Structure of Weak Interactions and Unwanted Processes

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A general discussion of the forbidden reactions such as $\mu\rightarrow e+\gamma$, $\mu\rightarrow e+e+e$, $\mu+N\rightarrow e+N$, $\mu\rightarrow e+\gamma+\gamma$, etc. which would arise from a possible nonlocal structure of the weak interactions is given. It is shown, by a canonical transformation, that the dominating terms of the structure, in an expansion in terms of the inverse of the average intermediate mass, do not contribute to any forbidden reaction. A discussion of the $\mu-e$ conversion process for a bound μ is given, and the rate is calculated for a particular model.

INTRODUCTION

THE possibility of a nonlocal structure for the weak interactions was first pointed out recently by Lee and Yang, who discussed the effects of such a structure on the μ-decay spectra.[1] The analysis by Lee and Yang, as well as the analysis by Bludman and Klein,[2] were phenomenological in character in that no particular hypotheses were made concerning the origin of the postulated nonlocal structure. In a later analysis by Byers and Peierls[3] a nonlocal structure for $\mu\rightarrow e+\nu+\bar{\nu}$ was assumed to arise from the exchange of a massive vector meson between the $(\mu\nu)$ and the $(e\nu)$ pairs. Though such a structure, if it exists, will be present in all weak processes, it is convenient to confine the discussion of its effects mainly to the processes resulting from the coupling of the $(\mu\nu)$, $(e\nu)$ pairs. For other processes the presence of strong interactions could make ambiguous the separation of the proper nonlocal effects.

For convenience we shall refer to the direct effects of the structure on the weak processes as to the primary effects of the structure. It was first pointed out by Feynman and Gell-Mann[4] that the existence of a structure would also lead to new processes, such as a possible muon decay into an electron and a gamma. Calculations of such a process, assuming the intermediate vector meson, were made by Feynman and Gell-Mann,[4] by Feinberg,[5] who also estimates the rate for muon decay into three electrons arising from internal conversion of the gamma, and by Ebel and Ernst.[6] Another process that will also occur through the structure is the absorption of a negative muon by a proton with the emission of an electron. For a negative muon in a bound orbit around the nucleus the process may occur incoherently on the single protons (and also possibly on the neutrons), and coherently through the direct conversion in the Coulomb field of the bound muon into an energetic electron. We shall refer to such indirect effects of the structure as to the secondary effects of the structure.

The main purpose of the present note will be a general discussion on the secondary effects of the structure. In Sec. 1 we discuss the general form of the nonlocality. In Secs. 2 and 3 we show that the largest contribution to the vertex does not produce any physical effect and it only amounts to a redefinition of the electron and muon states. In Sec. 4 we discuss the remaining contributions, and in Sec. 5 we apply the theory to the process of conversion of a bound negative muon into an electron, that would occur as a direct consequence of the existence of a structure.

1. LIMITATIONS ON THE STRUCTURE

We examine in this section what limitations are imposed on the structure from general requirements. The original nonlocal structure of the muon-decay interaction, specified by a kernel V, produces an effective muon-electron coupling, specified by a kernel K, simply related to V, as illustrated in Fig. 1. We write the effective Lagrangian for the effective muon-electron coupling in the form

$$\mathfrak{L}''(x)=-\int d\xi\,\bar{\varphi}(x-\tfrac{1}{2}\xi)K(\xi)\chi(x+\tfrac{1}{2}\xi)+\text{H.c.},\quad(1)$$

where φ and χ are the electron and muon field operators, respectively. We are interested in the general form of $K(\xi)$. Let us suppose that weak interactions are mediated through a coupling of the form

$$\mathfrak{L}'=J_\rho(x)\Phi_\rho(x)+\text{H.c.}\quad(2)$$

where J_ρ is the usual weak vector-axial[7] current and $\Phi_\rho(x)$ an unspecified four-vector constructed from the fields which participate in mediating the weak interactions. The interactions (2) gives rise to an effective Lagrangian

