(x)\\ &+\frac{1}{m_p+m_n}\frac{\partial}{\partial x_l}[\bar{\psi}_p(x)\gamma_l\gamma_4\psi_n(x)]A_4(x)\\ &+\frac{1}{m_p+m_n}\frac{\partial}{\partial x_4}[\bar{\psi}_p(x)\gamma_4\gamma_m\psi_n(x)]A_m(x).\end{aligned}\tag{II-4}$$

Here, as always, repeated Latin indices imply a sum from 1 to 3, while repeated Greek indices imply a sum from 1 to 4. Integrating by parts and using the identity and definition

$$\sigma_{lm}=\epsilon_{lmn}\sigma_n,\quad \boldsymbol{\gamma}=(\gamma_1,\gamma_2,\gamma_3),\tag{II-5}$$

we obtain

$$\begin{aligned}\bar{\psi}_p(x)\gamma_\mu\psi_n(x)A_\mu(x)=&\frac{1}{m_n+m_p}\left[\frac{\partial\bar{\psi}_p}{\partial x_4}\psi_n-\bar{\psi}_p\frac{\partial}{\partial x_4}\psi_n\right]A_4 &\text{(a)}\\ &+\frac{1}{m_n+m_p}[-2\bar{\psi}_p\,\mathrm{grad}\psi_n]\cdot\mathbf{A} &\text{(b)}\\ &+\frac{1}{m_n+m_p}[-i\bar{\psi}_p\boldsymbol{\sigma}\psi_n]\cdot\mathrm{curl}\mathbf{A} &\text{(c)}\\ &+\frac{1}{m_n+m_p}[-\bar{\psi}_p\psi_n]\,\mathrm{div}\mathbf{A} &\text{(d)}\\ &+\frac{1}{m_n+m_p}[\mathrm{div}(\bar{\psi}_p\boldsymbol{\gamma}\gamma_4\psi_n)]A_4 &\text{(e)}\\ &+\frac{1}{m_n+m_p}\left[\frac{\partial}{\partial x_4}(\bar{\psi}_p\gamma_4\boldsymbol{\gamma}\psi_n)\right]\cdot\mathbf{A}. &\text{(f)}\end{aligned}\tag{II-6}$$

Inspection of (II-6) shows that the first term is of zero order in (nucleon velocity)/c; the next three terms, first order in (nucleon velocity)/c or $|\mathbf{K}|/(m_p+m_n)$; and the last two terms second order in these parameters. Further repeated application of the identities (II-3) in Eq. (II-6) can be employed to remove any remaining Dirac operators, the resulting terms containing higher and higher powers of $1/(m_p+m_n)$.

The term (c) in (II-6) corresponds to a Dirac magnetic moment term which when added to the anomalous magnetic moment term in (I-4),

$$\begin{aligned}&i\left(\frac{\kappa_p-\kappa_n}{e}\right)\bar{\psi}_N(x)\sigma_{\mu\nu}\tau_+\psi_N(x)\frac{\partial A_\mu(x)}{\partial x_\nu}\\ &\qquad\cong-i\left(\frac{\kappa_p-\kappa_n}{e}\right)\bar{\psi}_p(x)\boldsymbol{\sigma}\tau_+\psi_n(x)\cdot\mathrm{curl}\mathbf{A},\end{aligned}$$

yields the total spin magnetic moment contribution. The term (b) in (II-6) depends explicitly on the nucleon velocities and gives rise to an orbital magnetic moment term. The numerical magnitude of the total spin magnetic moment term (a factor of nearly five enhancement over the Dirac magnetic moment term) suggests that this term contributes a correction to the first forbidden transitions, which is large compared to that contributed by the orbital magnetic moment term. An analogous conclusion is verified for allowed transitions in Gell-Mann's paper.[3]

The axial vector current in the $\mathcal{H}^{(A)}(x)$ of (I-5) can also be analogously reduced, with the first term zero order in (nucleon velocity)/c, the next two terms first order in (nucleon velocity)/c or $|\mathbf{K}|/(m_p+m_n)$, and the last term second order in these parameters.

$$\bar{\psi}_p(x)\gamma_\mu\gamma_5\psi_n(x)A_\mu(x)=i\bar{\psi}_p(x)\gamma_4\boldsymbol{\sigma}\psi_n(x)\cdot\mathbf{A}(x)+\frac{1}{m_n+m_p}[2i\bar{\psi}_p(x)\boldsymbol{\sigma}\cdot\mathrm{grad}\psi_n(x)]A_4(x)+\frac{1}{m_n+m_p}[i\bar{\psi}_p(x)\boldsymbol{\sigma}\psi_n(x)]\cdot\mathrm{grad}A_4(x)+\frac{1}{m_n+m_p}\left\{\frac{\partial}{\partial x_4}[\bar{\psi}_p(x)\gamma_5\psi_n(x)]\right\}A_4. \quad \text{(II-7)}$$

