Pionic Decay Modes of Light Λ Hypernuclei*

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Decay probabilities are calculated for pionic modes of decay of the Λ hypernuclei $A \leqslant 5$. An effect of the Pauli principle omitted in previous calculations for the two-body π^- and π^0 modes is now included. An estimate for the total decay probability of all π^- (or π^0) modes based on the completeness relation is checked by comparison with calculations based on detailed models of the decay process for two particular systems. As a result, the present data on ${}_\Lambda H^4$ decay are now consistent with either (a) spin $J=0$ for any value p/s less than 1.5, or (b) spin $J=1$ for any value p/s greater than 1.2, a much weaker conclusion than that reached previously. In either case, if the $\Delta T=\frac{1}{2}$ rule is roughly correct for Λ decay, the two-body mode $\pi^0+\alpha$ will be prominent among ${}_\Lambda He^4$ decay modes. The π^0/π^- ratio will then be large (~ 1.5) for ${}_\Lambda He^4$ decay and small ($\sim\frac{1}{6}$) for ${}_\Lambda H^4$ decay, compared with the value $\sim\frac{1}{2}$ for ${}_\Lambda He^5$ and ${}_\Lambda H^3$ decay.

1. INTRODUCTION

IN a recent paper,[1] it has been pointed out that the branching ratio for the various decay modes of a hypernucleus will generally depend on both the spin of the hypernucleus and the nature of the Λ-decay interaction. Calculations were made of the total decay probabilities for all π^- and for all π^0 modes of ${}_\Lambda H^4$, ${}_\Lambda He^4$ and ${}_\Lambda H^3$ hypernuclei, in terms of the following nonrelativistic approximation to the Λ-decay interaction,[2]

$$H'=\int \bar{\psi}_\Lambda(x)\left[s\phi(x)-i\left(\frac{p}{q_\Lambda}\right)\boldsymbol{\sigma}\cdot\nabla\phi(x)\right]\times\psi_p(x)d^3x+\text{H.c.}$$
$$+\int\bar{\psi}_\Lambda(x)[s_0\phi_0(x)-i(p_0/q_\Lambda)\boldsymbol{\sigma}\cdot\nabla\phi_0(x)]\times\psi_n(x)d^3x+\text{H.c.},\quad(1.1)$$

as well as of the partial rates for the two-body π^- and π^0 modes. Comparison with the experimental data[3] on ${}_\Lambda H^4$ decay led to the conclusion that the high relative frequency [greater than $60(\pm10)\%$ of all π^- modes of ${}_\Lambda H^4$ decay, at that time] observed for the two-body mode

$${}_\Lambda H^4 \rightarrow \pi^- + He^4 \quad (1.2)$$

could not be accounted for unless the ${}_\Lambda H^4$ hypernucleus had zero spin and the ratio p/s of the interaction (1.1) for the $\Lambda\rightarrow p+\pi^-$ decay were less than unity. Leon[4] subsequently made a similar calculation for ${}_\Lambda H^3$ decay using a rather detailed ${}_\Lambda H^3$ wave function and arrived at similar conclusions from the less complete data available on ${}_\Lambda H^3$ decay.

On the other hand, these estimates for the branching ratios of two-body modes for ${}_\Lambda H^3$ and ${}_\Lambda H^4$ decay were not as large as the best values given by the present experimental data[3] for any value of p/s or of the hypernuclear spins, although this could well have been the result of statistical fluctuations in the data. Because of this discrepancy, and the far-reaching consequences of the conclusions[1,4] which had been reached, these estimates have been carefully re-examined in the present work. Estimates of the total rate for π^- (or π^0) decay modes of the light hypernuclei are made in Sec. 3, both on the basis of the completeness relations and using particular models (with and without inclusion of the nuclear interactions in the final state), and where comparison is possible, the various estimates are found to show substantial agreement. However, in the course of the work, several errors were found in the previous results, and these are discussed and corrected here.

The estimates now obtained for the ${}_\Lambda H^4$ and ${}_\Lambda H^3$ two-body branching ratios are substantially larger than those given before[1] and allow agreement with the experimental values for either value ($S\pm\frac{1}{2}$) of the hypernuclear spin (S being the spin of the nuclear core), provided a corresponding choice is made for the ratio p/s. For the lower spin values ($S-\frac{1}{2}$), the data require for agreement that $p/s<1.5$; for the higher spin values ($S+\frac{1}{2}$), a ratio $p/s>1.2$ is required. The evidence on the nonmesonic decay rates for light hypernuclei suggests[5] that the former of these situations holds true, but the basis of this argument is not as certain as one would like. At the present stage, our results only set up a relationship between the spin dependence of Λ-nucleon forces and the nature of the Λ-decay interaction, which is required to account for the high frequency observed for two-body modes in the decay of light

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[1] R. H. Dalitz, Phys. Rev. **112**, 605 (1958).

[2] A more satisfactory approximation, invariant under Galilean transformations, could be obtained from (1.1) by modifying $\bar{\psi}_\Lambda(x)\boldsymbol{\sigma}\cdot\nabla\phi(x)\psi_p(x)$ by the addition of the term

$$-(\partial\phi/\partial t)\{\bar{\psi}_\Lambda(x)\boldsymbol{\sigma}\cdot(\nabla/2m)\psi_p(x)-[\nabla/2m_\Lambda\bar{\psi}_\Lambda(x)]\cdot\boldsymbol{\sigma}\psi_p(x)\},$$

with a similar modification for the term depending on $\phi_0(x)$. However, after an appropriate redefinition of the coefficient p, the results of the present paper would remain substantially unchanged.

[3] Ammar, Levi-Setti, Limentani, Schlein, Slater, and Steinberg, Nuovo cimento (to be published).

[4] M. Leon, Phys. Rev. **113**, 1604 (1959). We note also the recent calculations of L. E. Picasso and S. Rosati [Nuovo cimento **11**, 711 (1959)] who have used a less accurate ${}_\Lambda H^3$ wave function and who have neglected nuclear forces and the Pauli principle in the final three-body states.