$$\mathfrak{L}'(x)=-\int d\xi\,J_\rho^\dagger(x-\tfrac{1}{2}\xi)V_{\rho\sigma}(\xi)J_\sigma(x+\tfrac{1}{2}\xi)+\text{H.c.}\quad(3)$$

which couples the weak vector-axial current J_ρ[7] with

[1] T. D. Lee and C. N. Yang, Phys. Rev. **108**, 1611 (1957).
[2] S. Bludman and A. Klein, Phys. Rev. **109**, 550 (1958).
[3] N. Byers and R. E. Peierls, Nuovo cimento **10**, 520 (1958).
[4] R. P. Feynman and M. Gell-Mann, reported at the Stanford Meeting of the American Physical Society, December, 1957 [Bull. Am. Phys. Soc. **2**, 391 (1957)].
[5] G. Feinberg, Phys. Rev. **110**, 1482 (1958).
[6] H. E. Ebel and F. Ernst (unpublished).
[7] R. E. Marshak and E. C. G. Sudarshan, Proceedings of the Padua-Venice Conference on Mesons and Newly Discovered Particles, 1957 (to be published); R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

itself at different space-time points. We can write V in the form

$$V_{\rho\sigma}(\xi)=\langle 0|T[\phi_\rho(\xi/2)\phi_\sigma^\dagger(-\xi/2)]|0\rangle. \qquad (4)$$

From known arguments[8] one then derives the representation

$$V_{\rho\sigma}(\xi)=\int dm^2\,[\rho_1(m^2)\Delta_F(\xi_1 m^2)+\rho_2(m^2)\Delta_F^{\rho\sigma}(\xi_1 m^2)], \qquad (5)$$

where ρ_1 and ρ_2 are unknown spectral functions. The kernel K, is given, apart from constants, by

$$K(\xi)=\gamma_\rho S_F^{(\nu)}(\xi)\gamma_\sigma V_{\rho\sigma}(-\xi)a, \qquad (6)$$

where $a=\frac{1}{2}(1+\gamma_5)$ and $S_F^{(\nu)}$ is the neutrino propagator. From (6) and (5) it follows that $K(\xi)$ can be written as

$$K(\xi)=(\gamma\xi)F(\xi^2)a, \qquad (7)$$

where F is an unknown function. It may also be of interest to report the effective interaction, H', for $\mu\rightarrow e+\nu+\bar{\nu}$ as derived from (3) and (5).

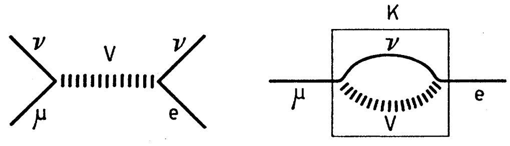
$$H'=(8)^{\frac{1}{2}}[G(\zeta^2)(\bar{e}\gamma_\lambda a\nu)(\bar{\nu}\gamma_\lambda a\mu)-g(\zeta^2)(\bar{e}a\nu)(\bar{\nu}a\mu)], \qquad (8)$$

where

$$(8)^{\frac{1}{2}}G(\zeta^2)=\int dm^2\frac{\rho_1(m^2)+\rho_2(m^2)}{\zeta^2+m^2+i\epsilon}, \qquad (9)$$

$$(8)^{\frac{1}{2}}g(\zeta^2)=m_e m_\mu\int dm^2\frac{\rho_2(m^2)}{m^2(m^2+\zeta^2+i\epsilon)}, \qquad (9')$$

ζ being the momentum transfer between the electron and the muon. If $\rho_1(m^2)$ or $\rho_2(m^2)$ have contributions also from low masses m^2 contained in the physical interval for ζ^2, the contribution from the poles in (9) and (9′) may produce effects simulating, in a reversible theory, an apparent lack of time-reversal invariance. On the other hand, such contributions from low mass values would also manifest themselves in a strong momentum dependence of the coupling constant, according to (9) and (9′). Such a strong momentum dependence is certainly excluded by the present data.[9]



(a) (b)

FIG. 1. (a) Nonlocal structure for $\mu\rightarrow e+\nu+\bar{\nu}$. The nonlocality is specified by a kernel V. (b) The induced $\mu-e$ nonlocal vertex, specified by the kernel K.