Detailed inspection of (II-6) and (II-7), along with the anomalous magnetic moment term in (I-4), with a view to the ultimate retardation expansion of the matrix elements of the lepton field covariant $A_\mu(x)$ (II-2), show that the lowest order terms [in (nucleon velocity)/c and $|\mathbf{K}|/(m_n+m_p)$] appearing in $\mathcal{H}(x)=\mathcal{H}^{(V)}(x)+\mathcal{H}^{(A)}(x)$ and having nonvanishing matrix elements for first forbidden unique transition are

(1) $-iC_A[\bar{\psi}_p(x)\gamma_4\boldsymbol{\sigma}\psi_n(x)]\cdot\mathbf{A}(x)$

(2) $-i\dfrac{C_A}{2m}[\bar{\psi}_p(x)\boldsymbol{\sigma}\psi_n(x)]\cdot\mathrm{grad}A_4(x)$

(3) $-i\dfrac{C_V}{2m}[\bar{\psi}_p(x)\boldsymbol{\sigma}\psi_n(x)]\cdot\mathrm{curl}\mathbf{A}(x)$

(4) $-iC_V\dfrac{(\kappa_p-\kappa_n)}{e}[\bar{\psi}_p(x)\boldsymbol{\sigma}\psi_n(x)]\cdot\mathrm{curl}\mathbf{A}(x)$

$$m\equiv m_p\cong m_n. \quad \text{(II-8)}$$

Terms other than those listed in (II-8) yield comparably negligible contributions to first forbidden unique transitions.

III. DIFFERENTIAL TRANSITION RATE

Equation (II-8) yields for the pertinent form of the matrix element giving the first forbidden unique transition amplitude for the conversion of a nuclear state $|N\rangle$ into a nuclear state $|N'\rangle$ with the emission of a negatron and an antineutrino:

$$\langle f|H_I|i\rangle=-iC_A\int\langle N'|\bar{\psi}_p(x)\boldsymbol{\sigma}\psi_n(x)|N\rangle\cdot\langle e\bar{\nu}|\mathbf{A}(x)|0\rangle d^3x-i\frac{C_A}{2m}\int\langle N'|\bar{\psi}_p(x)\boldsymbol{\sigma}\psi_n(x)|N\rangle\cdot\langle e\bar{\nu}|\mathrm{grad}A_4(x)|0\rangle d^3x-i\left(\frac{\mu_p-\mu_n}{e}\right)C_V\int\langle N'|\bar{\psi}_p(x)\boldsymbol{\sigma}\psi_n(x)|N\rangle\cdot\langle e\bar{\nu}|\mathrm{curl}\mathbf{A}(x)0\rangle d^3x,$$

where

$$\mu_p\equiv(e/2m)+\kappa_p=2.79(e/2m);\quad \mu_n\equiv\kappa_n=-1.91(e/2m). \quad \text{(III-1)}$$

As already mentioned, only those terms leading to $\Delta J=2$, "yes" nuclear transitions to lowest order in (nucleon velocity)/c and $|\mathbf{K}|/(m_n+m_p)$ are included in (III-1). Our procedure, where the term $\sim(\mu_p-\mu_n)$ is evaluated with the use of explicitly given operators, is to be contrasted with Gell-Mann's[3] procedure, where the evaluation is carried out on the basis of the comparison of corresponding beta emission and gamma emission matrix elements.

The use of the expressions (II-2) and

$$\langle e\bar{\nu}|\mathrm{curl}\mathbf{A}(x)|0\rangle=-i\mathbf{K}\times(\mathbf{J}e^{-i\mathbf{K}\cdot\mathbf{x}})$$
$$\langle e\bar{\nu}|\mathbf{A}(x)|0\rangle=\mathbf{J}e^{-i\mathbf{K}\cdot\mathbf{x}}=\mathbf{J}[1-i\mathbf{K}\cdot\mathbf{x}+\cdots] \quad \text{(III-2)}$$

enables us to write (III-1) in the form:

$$\langle f|H_I|i\rangle=-C_A\left\langle N'\left|\int\bar{\psi}_p(x)\mathbf{L}\cdot\boldsymbol{\sigma}\mathbf{x}\cdot\mathbf{K}\psi_n(x)d^3x\right|N\right\rangle$$

$$\mathbf{L}\equiv\mathbf{J}-i\frac{\mathbf{K}}{2m}J_4-i\left(\frac{\mu_p-\mu_n}{e}\right)\frac{C_V}{C_A}\mathbf{K}\times\mathbf{J}. \quad \text{(III-3)}$$