[5] R. H. Dalitz, Revs. Modern Phys. **31**, 823 (1959).

hypernuclei. Further knowledge of either of these physical factors would allow conclusions to be drawn concerning the other.

The Pauli principle is found to have a strong effect on the partial lifetimes for the π^- (or π^0) modes of these hypernuclei. These partial lifetimes depend quite considerably on the value of p/s and on the spin of the hypernucleus, and may vary quite markedly with the hypernuclear species. The variation of the π^0/π^- ratio is found to be especially strong since the π^0 mode is generally suppressed when the π^- modes are enhanced, and *vice versa*. For ${}_\Lambda\mathrm{H}^4$ and ${}_\Lambda\mathrm{He}^4$, it appears that the measurement of the total lifetime may now be feasible, by observation of the spatial distribution of the decay of the hypernuclei produced in the reactions

$$K^-+\mathrm{He}^4\rightarrow\begin{cases}{}_\Lambda\mathrm{He}^4+\pi^-, & (1.3\mathrm{a})\\ {}_\Lambda\mathrm{H}^4+\pi^0. & (1.3\mathrm{b})\end{cases}$$

Examples of the former reaction have recently been reported by Block *et al.*[6] Observations of the decay processes of the hypernuclei produced in these reactions will then allow also the determination of their partial lifetimes for the π^- modes, and of their nonmesonic decay rates. Knowledge of the nonmesonic decay rates for ${}_\Lambda\mathrm{H}^4$ and ${}_\Lambda\mathrm{He}^4$, as well as for ${}_\Lambda\mathrm{He}^5$, would throw a great deal of light on the spin dependence of the $\Lambda+p\rightarrow n+p$ and $\Lambda+n\rightarrow n+n$ interactions.

2. MATRIX ELEMENTS FOR THE HYPERNUCLEAR PIONIC DECAY MODES

For definiteness, we shall speak specifically of π^- modes in the present section. The corresponding expressions for π^0 modes may be obtained from the expressions given here by replacing (s,p) by (s_0,p_0) and by interchanging proton and neutron coordinates. We consider generally the decay of a hypernucleus of N neutrons and P protons, with wave functions $\phi(\Lambda;1,2\cdots P;1,\cdots N)$, to the nuclear states $\Phi_n(1,2,\cdots,P+1;1,2,\cdots N)$ with the emission of a negative pion into the state $\varphi_{\mathbf{q}}(x)$ of outgoing momentum $\mathbf{q}$. The wave functions ϕ and Φ_n are antisymmetric with respect to the proton labels and the neutron labels separately, and are normalized to unity for integration over all particle coordinates over all space. From the interaction Hamiltonian (1.1), the matrix element $M_n(\mathbf{q})$ for decay leading to the nuclear state Φ_n is given by

$$M_n(\mathbf{q})=\Big\langle\phi(\Lambda;1,\cdots P;1,\cdots N)\Big|\times\int\bar{\psi}_\Lambda(x)[s\varphi_{\mathbf{q}}(x)-i(p/q_\Lambda)\boldsymbol{\sigma}\cdot\nabla\varphi_{\mathbf{q}}(x)]\times\psi_p(x)d^3x\Big|\Phi_n(1,2,\cdots P+1;1,\cdots N)\Big\rangle,\quad(2.1)$$

where $\psi_p(x)$ annihilates a proton at the coordinate x and $\bar{\psi}_\Lambda(x)$ creates a Λ particle at this point. The evaluation of this matrix element may be carried out using the following relations given by Fock[7]:

$$\begin{aligned}\psi_p(x)\Phi_n(1,2\cdots P+1;1,\cdots N)&=(P+1)^{-\frac{1}{2}}[\Phi_n(x,1,2\cdots P;1,\cdots N)-\Phi_n(1,x,2\cdots P;1,\cdots N)\cdots\\&\quad+(-1)^P\Phi_n(1,2\cdots P,x;1,\cdots N)]\\&=(P+1)^{\frac{1}{2}}\Phi_n(x,1,2\cdots P;1,\cdots N),\end{aligned}\quad(2.2\mathrm{a})$$

$$\psi_\Lambda(x)\phi(\Lambda;1,\cdots P;1,\cdots N)=\phi(x;1,\cdots P;1,\cdots N),\quad(2.2\mathrm{b})$$

and leads to the result

$$M_n(\mathbf{q})=(P+1)^{\frac{1}{2}}\int\phi^*(x,1,2\cdots P;1,\cdots N)\times[s\varphi_{\mathbf{q}}(x)-i(p/q_\Lambda)\boldsymbol{\sigma}\cdot\nabla\varphi_{\mathbf{q}}(x)]\times\Phi_n(x,1,\cdots P;1,\cdots N)d^3xd^3x_1\cdots.\quad(2.3)$$

The presence of the factor $(P+1)^{\frac{1}{2}}$ represents a consequence of the Pauli principle for the protons of the final system, and leads to an enhancement of the rate for two-body decay. This factor was inadvertently overlooked in the earlier calculations.[1,4]

In obtaining expression (2.3), it has been tacitly assumed that the final state of pion and nucleons may be represented by the product of a pion wave function and a nuclear wave function. The pion wave function $\varphi_{\mathbf{q}}(x)$ may generally be chosen to allow for the distortion of the outgoing pion wave by the mean (optical model) potential resulting from the pion scattering and absorption by the nucleons. For the light systems to be considered here, it is reasonable to neglect these distortions and to represent $\varphi_{\mathbf{q}}(x)$ by a plane wave of momentum $\mathbf{q}$.