[8] H. Lehman, Nuovo cimento **9**, 342 (1954).
[9] H. Kruger and K. M. Crowe, Phys. Rev. **113**, 341 (1954).

2. LOWEST APPROXIMATION TO THE ELECTRIC CURRENT

According to the Lagrangian L'', (1), a muon can be annihilated at $x+\frac{1}{2}\xi$ and an electron created at $x-\frac{1}{2}\xi$. A unit of charge is thus carried between the two points, giving rise, in the spirit of minimal electromagnetic interactions, to an additional term in the electromagnetic interactions. This term is, however, not fixed uniquely from the form of the original nonlocal Lagrangian, because of the many possible trajectories for the electric charge between the two points. To preserve gauge invariance one finds that the Lagrangian (1) has to be replaced by[10]

$$\mathcal{L}''(x)=-\int d\xi\,\bar{\varphi}(x-\tfrac{1}{2}\xi)K(\xi)\chi(x+\tfrac{1}{2}\xi)\times\exp\left[-ie\int_{x-\frac{1}{2}\xi}^{x+\frac{1}{2}\xi}A_\mu(\eta)d\eta_\mu\right], \qquad (10)$$

where the integral is taken over an arbitrary trajectory from $x-\frac{1}{2}\xi$ to $x+\frac{1}{2}\xi$. The prescription (10) is the minimal modification to preserve gauge invariance. Any of our conclusions follows however from gauge invariance alone, except for the particular model considered in Sec. (4). We make the assumption that the spectral functions ρ in (5) become appreciable only for very high values of m^2 as compared to the momentum transfers involved in the processes. Accordingly we can simplify the form of the Lagrangian (10) by expanding φ, χ and the integral over A_μ in powers of ξ and keeping the lowest powers. As we have shown that K is an odd function of ξ, the zero-order term will give no contribution, and the same holds for all even orders. Keeping the first order terms, one obtains

$$\mathcal{L}''(x)=\tfrac{1}{2}\mathcal{A}\bar{\varphi}(x)(\gamma\overleftarrow{\boldsymbol{\partial}}-\gamma\overrightarrow{\boldsymbol{\partial}})\chi(x), \qquad (11)$$

where $\mathcal{A}$ is defined by

$$\mathcal{A}_{\gamma\rho}=\int d\xi\,\xi_\rho(\gamma\xi)F(\xi^2).$$

The form of the Lagrangian (11) is independent of the choice of the trajectory between $x-\frac{1}{2}\xi$ and $x+\frac{1}{2}\xi$, and contains the electromagnetic fields in the usual gauge-invariant combinations

$$\overrightarrow{\boldsymbol{\partial}}=\frac{\overrightarrow{\partial}}{\partial x}-ie\mathbf{A} \quad\text{and}\quad \overleftarrow{\boldsymbol{\partial}}=\frac{\overleftarrow{\partial}}{\partial x}+ie\mathbf{A}.$$

We can take our system as consisting of electrons, muons, and photons with the interaction term (11) in addition to their usual electromagnetic interaction. The

[10] C. Bloch, Kgl. Danske Videnskab. Sebskab, Mat.-fys. Medd. **27**, 8 (1952).

total electromagnetic interaction is now given by $iejA$, where

$$iej(x)=ie[\bar{\varphi}\gamma\varphi+\bar{\chi}\gamma\chi+\alpha\bar{\varphi}\gamma a\chi+\alpha\bar{\chi}\gamma a\varphi]. \quad (12)$$

In (12) we express the Heisenberg fields φ and χ in terms of the "in" fields $\varphi^{(0)}$ and $\chi^{(0)}$, keeping only zero-order terms in e and first order terms in A. We find

$$j(0)=\bar{\varphi}^{(0)}(0)\gamma\varphi^{(0)}(0)+\bar{\chi}^{(0)}(0)\gamma\chi^{(0)}(0)$$
$$+\alpha\bar{\varphi}^{(0)}(0)\gamma a\varphi^{(0)}(0)$$
$$+\alpha\bar{\varphi}^{(0)}(0)\gamma\int S_R{}^{(e)}(-x)\left(\gamma\frac{\overrightarrow{\partial}}{\partial x}\right)a\chi^{(0)}(x)dx$$
$$-\alpha\int\bar{\varphi}^{(0)}(x)a\left(\gamma\frac{\overleftarrow{\partial}}{\partial x}\right)S_A{}^{(\mu)}(x)dx\gamma\chi^{(0)}(0). \quad (13)$$

