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Electron Scattering by Noble Gases in the Limit of Zero Energy*

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The elastic scattering at zero energy by argon depends strongly on the polarization tail, since the contribution to the scattering length by forces inside the atom cancel. Assuming this is true for all noble gases, one can approximate their zero-energy elastic cross section by $3.52\times10^{-16}(p/r_0)^2$ cm², where both p, the polarizability, and r_0, the size of the atom, are in atomic units. Using published values of the polarizability and scaling r_0 as the cube root of the atomic number from $r_0=8$ for argon, we find that this expression is consistent with existing experimental data. It is conjectured that the success of this approximation is a result of the Pauli exclusion principle.

IN a recent calculation[1] it was noted that the polarization tail is the dominant factor in determining the elastic scattering of low-energy electrons by argon. The contributions to the scattering by forces within the atom canceled because of a seemingly fortuitous periodicity of the wave function. That is, the zero-energy cross section would have vanished if we had neglected the forces at larger radii than about 8 atomic units, which is effectively the outer edge of the atomic charge distribution. The attractive polarization force outside the atom by itself leads in the case of argon to a zero-energy cross section of 7.5×10^{-16} cm² in agreement with measurements by Ramsauer and Kollath,[2] and Pack and Phelps.[3]

Since the polarization tail determines the cross section, it is possible to estimate this elastic scattering by a perturbation technique. The difficult problem of determining the forces on the electron inside the atom does not have to be solved. It is only necessary to assume that the forces inside the atom do not contribute to the cross section. Then the perturbation method yields a simple expression in terms of the size of the atom and its polarizability. Using the atomic size determined in the detailed argon calculation, one can scale to the other noble gases and readily predict their zero energy elastic scattering cross sections.

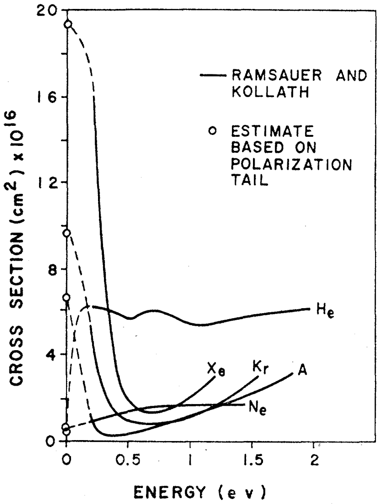

FIG. 1. Elastic scattering cross section of electrons by noble gas atoms as a function of energy. The solid lines correspond to the measurements by Ramsauer and Kollath; the circles at zero energy are given in Table I of this note.

The cross section (Q) for elastic scattering in the limit of zero energy can be expressed in terms of the logarithmic derivative (G_0'/G_0) of the zero order partial wave solution of the Schrödinger equation (G_0) as[4]

$$
\begin{aligned}
Q/4\pi a_0^2 &= \lim_{r\to\infty}\left[(G_0/G_0')_r - r\right]^2 \\
&\doteq \Big\{ (G_0/G_0')_{r_0} - r_0 + \left[p/(G_0')^2_{r_0}\right] \\
&\qquad\qquad \times \int_0^\infty G_0^2 r^{-4} dr \Big\}^2, \qquad (1) \\
&\approx (p/r_0)^2
\end{aligned}
$$

where a_0 is the Bohr radius (0.529×10^{-8} cm), p is the polarizability in atomic units (a.u.), and r_0 (a.u.) is the effective radius of the atomic charge distribution (see Table I). We have first used the approximation that the wave function outside the atom is not changed appreciably by the polarization tail, and second, the assumption that the cross section resulting from forces inside the atom is zero so that $(G_0)_r = r(G_0')_{r_0}$ outside the atom for zero energy. The perturbation result [Eq. (1)] fits the argon numerical integration solution using the values $p=11$ a.u. and $r_0\doteq 8$ a.u.

To apply this equation to other noble gases, one needs only their polarizability and atomic size. Using

TABLE I. Zero energy elastic scattering cross sections for noble gases.[a]

atom	Z	p (a.u.)	r_0 (a.u.)	Q (cm²) $\times10^{16}$
He	2	1.4	3.8	0.44
Ne	10	2.6	6.6	0.57
A	18	11	8.0	6.6
Kr	36	17	10	9.6
Xe	54	27	11	19

[a] Z = atomic number, p = polarizability, r_0 = effective atomic radius, Q = cross section for elastic scattering.

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[1] B. Kivel, Phys. Rev. **116**, 926 (1959).

[2] V. C. Ramsauer and R. Kollath, Ann. Physik **3**, 536 (1929).