In the calculation of the two-body decay rates for the light hypernuclei, the wave functions for the final nuclei H^3, He^3, and He^4 may be adequately approximated by a product of spin and space wave functions. Similarly, the wave functions for the hypernuclei $A\leqslant5$ may be represented by the product of a space wave function describing the motion of the Λ particle relative to the nuclear core, a space wave function describing the core nucleus, and an over-all spin wave function. As discussed previously,[1] with these approximations the partial lifetime for a two-body decay mode separates into the following factors, a sticking probability $F^2(q)$, a spin factor $\mathcal{S}$, the additional factor $(P+1)$ of expression (2.3), and some phase-space factors. Explicitly, the partial decay rate is given by

$$R=(P+1)\mathcal{S}F^2(q)2q/(1+\omega_q/M_r),\quad(2.4)$$

where ω_q is the total pion energy and M_r is the mass of the recoil nucleus.

[6] Block, Brucker, Hughes, Kikuchi, Meltzer, Anderson, Pevsner, Harth, Leitner, and Cohn, Phys. Rev. Letters 3, 291 (1959).

[7] V. Fock, Z. Physik 75, 622 (1932); R. Becker and G. Liebfried, Phys. Rev. 69, 34 (1946).

For $_\Lambda H^4$ and $_\Lambda He^4$ decay, the function $F(q)$ has already been calculated[1,8] assuming Gaussian forms $\exp(-\frac{1}{2}\alpha_A \sum_{ij}' r_{ij}^2)$ for the space wave functions of the nuclear systems $A=3$ and 4. For $_\Lambda H^4 \to \pi^- + He^4$, the decay rate relative to that for π^- decay of the free Λ particle then has the values

$$\text{(a) } J=0:\quad 2\frac{q}{q_\Lambda}\frac{1+\omega_\Lambda/M_1}{1+\omega_q/M_4}\frac{s^2}{s^2+p^2}F^2(q)=\frac{1.41s^2}{s^2+p^2}$$

$$\text{(b) } J=1:\quad \frac{2}{3}\left(\frac{q}{q_\Lambda}\right)^3\frac{1+\omega_\Lambda/M_1}{1+\omega_q/M_4}\frac{p^2}{s^2+p^2}F^2(q)=\frac{0.76p^2}{s^2+p^2}, \tag{2.5}$$

according as $J=0$ or 1 for ground state $_\Lambda H^4$. The final values given correspond to $q/q_\Lambda=1.3$ and $F^2(q)=0.46$, the latter being appropriate to $B_\Lambda=1.8$ Mev for $_\Lambda H^4$. For $J=0$, the rate of two-body decay alone already exceeds the rate of free Λ decay through the s channel. For the process $_\Lambda He^4 \to \pi^0 + He^4$, after making the appropriate changes in (2.3), the decay rate takes the value 1.48 $s_0^2/(s^2+p^2)$, or 1.04 $p_0^2/(s^2+p^2)$, relative to the π^- rate for free Λ decay, according as $J=0$ or 1. Here the values $q_0/q_\Lambda=1.45$ and $F^2(q_0)=0.45$ have been used.

The two-body modes of $_\Lambda H^3$ decay are

$$_\Lambda H^3 \to \begin{cases} He^3+\pi^-, & (2.6a)\\ H^3+\pi^0. & (2.6b)\end{cases}$$

The decay rate for the mode (2.6a) is given by the appropriate expression

$$\text{(a) } J=\tfrac{1}{2}:\quad 2F^2(q)[\tfrac{3}{4}s^2+\tfrac{1}{12}p^2(q/q_\Lambda)^2]2q/(1+\omega_q/M_3),$$
$$\text{(b) } J=\tfrac{3}{2}:\quad \tfrac{2}{3}F^2(q)\tfrac{1}{3}p^2(q/q_\Lambda)^2 2q/(1+\omega_q/M_3); \tag{2.7}$$

that for the mode (2.6b) by the corresponding expression obtained by insertion of s_0, p_0, and q_0. With $B_\Lambda=0.25$ Mev for $_\Lambda H^3$, Leon[4] has obtained the value $F^2(q)=0.37$, using a $_\Lambda H^3$ wave function[9] which includes correlations between Λ particle and nucleons and an exponential wave function $\exp[-\alpha(r_{12}+r_{23}+r_{31})]$ for He^3.

It is of interest to remark here that the prediction of the $\Delta T=\frac{1}{2}$ rule for Λ decay[5] is $s/s_0=p/p_0=-\sqrt{2}$. This prediction is not in conflict with the ratio $(s^2+p^2)/(s_0^2+p_0^2)=1.81\pm0.2$ obtained experimentally for free Λ decay.[10] This rule predicts a π^-/π^0 ratio of 2 for the $_\Lambda H^3$ modes (2.6). A verification of this prediction for $_\Lambda H^3$ decay would provide evidence supporting this rule for the s and p channels of Λ decay separately.

For the hypernuclei of the nucleon $1p$ shell, the corresponding discussion for two-body modes such as

$$\begin{aligned} {}_\Lambda He^7 &\to \pi^- + Li^7,\\ {}_\Lambda Li^7 &\to \pi^- + Be^7,\\ {}_\Lambda Li^8 &\to \pi^- + Be^8,\end{aligned} \tag{2.7}$$

is more complicated and will not be discussed in detail here. In these cases, the proton resulting from the Λ decay is captured into the p shell. As a result, the s-channel Λ-decay interaction leads necessarily to the emission of a p-wave pion in these two-body modes, and an initial hypernuclear state of spin J can only reach final nuclear states of spin $J+1$, J or $J-1$ ($J=0\to0$ being completely forbidden) in the approximation considered here. Similarly the p-channel Λ-decay interaction allows the emission of both s-wave and d-wave pions, so that final states of spin $J-2, J-1, \cdots J+2$ can be reached[11] (here only $J=0\to1$ and $J=1\to0$ are completely excluded).