The nuclear matrix element involves the inner product of two Cartesian tensors $\sigma_l x_m$ and $L_l K_m$. However, since $|N\rangle$ and $|N'\rangle$ differ by two units of total angular momentum and in parity, only the symmetric traceless part of the operator $\sigma_m x_l$ yields a nonvanishing contribution. Consequently,

$$\langle f|H_I|i\rangle=-C_A\Lambda_{lm}\left\langle N'\left|\int\bar{\psi}_p(x)D_{lm}^{(2)}(\mathbf{x},\boldsymbol{\sigma})\psi_n(x)d^3x\right|N\right\rangle,$$

$$\Lambda_{lm}\equiv\tfrac{1}{2}(L_lK_m+L_mK_l)-\frac{1}{3}\left(\sum_{k=1}^{3}L_kK_k\right)\delta_{lm}, \quad \text{(III-4)}$$

$$D_{lm}^{(2)}\equiv\tfrac{1}{2}(\sigma_lx_m+\sigma_mx_l)-\frac{1}{3}\left(\sum_{k=1}^{3}\sigma_kx_k\right)\delta_{lm}.$$

In the situation in which the electrons emanate from randomly oriented parent nuclei and the final nuclear orientation is not observed, the transition rate of interest is

$$\frac{1}{2J+1}\sum_{s_L}\sum_{MM'}|\langle f|H_I|i\rangle|^2, \quad \text{(III-5)}$$

where the sum is over all initial and final nuclear magnetic quantum numbers, and over the final lepton spins s_L. The sum over M, M' decouples the nuclear and lepton parts in (III-5) and we obtain

$$\frac{1}{2J+1}\sum_{sL}\sum_{MM'}|\langle f|H_I|i\rangle|^2=|C_A|^2[\sum_{sL}\sum_{lm}\Lambda_{lm}\Lambda_{lm}^*]\times\left(\frac{32\pi}{75}|\langle\alpha J||0||\alpha' J'\rangle|^2\right), \quad \text{(III-6)}$$

where α, α' refer to the energy, parity, $\cdots$ quantum numbers of the states $|N\rangle$, $|N'\rangle$ and $|J'-J|=2$.

The only factor in (III-6) which is involved in the emitted electron momentum distribution is the sum $\sum_{sL}\sum_{lm}\Lambda_{lm}\Lambda_{lm}^*$, whose value can be found by standard techniques. We first note the identity

$$\sum_{sL}\sum_{lm}\Lambda_{lm}\Lambda_{lm}^*=\sum_{sL}[|\mathbf{L}|^2|\mathbf{K}|^2]+\tfrac{1}{6}\sum_{sL}[|\mathbf{L}\cdot\mathbf{K}|^2], \quad \text{(III-7)}$$

and then carry out the lepton spin sums, with the result

$$\sum_{sL}[|\mathbf{L}|^2]=\frac{2}{E_eE_\nu}[3E_eE_\nu-\mathbf{q}_e\cdot\mathbf{q}_\nu]+\frac{8\mu}{E_eE_\nu}[(\mathbf{K}\cdot\mathbf{q}_\nu)E_e-(\mathbf{K}\cdot\mathbf{q}_e)E_\nu]+\frac{4}{2mE_eE_\nu}[(\mathbf{K}\cdot\mathbf{q}_\nu)E_e+(\mathbf{K}\cdot\mathbf{q}_e)E_\nu], \quad \text{(III-8)}$$

$$\sum_{sL}[|\mathbf{L}\cdot\mathbf{K}|^2]=\frac{2}{E_eE_\nu}[q_e^2\mathbf{q}_\nu\cdot\mathbf{K}+q_\nu^2\mathbf{q}_e\cdot\mathbf{K}+E_\nu E_eK^2]+\frac{4K^2}{2mE_eE_\nu}[\mathbf{K}\cdot\mathbf{q}_\nu E_e+E_\nu\mathbf{K}\cdot\mathbf{q}_e].$$