[3] J. L. Pack and A. V. Phelps, Bull. Am. Phys. Soc. Ser. II, 4, 317 (1959).

[4] B. Kivel, Research Note 129, AVCO-Everett Research Laboratory, Everett, Massachusetts, 1959 (unpublished).

known polarizabilities[5] and scaling the atomic size according to the Thomas-Fermi model of the atom as the cube root of the atomic number ($r_0 \propto Z^{\frac{1}{3}}$), we find the zero energy cross sections given in Table I. Because of a faster increase of polarizability than atomic size the zero energy cross section increases with atomic number.

In Fig. 1 the cross section values from Table I are compared with the experimental results of Ramsauer and Kollath. With the exception of He, our values are seen to fit smoothly to their measurements. In the case of He a low value is substantiated by swarm measurements of Townsend and Bailey,[6] which indicate a strong decrease in cross section as the mean electron energy approaches zero.

The conclusion drawn from this agreement is that the polarization force does indeed determine the elastic scattering of electrons in the neighborhood of zero energy. As a corollary electrons in the limit of zero energy are not scattered within the closed shells of the noble gas atoms. This is viewed as a consequence of the Pauli exclusion principle. It appears then that the occurrence of the Ramsauer-Townsend minimum for the noble gas atoms is not fortuitous but rather a result of their closed shells.

Note added in proof.—The predicted elastic cross sections are contradicted by measurements of spectral line shifts. The line shift depends on the interaction between the bound electron and a neighbor atom. This work, which is summarized by Massey and Burhop,[6] on page 178, was brought to our attention by A. V. Phelps. The cross section for argon is in agreement with our calculation using conventional methods.[1]

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[5] J. H. Van Vleck, *Electric and Magnetic Susceptibilities* (Oxford University Press, London, 1932), p. 225.

[6] H. S. W. Massey and E. H. S. Burhop, *Electronic and Ionic Impact Phenomena* (Oxford University Press, London, 1952), p. 28.

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Radiative Proton Capture by O^{18}

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A search for gamma-emitting levels in F^{19} between 8.3 and 9.8 Mev of excitation energy has been made by bombarding a NiO target (39% O^{18}) with protons over the energy range 360 to 1960 kev. Resonances in the gamma-ray yield were observed with the use of a 3-in. diam×3-in. sodium iodide crystal and a 20-channel pulse-height analyzer. The results are as follows:

Resonance energy (kev)	Resonance width (kev)	γ-ray energies (Mev)
630±2	2.6±1.0	8.5 ±0.2
849±3	40±5	8.5 ±0.3, 7.2±0.2, 4.8±0.2, 4.3 ±0.2(?), 2.4±0.2(?)
1169±2	≤0.9	8.8 ±0.2, 7.7±0.2, 6.27±0.05, 3.67±0.05, 2.59±0.05, 1.24±0.05
1399±5	<15	···
1685±5	<15	···
1769±2	4.0±1.0	9.4 ±0.3
1931±2	1.5±1.0	9.8 ±0.2, 8.45±0.10

The angular distribution of the 6.27-Mev gamma ray from the 1169-kev resonance was measured, and the results are listed in tabular form. The results of the present experiment are compared with other experiments using the $O^{18}(p,\gamma)F^{19}$, $O^{18}(p,\alpha)N^{15}$, and the $O^{18}(d,n)F^{19}$ reactions.

I. INTRODUCTION

THE region of excitation in F^{19} from about 6 Mev to about 10 Mev is conveniently accessible only by means of nuclear reactions on the target nuclide, O^{18}. Accordingly, a program was begun at this laboratory to investigate this region from about 6 Mev to about 9 Mev by means of the $O^{18}(d,n\gamma)F^{19}$ reaction, and the region from about 8 Mev to about 10 Mev by means of the $O^{18}(p,\gamma)F^{19}$ reaction. The results of the experiments using the $O^{18}(d,n\gamma)F^{19}$ reaction have been reported in abstract[1] and final[2,3] form. The results of the $O^{18}(p,\gamma)F^{19}$ reaction have been reported as an abstract account[4] and are herein presented in complete form.

In the region of interest to the present experiment, two states are known from previous experiments with

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[1] H. D. Holmgren and J. W. Butler, Phys. Rev. **99**, 655(A) (1955).

[2] J. W. Butler and H. D. Holmgren, Phys. Rev. **112**, 461 (1958).

[3] Butler, Fagg, and Holmgren, Phys. Rev. **113**, 268 (1959).

[4] J. W. Butler and H. D. Holmgren, Phys. Rev. **99**, 1649(A) (1955).