3. TOTAL RATES FOR π^- AND π^0 MODES OF HYPERNUCLEAR DECAY

In this section several estimates will be made of the total rates for the pion modes of hypernuclear decay, in order to assess their reliability by comparisons between them. Within the approximations of this paper, the total rate for π^- decay of a hypernucleus of wave function ϕ is given by the expression

$$R(\pi^-)=\frac{2\pi}{2J+1}\sum_n \delta\left(\omega_q+\frac{q^2}{2M_r}+E_n-Q_\Lambda+B_\Lambda\right)\times[|M_n(q)|^2]_{\text{Av}}q^2dq/\pi\omega_q, \tag{3.1}$$

where $M_n(q)$ denotes the matrix element (2.3) for decay to a final nuclear state of wave function Φ_n, the sum n is taken over all states of $(P+1)$ protons and N neutrons (M_r being the mass of the final nuclear system), and E_n denotes the energy of the nucleons in their barycentric system, relative to the total energy of a free proton and the ground state of the nucleus (N,P). First, an estimate will be obtained for (3.1) by the use of completeness relations; then evaluations of (3.1) will be made for some specific hypernuclides with the use of particular models.

[8] R. H. Dalitz and B. W. Downs, Phys. Rev. **111**, 967 (1958).
[9] B. W. Downs and R. H. Dalitz, Phys. Rev. **114**, 593 (1959).
[10] This ratio is a combination of the value recently reported by Crawford, Cresti, Douglass, Good, Kalbfleisch, Stevenson, and Ticho [Phys. Rev. Letters **2**, 266 (1959)] and the values of other groups quoted by Glaser, Good, and Morrison, *Proceedings of the 1958 Annual International Conference on High-Energy Physics at* CERN, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958), p. 270.

[11] The conflicting statement made concerning $_\Lambda Li^8$ decay on p. 980 of reference 8 is incorrect. In the decay $_\Lambda Li^8 \to \pi^- + Be^8$, the transitions $2\to2$, $1\to0$, or $1\to2$ to the ground state and first excited state of Be^8 are allowed through the s-channel Λ-decay interaction, and the transitions $J=0\to0$, $0\to2$, $2\to0$, and $2\to2$ (as well as $2\to4$ to the second excited state) are allowed through the p-channel interaction. In the six examples observed for this decay mode, none leads to ground state Be^8; the c.m. α-α energies observed for these events are widely dispersed and show no marked correlation with the known energy levels of Be^8.

(a) Use of the Completeness Relation

The empirical spectra[3] of π^- momenta from the decay of light hypernuclei show a strong peaking near the upper end of the continuous spectrum. In physical terms, this feature reflects (i) the low binding energy B_Λ of the Λ particle, which means that rather low momenta predominate in the Λ-momentum distribution within these hypernuclei and (ii) the fact that the π^- meson suffers little scattering in its escape from the system. It suggests that, in the matrix elements M_n of the expression (3.1), the replacement of q by a suitable mean value $\bar{q}$ should be quite a good approximation. The term E_n may then be replaced by the corresponding mean $\bar{E}$ in the energy conservation relation and, if the summation n is then extended over all states, whether or not energetically accessible, the completeness relation

$$\begin{aligned}\sum\nolimits_n \Phi_n(1,2,\cdots n_P;\alpha,\beta,\cdots\eta_N)&\\ \times\Phi_n^*(1',2',\cdots n_P';\alpha',\beta',\cdots\eta_N')&\\ =\frac{1}{N!(P+1)!}\Big[\sum_{\mathcal{P}}\epsilon(\mathcal{P})\mathcal{P}_P\delta_{11'}{}^{\sigma}\delta_{22'}{}^{\sigma}\cdots&\\ \times\delta(\mathbf{r}_1-\mathbf{r}_1')\delta(\mathbf{r}_2-\mathbf{r}_2')\cdots\Big]&\\ \times\Big[\sum_{\mathcal{P}}\epsilon(\mathcal{P})\mathcal{P}_N\delta_{\alpha\alpha'}{}^{\sigma}\delta_{\beta\beta'}{}^{\sigma}\cdots\delta(\mathbf{r}_\alpha-\mathbf{r}_\alpha')\cdots\Big],&\quad(3.2)\end{aligned}$$

may be used for the evaluation of the total rate (3.1) in closed form. In the relation (3.2), the summations are to be taken over all permutations $\mathcal{P}_P$ and $\mathcal{P}_N$, respectively, of the $(P+1)$ proton labels $(1,2,\cdots n_p)$ and the N neutron labels $(\alpha,\beta,\cdots\eta_N)$, the coefficients $\epsilon(\mathcal{P})$ being the signatures of the corresponding permutations $\mathcal{P}$. These approximations and the use of relation (3.2) reduce expression (3.1) to the form given previously,

$$\begin{aligned}R(\pi^-)=\Bigg\{s^2+\left(\frac{\bar{q}}{q_\Lambda}\right)^2p^2&\\ -\frac{\eta(\bar{q})}{2J+1}\mathrm{Tr}\left[s^2P_{\Lambda1}{}^{\sigma}+\frac{p^2}{3}\left(\frac{\bar{q}}{q_\Lambda}\right)^2(2-P_{\Lambda1}{}^{\sigma})\right]\Bigg\}&\\ \times2\bar{q}/(1+\bar{\omega}_q/M_r),&\quad(3.3)\end{aligned}$$

in which the trace is to be taken over the spin states of the initial hypernucleus, the suffix 1 referring to any proton of this hypernucleus. In deriving (3.3), use was made of the antisymmetry of the hypernuclear wave function ϕ with respect to both proton and neutron labels. The function $\eta(q)$ denotes the overlap integral

$$\begin{aligned}\eta(q)=\int\phi^*(\Lambda;1,2,\cdots P;\alpha,\beta,\cdots N)&\\ \times\exp[i\mathbf{q}\cdot(\mathbf{r}_\Lambda-\mathbf{r}_1)]\phi(1;\Lambda,2,\cdots P;\alpha,\beta,\cdots N)&\\ \times d^3x_\Lambda d^3x_1\cdots d^3x_P d^3x_\alpha\cdots d^3x_N.&\quad(3.4)\end{aligned}$$