If one now makes use of the equations of motions for the "in" fields, $[\gamma(\partial/\partial x)+m_e]\varphi^{(0)}(x)=0$, and similarly for $\chi^{(0)}(x)$ and of $[\gamma(\partial/\partial x)+m]S(x)=-\delta(x)$, one can verify that the last three terms in (13) cancel exactly and the total current j is thus given to this order by the sum of the electron and muon currents alone. This result implies in particular that the Lagrangian (11) does not contribute to $\mu\rightarrow e+\gamma$, and the process is due to the higher order approximations of the Lagrangian (10). This fact was already noted by Gell-Mann and Feynman.[11] It makes clear the smallness of the calculated rate for the calculated rate for $\mu\rightarrow e+\gamma$. It is also clear from our derivation that (11) does also not contribute to $\mu\rightarrow e+e+e$, and to $\mu+p\rightarrow e+p$. The same is true also for more complicated processes such as $\mu\rightarrow e+\gamma+\gamma$. It is easy to show that such results are all consequences of a theorem that we state and prove in the next section.

3. EQUIVALENCE THEOREM

In this section we show that our system of electrons, muons, and photons, with their electromagnetic interactions, plus the interaction term (11), is equivalent up to first order in such interaction to the system of electrons, muons, and photons interacting with their ordinary electromagnetic interaction only.

The total Lagrangian,

$$\mathcal{L}=-\bar{\varphi}(\gamma\partial+m_e)\varphi-\bar{\chi}(\gamma\partial+m_\mu)\chi-\tfrac{1}{4}F_{\mu\nu}F_{\mu\nu}$$
$$-\tfrac{1}{2}[\alpha\bar{\varphi}(\gamma\overrightarrow{\boldsymbol{\partial}}-\gamma\overleftarrow{\boldsymbol{\partial}})a\chi+\text{H.c.}], \quad (14)$$

can be simplified at the lowest order in α by transforming the last term into

$$\tfrac{1}{2}[\alpha(m_\mu\bar{\varphi}\bar{a}\chi+m_e\bar{\varphi}a\chi)+\text{H.c.}] \quad (15)$$

by use of the equation of motion satisfied by φ and χ at zero order in α. Let us introduce a spinor Ψ with the electron field and the muon field as components,

$$\Psi=\begin{bmatrix}\varphi\\ \chi\end{bmatrix}. \quad (16)$$

The Lagrangian (14) can be written as

$$\mathcal{L}=-\bar{\Psi}[\gamma\partial+M]\Psi-\tfrac{1}{4}F_{\mu\nu}F_{\mu\nu}, \quad (17)$$

where

$$M=M_0+M_1+i\gamma_5M_2, \quad (18)$$

and the matrices M_0, M_1, and M_2 are two-by-two matrices in the space of the spinors Ψ. In terms of Pauli matrices,

$$M_0=\tfrac{1}{2}(m_\mu+m_e)-\tfrac{1}{2}(m_\mu-m_e)\sigma_z, \quad (19)$$

$$M_1=\tfrac{1}{4}(m_\mu+m_e)[\mathbf{R}(\alpha)\sigma_x-\mathbf{I}(\alpha)\sigma_y], \quad (19')$$

$$M_2=-\tfrac{1}{4}(m_\mu-m_e)[\mathbf{R}(\alpha)\sigma_y-\mathbf{I}(\alpha)\sigma_x]. \quad (19'')$$