In Eq. (III-8),

$$\mu\equiv\left(\frac{\mu_p-\mu_n}{e}\right)\frac{C_V}{C_A}\cong-\frac{4.7}{2m}(0.84)\cong-\frac{4.0}{2m};\mathbf{q}_{e\nu},\ E_{e\nu}$$

is the momentum energy of the indicated lepton. Equations (III-6)–(III-8) yield for the electron-neutrino momentum spectrum and angular correlation:

$$[\sum_{sL}\sum_{lm}\Lambda_{lm}\Lambda_{lm}^*]q_e^2q_\nu^2d(\cos\theta)\equiv(S_1+S_2)q_e^2q_\nu^2d(\cos\theta),$$

$$S_1\equiv\frac{1}{E_eE_\nu}\{K^2(3E_eE_\nu-\mathbf{q}_\nu\cdot\mathbf{q}_e)+\tfrac{1}{3}(q_e^2\mathbf{q}_\nu\cdot\mathbf{K}+q_\nu^2\mathbf{q}_e\cdot\mathbf{K}+E_\nu E_eK^2)\}, \quad \text{(III-9)}$$

$$S_2\equiv\frac{1}{E_eE_\nu}\{K^2[4\mu+(8/3)/2m](\mathbf{K}\cdot\mathbf{q}_\nu)E_e+K^2[(8/3)/2m-4\mu](\mathbf{K}\cdot\mathbf{q}_e)E_\nu\},$$

where $\theta\equiv$angle between electron and neutrino momenta; whence, integrating over θ, the electron momentum spectrum $D(q_e)$ is

$$D(q_e)=\left\{\frac{20}{3}(q_e^2+q_\nu^2)+\left(4\mu+\frac{8}{3}\Big/\frac{1}{2m}\right)\times[2(q_e^2+q_\nu^2)q_\nu+\tfrac{4}{3}q_e^2q_\nu]+\left[\left(\frac{8}{3}\Big/\frac{1}{2m}\right)-4\mu\right]\times\left[2(q_e^2+q_\nu^2)q_e\frac{q_e}{E_e}+\frac{4}{3}\frac{q_e^2q_\nu^2}{E_e}\right]\right\}q_e^2q_\nu^2. \quad \text{(III-10)}$$

The result (III-10) applies to negatron beta decay. For positron beta decay the result is the same except that μ is replaced by $-\mu$.

Up to now, we have neglected the effect of the Coulomb field on the emitted electron. As far as the electron momentum spectral shape is concerned, only the Coulomb corrections to the main term in (III-10), *viz.*, $(20/3)(q_e^2+q_\nu^2)$, need be here considered; we represent the correction as a multiplicative factor $C(q_e)$:

$$(20/3)(q_e^2+q_\nu^2)\rightarrow(20/3)(q_e^2+q_\nu^2)C(q_e/q_m), \quad \text{(III-11)}$$

where q_m=maximum electron momentum. Table I gives values of $C(q_e/q_m)$ calculated for $Z=8$

$$({}_7\mathrm{N}^{16}\rightarrow{}_8\mathrm{O}^{16}+e^-+\bar{\nu}).$$

These values were obtained from a linear extrapolation of values computed for $Z=10$, 12, and 14 from the extensive tables of Rose, Perry, Dismuke, and Bell.[11,12] Thus, Eq. (III-10) becomes

$$D(q_e)=\left\{\frac{20}{3}(q_e^2+q_\nu^2)C(q_e)+\left(4\mu+\frac{8}{3}\Big/2m\right)\times[2(q_e^2+q_\nu^2)q_\nu+\tfrac{4}{3}q_e^2q_\nu]+\left(\frac{8}{3}\Big/2m-4\mu\right)\times\left[2(q_e^2+q_\nu^2)q_e\frac{q_e}{E_e}+\frac{4}{3}\frac{q_e^2q_\nu^2}{E_e}\right]\right\}q_e^2q_\nu^2. \quad \text{(III-12)}$$

IV. EXPERIMENTAL CONSEQUENCES

For beta transitions involving very low energy transfer to the lepton field, $q_m/2m\cong0$ and $q_m\mu\cong0$. Since $q_e\leqq q_m$, and $q_\nu\leqq(q_m^2+m_e^2)^{\frac{1}{2}}-m_e$, the electron momentum spectrum (III-12) takes the limiting form:

$$D^{(0)}(q_e)=(20/3)(q_e^2+q_\nu^2)q_e^2q_\nu^2C(q_e), \quad \text{(IV-1)}$$

which is recognized as the conventional first forbidden unique spectral shape.

Since, as noted after Eq. (III-8), $\mu\cong4.0/2m$, we may drop $(8/3)/2m$ compared to $4\mu=16.0/2m$. Also, the corrective terms $\propto\mu$ are at all appreciable only for

[11] Rose, Perry, and Dismuke, Oak Ridge National Laboratory ORNL-1459 (unpublished).

[12] Dismuke, Rose, Perry, and Bell, Oak Ridge National Laboratory ORNL-1222 (unpublished).