For ${}_\Lambda H^4$ and ${}_\Lambda He^4$, the product wave function

$$\begin{aligned}\phi(\Lambda;1;\alpha\beta)=D\exp[-\tfrac{1}{2}\alpha_3(r_{1\alpha}{}^2+r_{1\beta}{}^2+r_{\alpha\beta}{}^2)]&\\ \times u(|\mathbf{r}_\Lambda-\tfrac{1}{3}(\mathbf{r}_1+\mathbf{r}_\alpha+\mathbf{r}_\beta)|)&\quad(3.5)\end{aligned}$$

considered previously[1,8] may be used for the evaluation of $\eta(\bar{q})$. With $u(R)$ normalized to unity for integration over all space, and with the corresponding normalization constant D, $\eta(\bar{q})$ reduces to the integral

$$\begin{aligned}\eta(\bar{q})=(\tfrac{3}{2})^6(2\pi\alpha_3{}^3)^{\frac{1}{2}}\int_0^\infty\int_0^\infty\left[\int_{(R-S)^2}^{(R+S)^2}\exp(27\alpha_3x/32)\right.&\\ \left.\times\left(\frac{\sin Q\sqrt{x}}{Q\sqrt{x}}\right)dx\right]F(R)F(S)dRdS,&\quad(3.6)\end{aligned}$$

where $F(R)=Ru(R)\exp(-9\alpha_3R^2/4)$ and $Q=3\bar{q}/4$. This integral has been evaluated numerically using the wave functions $u(R)$ computed for $B_\Lambda=1.9$ Mev and 2.5 Mev in the earlier work[8] on the binding energy of the $({}_\Lambda H^4,{}_\Lambda He^4)$ doublet. The results[12] may be adequately represented by a quadratic in Q (expressed in unit 100 Mev/c),

$$\begin{aligned}&\text{(a) } B_\Lambda=1.9\text{ Mev},\\ &\qquad\eta(\bar{q})=0.744-0.466Q^2+0.172Q^4;\\ &\text{(b) } B_\Lambda=2.5\text{ Mev},\qquad\qquad\qquad\qquad(3.7)\\ &\qquad\eta(\bar{q})=0.794-0.475Q^2+0.168Q^4.\end{aligned}$$

For $\bar{q}=94$ Mev/c, these expressions (3.7a) and (3.7b) lead to the values 0.57 and 0.61 for η, respectively. For ${}_\Lambda He^5$ decay, with $B_\Lambda=2.8$ Mev, an extrapolation leads to the estimate $\eta\simeq0.66$, which we shall use below. For ${}_\Lambda H^3$, the integral (3.4) has been carried through by Leon.[4]

As a result of the smallness of the energy release in Λ decay, this estimate of the total π^- (or π^0) decay rate from the completeness relation suffers from certain defects, for which some correction may be made:

(i) Transitions to final bound states of the nucleus $(N, P+1)$ are frequently a large fraction of the decay modes, as we have noted above for ${}_\Lambda H^4$ decay. For these terms, which correspond to negative values of E_n, the value of q is much larger than the mean value $\bar{q}$ appropriate to the continuum pion distribution. Their contribution is substantially underestimated if the replacement $q=\bar{q}$ is made for them; this is especially the case for the p-channel terms, for which the probability contains an additional factor $(q/q_\Lambda)^2$. This may be allowed for by taking the value of $\bar{q}$ appropriate to

[12] These results may be compared with the evaluation of reference 1 using the approximate wave function (2.20) discussed there for $B=1.8$ Mev, which leads to the corresponding expression $\eta(\bar{q})=0.821-0.482Q^2+0.159Q^4$. Since this Gaussian approximation to u is more compact than the exact form of u, it naturally leads to somewhat larger values for $\eta(\bar{q})$. For $\bar{q}=94$ Mev/c, the approximate form of u leads to $\eta(\bar{q})=0.63$. It should be mentioned here that Eqs. (2.19) and (2.21) of reference 1 contain several misprints. The first factor of Eq. (2.19) should read $(\frac{3}{4})^3$; the first two factors of Eq. (2.21) should be $(\frac{3}{4})^3(9\alpha_3/2)^{\frac{3}{2}}$.

the continuum distribution and then adding for each bound state term the difference between its contribution to the sum (3.1) for the correct value of q and that for the value $\bar{q}$, as was done already in reference 1.

(ii) For s-channel decay, the reduction in the final phase space available, resulting from the energy given to the final nucleons, is one factor tending to reduce the decay rate for a bound Λ particle from that for free Λ decay. This is one of the effects which the choice of a mean $\bar{q}$ less than q_Λ takes into account, at least roughly. For p-channel decay, to this effect there is added the effect of the factor $(q/q_\Lambda)^2$ contributed by the matrix element to the decay probability. Although the physical argument underlying the mean value approximation suggests that the mean value of $(q/q_\Lambda)^2$ may be obtained rather unambiguously from the peaked distribution of q, either from observation or from calculations based on a simplified model, it is not necessarily the case (as we shall see from examples below) that this value of $\bar{q}$ leads to an adequate representation of the former effect. However, since the Pauli principle generally produces a much larger modification of the decay rate, we shall generally adopt for $\bar{q}$ the value at the maximum of the q distribution.

(iii) After the approximation $q=\bar{q}$ in the sum of Eq. (3.1) over states $E_n>0$, the extension of the sum to energetically inaccessible states clearly adds to its value. The relative magnitude of this addition will depend, for example, on the character of the nuclear interactions as function of the energy E_n. Since the accessible energies E_n only run up to about 30 Mev, it is quite possible for matrix elements to particular energetically-inaccessible states to be appreciable, under suitable circumstances; for example, if there exists an appropriate resonance state in this region. Since the extended sum is independent of the nature of the interactions in the final nuclear states, it then follows that the restricted sum leading to the actual decay rate (3.1) may have some dependence on the particular features of the nuclear interactions for the final $(N, P+1)$ system. However, since final nuclear states over a considerable range of angular momenta contribute appreciably, it is reasonable to expect that the sensitivity of the total decay rate to the particular structure of the resonance states of the final system should be quite weak.