These equations show that the commutator of M_0 with M_1 (M_2) gives essentially M_2 (M_1), while the anticommutator gives essentially M_1 (M_2). We can now show that there exists a unitary matrix U such that by transforming the field operators according to $\Psi=U\Psi'$ the Lagrangian takes the form:

$$\mathcal{L}=-\Psi'[\gamma\partial+M_0]\Psi'-\tfrac{1}{4}F_{\mu\nu}F_{\mu\nu}, \quad (20)$$

as in the absence of the interaction term (11). In other words, the addition of the interaction (11) amounts, up to the first order, only to a formal redefinition of the electron and muon states, and of course no redefinition of the masses is necessary at this order. Let us write

$$U=1+iU_1+i\gamma_5U_2, \quad (21)$$

where U_1 and U_2 must be Hermitian and of the order of α Eq. (20) is then equivalent to (17) provided there exist two Hermitian matrices U_1 and U_2 such that

$$i[M_0,U_1]=-M_1, \quad (22)$$

$$\{M_0,U_2\}=-M_2. \quad (22')$$

We have seen that one can satisfy these two conditions by taking U_1 of the form (constant)$\times M_2$ and U_2 of the form (constant)$\times M_2$.

4. THE ELECTROMAGNETIC VERTEX

The general form of the $\mu\rightarrow e-\gamma$ vertex will be

$$A_\alpha(k)\langle e|J_\alpha|\mu\rangle$$
$$=A_\alpha(k)\bar{u}_L{}^{(e)}[a(k^2)\sigma_{\alpha\beta}k_\beta+ib(k^2)\gamma_\alpha+c(k^2)k_\alpha]u^{(\mu)}, \quad (23)$$

where $k=p^\mu-p^e$, $u_L{}^{(e)}$ is the spinor for the left-handed electron (we neglect m_e), $u^{(\mu)}$ is the spinor for the muon, and a, b, and c are form factors depending on k^2. Gauge invariance requires $k_\alpha\langle e|J_\alpha|\mu\rangle=0$, which gives, using the Dirac equation, $\bar{u}_L{}^{(e)}[-b(k^2)(m_\mu-m_e)+c(k^2)k^2]u^{(\mu)}$

[11] M. Gell-Mann and R. P. Feynman, *1958 Annual International Conference on High-Energy Physics at CERN*, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958), p. 261.

$=0$. The general form of the vertex is then

$$A_\alpha(k)\bar{u}_L^{(e)}\left\{a(k^2)\sigma_{\alpha\beta}k_\beta + c(k^2)\left[k_\alpha+\frac{ik^2}{m_\mu-m_e}\gamma_\alpha\right]\right\}u^{(\mu)}. \quad (24)$$

For real photons $A_\alpha k_\alpha=0$ and $k^2=0$, so that only the $\sigma_{\alpha\beta}$ term contributes.[5]

The discussion in the preceding sections shows that the main contribution to the secondary effects produced by the structure comes from the third order terms in the multipole expansion of (10). Such terms cannot be calculated unambiguously because of their dependence on the integration path from $x-\frac{1}{2}\xi$ to $x+\frac{1}{2}\xi$. A different approach to the problem with specific physical models[5,6] does also not lead to unambiguous predictions because of the delicate dependence of the result on the treatment of the divergent integrals. It may be instructive here to derive the effective Lagrangian with the simplest assumption of a straight-line path for the current through the nonlocality; such a choice makes the field A only appear in the combinations $\overleftarrow{\partial}$ and $\overrightarrow{\partial}$. Let us neglect the electron mass and call φ_L the left-handed electron, $\varphi_L=a\varphi$. The Lagrangian (10) can then be approximated as

$$\mathcal{L}''(x)=\tfrac{3}{8}B\eta_{\rho\sigma\tau\lambda}\bar{\varphi}_L[-\tfrac{1}{3}\overleftarrow{\partial}_\rho\overleftarrow{\partial}_\sigma\overleftarrow{\partial}_\tau+\overleftarrow{\partial}_\rho\overleftarrow{\partial}_\sigma\overrightarrow{\partial}_\tau - \overleftarrow{\partial}_\rho\overrightarrow{\partial}_\sigma\overrightarrow{\partial}_\tau+\tfrac{1}{3}\overrightarrow{\partial}_\rho\overrightarrow{\partial}_\sigma\overrightarrow{\partial}_\tau]\gamma_\lambda\chi, \quad (25)$$