TABLE I. Values of $C(q_e)$ calculated for $Z=8$.

(q_e/q_m)	$C(q_e/q_m)$	(q_e/q_m)	$C(q_e/q_m)$
0.40	0.9900	0.75	0.9853
0.45	0.9893	0.80	0.9854
0.50	0.9886	0.85	0.9847
0.55	0.9879	0.90	0.9838
0.60	0.9867	0.95	0.9830
0.65	0.9866	1.00	0.9820
0.70	0.9861		

relatively high-energy beta transitions, where $q_e \cong E_e$ and $q_\nu \cong q_m - q_e$ over most of the spectrum; thus Eq. (III-12) becomes

$$D(q_e)=\left\{\frac{20}{3}(q_e^2+q_\nu^2)C(q_e)+8\lambda[(q_e^2+q_\nu^2)-\tfrac{2}{3}q_eq_\nu]\times\left(\frac{q_\nu-q_e}{q_m}\right)\right\}q_e^2q_\nu^2$$

with

$$\lambda \equiv \mu q_m = \frac{\mu_p-\mu_n}{e}\frac{C_V}{C_A}q_m = -2.0\frac{q_m}{m}. \tag{IV-2}$$

Equation (IV-2) shows that the correction terms $\propto \mu$ give no contribution at the "midpoint" of the electron momentum spectrum where $q_\nu \cong q_e = \frac{1}{2}q_m$.

Introducing the ratio

$$R(q_e)=D(q_e)/D^0(q_e), \tag{IV-3}$$

we find from Eqs. (IV-1) and (IV-2) that

$$\begin{aligned} R(q_e=\tfrac{1}{2}q_m)&=1,\\ R(q_e=q_m)&=1-(6/5)\frac{\lambda}{C(q_m)}\cong 1-(6/5)\lambda, \end{aligned} \tag{IV-4}$$

and from Eq. (III-12) that

$$R(q_e=0)\cong[1+(6/5)\lambda]. \tag{IV-5}$$

The function $R(q_e)$ is practically linear over a large portion of the spectrum. The deviation of $R(q_e)$ from unity by the predicted amount (IV-4) will indicate the presence of the anomalous magnetic moment term of (I-4) in the interaction Hamiltonian generating nucleon beta decay.

The most favorable condition for the experimental verification of the correction

$$\propto \lambda = \mu q_m = [(\mu_p-\mu_n)/e](C_V/C_A)q_n$$

is found in a nuclear transition with high-energy release in a low-Z element. The nucleus[13–15] ${}_7N^{16}$ appears to be especially favorable:

$$\begin{aligned} &{}_7N^{16}\rightarrow {}_8O^{16}+e^-+\bar{\nu},\\ &\tau_{\frac{1}{2}}=7.37\pm0.04\text{ sec};\quad q_m=10.4\text{ Mev}/c, \end{aligned} \tag{IV-6}$$

and we find from Eq. (IV-4) that

$$\begin{aligned} R(q_e=0)&\cong 1-0.03,\\ R(q_e=q_m)&\cong 1+0.03, \end{aligned} \tag{IV-7}$$

a deviation which may be detectable with present experimental techniques. The feasibility of the experiment is now being investigated at Washington University.

The effect of inner bremsstrahlung on $R(q_e)$ in ${}_7N^{16}\rightarrow{}_8O^{16}$ has been estimated and found to be considerably smaller than the 3% correction $\propto \mu$ exhibited in Eqs. (IV-4), (IV-5), and (IV-7).

V. DISCUSSION AND CONCLUSIONS

The nucleon-structural effect (above correction $\propto \mu$) on the first forbidden unique electron momentum spectrum of N^{16}, arising from the lepton-conserved isovector current coupling, thus amounts to an approximate 3% deviation when reckoned from the spectral mid-point to the spectral end point. On the other hand, in the B^{12}, N^{12} allowed spectra the corresponding spectral mid-point to spectral end-point deviation is 5%. However, in the first forbidden unique transitions, the lifetimes are longer by a factor of 100 than in allowed transitions of comparable energy release, and the over-all spectral shapes are relatively flatter. These factors offer distinct experimental advantages in detecting small spectral shape corrections. Unfortunately, ${}_9F^{16}$ is probably proton unstable so that one will not be able to test in this case the change in sign of the nucleon-structural correction in passing from a negatron to the corresponding positron emitter. Altogether, however, precise studies of electron momentum spectral shapes in first forbidden unique transitions seem to offer new possibilities for the detection of small effects associated with any coupling of leptons to a conserved isovector current.

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