(b) Decay of $_\Lambda He^5$

The only π^- mode known for $_\Lambda He^5$ decay is

$$_\Lambda He^5 \rightarrow \pi^- + p + He^4. \quad (3.8)$$

The nuclear interactions in the p-He^4 system are known from analysis of the p-He^4 scattering data to be strong, a sharp $p_{\frac{3}{2}}$ resonance being found at c.m. energy 1.8 Mev and a broad $p_{\frac{1}{2}}$ resonance over the range 5–10 Mev. A calculation of the characteristics of this decay mode (3.8) has recently been made by Byers and Cottingham,[13] who have taken into account the influence of the p-He^4 forces in the $s_{\frac{1}{2}}$, $p_{\frac{1}{2}}$, and $p_{\frac{3}{2}}$ states. Their calculation does not take into account the Pauli principle in an explicit way, for the proton and the alpha particle are treated as independent structureless particles whose interaction is described by a set of phenomenological potentials. However, they use a strongly attractive $s_{\frac{1}{2}}$ potential which reproduces the known phase shifts for $s_{\frac{1}{2}}$ p-He^4 scattering. This $s_{\frac{1}{2}}$ potential implies a bound state for the p-He^4 system, which is naturally omitted in the sum over the final p-He^4 states reached in the decay (3.8); it is this omission which suppresses the total rate of decay and which is equivalent to the imposition of the Pauli principle relating p with the constituent protons of the alpha particle. The Λ-He^4 wave function used corresponds to $B_\Lambda=3.1$ Mev and a square well potential of radius 3.1 fermis. The calculation treats the pion nonrelativistically, an approximation which affects only slightly the ratio R_b/R_f of the decay probabilities for the bound and the free Λ particle. Their calculation[14] leads to the following result for the total π^- decay rate,

$$R_b(\pi^-)/R_f(\pi^-)=0.43(s^2+0.70p^2)/(s^2+p^2). \quad (3.9)$$

The reduction in the decay probability is especially strong for pure p-channel decay, by the factor 0.30. After replacing (s,p) by (s_0,p_0), the corresponding ratio $R_b(\pi^0)/R_f(\pi^0)$ will also be closely given by (3.9).

The total decay rate was also calculated using the same Λ-He^4 wave function but neglecting both the Pauli principle and the final state forces, with the result

$$R_b(\pi^-)/R_f(\pi^-)=0.866(s^2+0.78p^2)/(s^2+p^2). \quad (3.10)$$

The first factor represents the reduction in phase space for the s channel decay and the mean value of $(q/q_\Lambda)^2$ is given by 0.78. In considering the estimate (3.3), we will therefore use a mean value $\bar{q}/q_\Lambda=0.9$, which is also not inconsistent with the empirical data on $_\Lambda He^5$ decay; however the phase space factor of (3.3) corresponding to this mean value is $(\bar{q}/q_\Lambda)(1+\omega_\Lambda/M)/(1+\bar{\omega}/M_4)\simeq 1.03$. With this mean value, Eq. (3.3) then leads to the estimate $1.03[1-\eta(\bar{q})](s^2+0.8p^2)/(s^2+p^2)$ for R_b/R_f which, with the earlier estimate $\eta(\bar{q})\simeq 0.66$ leads to a ratio smaller than (3.9) by a factor 0.80 for the s channel, 0.90 for the p channel. This degree of discrepancy may be partly due to the use of an $\eta(\bar{q})$ which was calculated for a different Λ-He^4 wave function (corresponding to a Gaussian potential), but probably reflects a real enhancement of the decay rate by the very strong final state interactions in this decay process. This comparison supports the view that, even with extremely strong final-state forces, the estimate (3.3)

[13] N. Byers and N. Cottingham (private communication, 1958); calculations on a similar basis have also been reported recently by Y. C. Tang [Nuovo cimento **10**, 780 (1958)] and by J. Szymanski [Nuovo cimento **10**, 834 (1958)].

[14] It is a pleasure to acknowledge the assistance given us by Dr. Byers and Dr. Cottingham in providing the details of their calculations from which the result (3.9) has been obtained.

based on the completeness relation will not be grossly in error.

(c) π^- Mode of $_\Lambda \dot{\mathrm{He}}^4$ Decay

The only appreciable π^- mode for $_\Lambda\mathrm{He}^4$ decay is

$$_\Lambda\mathrm{He}^4 \rightarrow \pi^- + p + \mathrm{He}^3. \tag{3.11}$$

Taking the value $\bar{q}/q_\Lambda = 0.9$, consistent with the observed pion spectrum,[15] the value of $\eta(\bar{q})$ is 0.58 for $B_\Lambda = 1.9$ Mev, and the total π^- decay rate given by the completeness relation is

$$R_b(\pi^-)/R_f(\pi^-) = 0.43(s^2+0.81p^2)/(s^2+p^2). \tag{3.12}$$

To provide an alternative estimate, a calculation has also been made in which the Pauli principle has been taken into account in the final state, although with neglect of the p-He^3 nuclear interactions.[16] The p-He^3 wave function used was therefore simply

$$\frac{1}{\sqrt{3}}\{\exp[i\mathbf{p}\cdot\mathbf{r}_1 + i(\mathbf{P}/3)\cdot(\mathbf{r}_2+\mathbf{r}_3+\mathbf{r}_\beta)]\chi_1\psi(2,3;\beta)$$
$$-\exp[i\mathbf{p}\cdot\mathbf{r}_2 + i(\mathbf{P}/3)\cdot(\mathbf{r}_1+\mathbf{r}_3+\mathbf{r}_\beta)]\chi_2\psi(1,3;\beta)$$
$$-\exp[i\mathbf{p}\cdot\mathbf{r}_3 + i(\mathbf{P}/3)\cdot(\mathbf{r}_1+\mathbf{r}_2+\mathbf{r}_\beta)]$$
$$\times\chi_3\psi(2,1;\beta)\}, \tag{3.13}$$