since the first order term (11) has been eliminated by a canonical transformation, that will not affect the remaining contributions up to the first order in weak interactions. In (25), $B\eta_{\rho\sigma\tau\lambda}$ is defined by

$$B\eta_{\rho\sigma\tau\lambda}\gamma_\lambda=\int d\xi\xi_\rho\xi_\sigma\xi_\tau(\gamma\xi)F(\xi^2), \quad (26)$$

which can be satisfied by taking for $\eta_{\rho\sigma\tau\lambda}$ the totally symmetric isotropic fourth rank tensor, $\delta_{\rho\sigma}\delta_{\tau\lambda}+\delta_{\rho\tau}\delta_{\sigma\lambda}+\delta_{\rho\lambda}\delta_{\sigma\tau}$. For the case of an external electromagnetic field the Lagrangian (25) can be written as

$$\mathcal{L}''(x)=\tfrac{1}{8}B\bar{\varphi}_L\{-6m_\mu^3+6m_\mu eF_{\alpha\beta}\sigma_{\alpha\beta} -4ie\gamma_\alpha(\partial F_{\alpha\beta}/\partial x_\beta)\}\chi. \quad (27)$$

The term $6m_\mu^3$ in (27) does not give rise to any transition. This is obvious for transitions between free-particle states, and one can see that it is also true for states in the external field by noting most easily that the solutions $\varphi(x)$ and $\chi(x)$ belonging to the same energy eigenvalue E are the eigensolutions of the same Hermitian operator $[\gamma_i(\partial/\partial x_i)-\gamma_4E-ie\gamma_\mu A_\mu]$, but they belong to the two different eigenvalues m_e and m_μ and thus they are orthogonal. The vertex from (27) corresponds to energy-independent a and c in (24) given by $a=(3iem_\mu B)/2(2\pi)^3$ and $c=-(em_\mu B)/2(2\pi)^3$.

5. PHYSICAL PROCESSES

The presence of a $\mu\rightarrow e-\gamma$ vertex (24) will give rise to unwanted processes such as $\mu\rightarrow e+\gamma$, $\mu\rightarrow e+e+e$, $\mu+N\rightarrow e+N$, $\mu\rightarrow e+\gamma+\gamma$, etc.[12] We shall fix our attention here on the process of conversion of a negative muon, captured in the lowest atomic orbit around a nucleus, into an energetic electron.[13] The process can occur by a coherent mechanism, i.e., the bound muon changes into an electron of same total energy, with the Coulomb field absorbing the excess momentum; or it can occur incoherently on a single proton in the nucleus. The general form of the matrix element for the coherent transition in the electric field of the nucleus is

$$\tfrac{1}{2}e\bar{u}_L^e[-B_1G_1\boldsymbol{\alpha}\cdot\mathbf{n}+B_2G_2\beta]u_\mu, \quad (28)$$

where $\mathbf{n}$ is the unit vector in the direction of the electron momentum (we use a plane wave for the electron); $\boldsymbol{\alpha}$ and β are Dirac matrices; G_1 and G_2 are given by

$$G_1=3im_\mu\int(\mathbf{r}\cdot\mathbf{m})E(r)\exp(-i\mathbf{r}\cdot\mathbf{n}m_\mu)f(r)d\mathbf{r}, \quad (29)$$

$$G_2=\int\rho(r)\exp(-i\mathbf{r}\cdot\mathbf{n}m_\mu)f(r)d\mathbf{r}, \quad (29')$$

where $E(r)$ is the electric field of the nucleus, $f(r)$ is the radial muon wave function, and $\rho(r)$ is the nuclear density; B_1 and B_2 depend on the specific form of the vertex (24). In the particular model leading to the Lagrangian (27) one has $B_2=B_1$. The probability of the coherent process can be written as

$$\mathcal{C}(Z)=\tfrac{1}{4}\left(\frac{e^2}{4\pi}\right)m_\mu^2|G_2(Z)|^2|B_2+3[1+F(Z)]B_1|^2. \quad (30)$$