where χ is the spin function of the final proton (momentum p), and ψ denotes the wave function of the He^3 nucleus (recoil momentum P). The wave function (3.5) was used for $_\Lambda\mathrm{He}^4$, the Gaussian form already used there for the nuclear core of the $_\Lambda\mathrm{He}^4$ wave function being chosen (with the appropriate spin factor) for ψ. With the axis of spin quantization along the direction of the outgoing pion, the matrix element (2.3) then takes essentially the same form for each of the initial spin states of $_\Lambda\mathrm{He}^4$, namely

$$A\int G(2,3;\beta)u(|\mathbf{r}_1-(\mathbf{r}_2+\mathbf{r}_3+\mathbf{r}_\beta)/3|)e^{i\mathbf{q}\cdot\mathbf{r}_1}(1-P_{12})$$
$$\times\exp[i\mathbf{P}\cdot(\mathbf{r}_2+\mathbf{r}_3+\mathbf{r}_\beta)/3]e^{i\mathbf{p}\cdot\mathbf{r}_1}G(2,3;\beta)$$
$$\times d^3r_1d^3r_2d^3r_3d^3r_\beta, \tag{3.14}$$

where P_{12} interchanges the suffices 1 and 2 standing to the right, momentum conservation requires

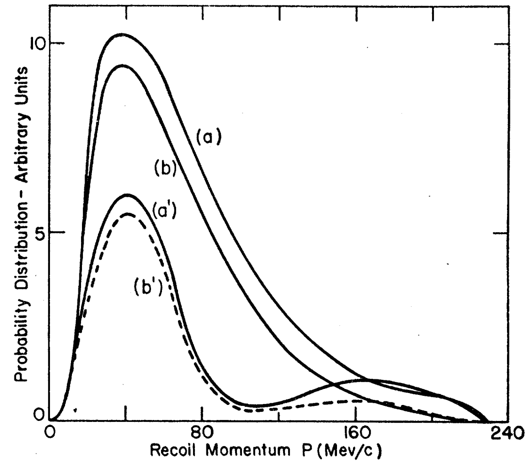

FIG. 1. The He^3 recoil momentum spectrum for $_\Lambda\mathrm{He}^4 \rightarrow \pi^- + p + \mathrm{He}^3$ decay, calculated with neglect of the forces between the final particles, for s-channel Λ decay both (a) neglecting, and (a′) taking into account the Pauli principle, and for p-channel decay both (b) neglecting, and (b′) taking into account the Pauli principle for the final nucleons.

$\mathbf{P}+\mathbf{p}+\mathbf{q}=0$, and the constant A is either $\pm s$ or $\pm pq/q_\Lambda$ according as the s-channel or p-channel interaction is effective. Inserting the normalized Gaussian form for G, expression (3.14) reduces to

$$A\bigg\{\int e^{i\mathbf{P}\cdot\mathbf{R}}u(R)d\mathbf{R}$$
$$-\left(\frac{9}{5}\right)^{\frac{3}{2}}\exp[-(1/10\alpha_3)(\mathbf{P}/3-\mathbf{p})^2]$$
$$\times\int\exp[-(9\alpha_3/10)R^2]e^{3i\mathbf{R}\cdot(\mathbf{q}-\mathbf{p})/5}u(R)d\mathbf{R}\bigg\}. \tag{3.15}$$

The matrix element (3.15) has been evaluated by numerical integration and leads to the recoil momentum distributions shown in Fig. 1. These distributions give greater weight to the lower recoil momenta ($\sim$50 Mev/c) than does the distribution obtained by omitting the second term of (3.15), which corresponds to neglecting the Pauli principle. The inclusion of the p-He^3 p-wave attraction may be expected to produce some increase of the probability of recoil momenta above 50 Mev/c. In Fig. 2 is shown the angular distribution obtained for the pions in the π^--p rest system, relative to the direction of their total momentum; a marked forward peaking appears for recoil momenta exceeding 100 Mev/c, in comparison with a backward peaking for lower momenta. Integration over these distributions leads to the total π^- decay rate.

$$R_b(\pi^-)/R_f(\pi^-) = 0.40(s^2+0.79p^2)/(s^2+p^2), \tag{3.16}$$

which agrees rather well with the estimate (3.12) from the completeness relation. Even for a system as lightly bound as $_\Lambda\mathrm{He}^4$, the suppression of the decay rate by

[15] Calculation of $R_b(\pi^-)/R_f(\pi^-)$ for $B\ =1.9$ Mev, with neglect of the Pauli principle and the nuclear forces in the final state, leads to the result 0.84 $(s^2+0.80p^2)/(s^2+p^2)$; the ratio of p to s contributions corresponds to a mean value $(\bar{q}/q_\Lambda)^2=0.8$.

[16] The information available on the p-He^3 interaction has been summarized recently by B. H. Bransden and H. H. Robertson [Proc. Phys. Soc. (London) **A72**, 770 (1958)], who have also made phase-shift calculations based on the resonating-group approximation. It appears that the p-wave attractions are a good deal weaker than for the p-He^4 system, no p-He^3 resonance appearing in this wave up to 10-Mev c.m. energy; the s-wave interactions are quite strong and the calculations suggest that the s-phase shifts may pass through 90° rather gradually somewhere between 10 and 20 Mev.

the Pauli principle is quite a strong effect, increasing the partial lifetime for π^- decay by a factor of greater than 3 for pure p-channel decay.