The dependence on Z is contained in $G_2(Z)$, defined by (29′), and in $F(Z)$, which can be easily derived from G_1 and G_2. An important simplification occurs in the limit in which one can take the muon wave function as a constant in the integrals. In this limit $G_2(Z)$ becomes the Fourier transform of the nuclear form factor at the relevant momentum transfer, as measured from electric scattering experiments, and $F(Z)$ is identically zero. This is the approximation used by Weinberg and Feinberg,[13] and it certainly holds for sufficiently light elements. With this approximation $\mathcal{C}(Z)$ has a maximum

[12] See, for instance, the review article by R. Gatto, in Fortschr. Physik 7, 147 (1959).

[13] After this paper was submitted, a note by S. Weinberg and G. Feinberg appeared in Phys. Rev. Letters 3, 111 (1959), dealing extensively with this problem. We have therefore abridged this section to deal only with a few points not examined in the paper by Feinberg and Weinberg, to which we refer for more complete discussion.

around $Z=30$ since $G_2(Z)$ is known to have a maximum there. For heavier elements $G_2(Z)$ is no longer approximated by the Fourier transform of the nuclear form factor, and $F(Z)$ cannot be neglected. One finds that $F(Z)$ is negative and very small for light elements but becomes large and positive for heavy elements. For Pb one finds for $F(Z)$ values between 2.5 and 5 depending on the nuclear model and on the muon wave function. Phenomenologically the most interesting case would be $B_2 \gg B_1$, since the term with B_1 is already severely limited from the present upper limit for $\mu \to e+\gamma$. In this case the only relevant form factor is $G_2(Z)$. The probability for the incoherent effect can easily be evaluated by noting that it occurs by the same mechanism as ordinary muon absorption with emission of a neutrino, and a close examination shows that the ratio of the two processes is a constant independent of Z, apart from negligible minor effects. For the model leading to the Lagrangian (27) one can compute the relation between the total branching ratio, R, for (bound μ^-)+nucleus $+e^-$, among all μ-captures, and the branching ratio R' for $\mu \to e+\gamma$ in μ-decay. One finds a general increase of R by increasing Z, essentially because of the importance of the term $F(Z)$. For $Z=10$ the relation is $R=0.9 \times 10^{-3}\, R'$, for $Z=30$, $R=2.8\times 10^{-3}\, R'$, as calculated with a Coulomb wave function and a Fermi shape for the nucleus, for Pb one finds $R=16\times 10^{-3}\, R'$ with the Wheeler wave function[14] and a square nuclear model.

[14] J. A. Wheeler, Revs. Modern Phys. **21**, 133 (1949).

6. CONCLUSION

A possible nonlocal structure of the weak interactions, for instance for $\mu \to e+\nu+\bar{\nu}$, would lead to primary effects such as deviations of the final spectra from those predicted in the local theory, and to secondary effects, namely to the existence of processes such as $\mu \to e+\gamma$, $\mu \to e+e+e$, $\mu+p \to p+e$, $\mu \to e+\gamma+\gamma$, forbidden in the local theory. Under the hypothesis, supported by the present experiments, that a possible structure, if it exists, is due to rather large intermediate masses, we can show that the dominant terms of the structure do not contribute to any secondary effect. In fact such terms can be transformed away through a canonical transformation. Their only effect consists in an unobservable redefinition of the electron and muon states. This circumstance makes the rates for secondary reactions such as $\mu \to e+\gamma$, $\mu \to e+e+e$, $\mu+N \to \mu+N$, $\mu \to e +\gamma+\gamma$, etc., anomalously small. The calculation of the next terms of the structure contributing to the secondary reactions cannot be performed unambiguously. With a simplified model we find figures of the order of 10^{-3} between the branching rate for muon captures giving muon-electron conversion and the branching rate for $\mu \to e+\gamma$ in free μ decay. Considering the present impossibility of producing reliable theoretical estimates for the rates of the secondary reactions, the most convenient way to conclude on a possible structure for weak interactions seems to be a very accurate measurement of the muon-decay spectra, unless the structure originates from very high intermediate manes.