For the π^- modes of ${}_\Lambda H^4$ decay and the π^0 modes of ${}_\Lambda He^4$ decay, which involve more complicated final nuclear interactions, no calculations have been made on the basis of particular models. From the experience above, our expectation is that the estimates based on the completeness relation should be quite reliable for the total decay rates, and these will now be given. With $J=0$ for ${}_\Lambda H^4$, the expression for $R_b(\pi^-)/R_f(\pi^-)$ is

$$\left(\frac{\bar{q}}{q_\Lambda}\right)\left\{s^2\left[1+\eta(\bar{q})+2\left(\frac{q}{\bar{q}}\right)F^2(q)-2F^2(\bar{q})\right]+\left(\frac{\bar{q}}{q_\Lambda}\right)^2 p^2[1-\eta(\bar{q})]\right\}\left(1+\frac{\omega_\Lambda}{M}\right)\Big/(s^2+p^2)\left(1+\frac{\bar{\omega}}{4M}\right), \quad (3.17a)$$

which takes the value $(1.84s^2+0.35p^2)/(s^2+p^2)$ with $\bar{q}/q_\Lambda=0.9$, $\eta(\bar{q})=0.58$ and the values of F^2 given previously.[1] For $J=1$, the expression is

$$\left(\frac{\bar{q}}{q_\Lambda}\right)\left\{s^2[1-\eta(\bar{q})]+p^2\left(\frac{\bar{q}}{q_\Lambda}\right)^2\times\left[1-\tfrac{1}{3}\eta(\bar{q})+\frac{2}{3}\left(\frac{q}{\bar{q}}\right)^3F^2(q)-\tfrac{2}{3}F^2(\bar{q})\right]\right\}\times\left(1+\frac{\omega_\Lambda}{M}\right)\Big/(s^2+p^2)\left(1+\frac{\bar{\omega}}{4M}\right), \quad (3.17b)$$

which takes the value $(0.43s^2+1.12p^2)/(s^2+p^2)$. For ${}_\Lambda He^4$, exactly corresponding expressions hold for the π^0 modes.

4. DISCUSSION AND CONCLUSION

First we consider the branching ratio for two-body modes in ${}_\Lambda H^4$ decay. The fraction of two-body events among the π^- modes of ${}_\Lambda H^4$ decay may be obtained as function of p/s and J from Eqs. (2.5) and (3.17), with the results

$$\begin{aligned}&\text{(a) } J=0:\quad 1.41s^2/(1.84s^2+0.35p^2),\\&\text{(b) } J=1:\quad 0.76p^2/(0.43s^2+1.12p^2),\end{aligned} \quad (4.1)$$

which have been plotted in Fig. 3. The empirical value[3] of this ratio now stands at 0.66 ± 0.06. The prediction for $J=0$ lies within two standard deviations of this value if $p/s\leqslant1.5$, that for $J=1$ if $p/s\geqslant1.2$. The ratio (4.1) is less sensitive to p/s than had been thought the case previously.[1] Decay through the channel for which the two-body mode is forbidden is strongly suppressed by the Pauli principle and, for the allowed channel, the two-body mode always contributes a large fraction of the decay events, both because of the enhancement of the two-body mode by the $(P+1)^{\frac{1}{2}}$ factor noted in Sec. 2 and, for the p channel, because of the large value of q/q_Λ. As a result, the only conclusions possible from these data alone are the (overlapping) limitations on p/s which correspond to each of the two possible spin values for the $({}_\Lambda H^4, {}_\Lambda He^4)$ doublet.

Observations on the (nonmesonic)/(π^- mesonic) ratio Q in the decay modes of Λ hypernuclei provide some evidence that p/s cannot appreciably exceed unity. Karplus and Ruderman[17] have shown that, if the p-channel amplitude were dominant in Λ decay, the nonmesonic decay processes may be expected to predominate over the π^- modes by a factor of order 20 for ${}_\Lambda He^4$ and ${}_\Lambda He^5$. There are many uncertainties[1,5,18] in the details of their argument but, even so, it appears unlikely that a ratio Q as small as the observed value[19] of 1.5 could hold if p/s were much greater than 1. On this basis, the above discussion of the ${}_\Lambda H^4$ branching ratio (4.1) favors $J=0$ for the ${}_\Lambda H^4$ ground state, although $J=1$ cannot yet be finally excluded.

Since the Pauli suppression is much weaker in ${}_\Lambda H^3$

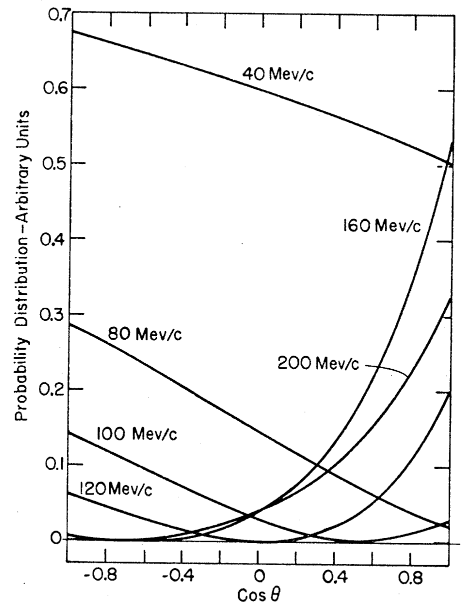

FIG. 2. The angular distribution of the π^- mesons emitted in ${}_\Lambda He^4 \to \pi^- + p + He^3$ decay (specified in the π^--p barycentric system relative to the direction of their total momentum) is plotted for various values of the He^3 recoil momentum P, for s-channel Λ decay. For p-channel Λ decay, the curves are to be multiplied by $(q/q_\Lambda)^2$, which is a function only of P. The effect of the Pauli principle on the final particles is included here, and the effect of their nuclear forces neglected, as discussed in the text.

[17] R. Karplus and M. Ruderman, Phys. Rev. **102**, 247 (1956).
[18] S. Treiman, *Proceedings of the 1958 International Conference on High-Energy Physics at* CERN, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958), p. 276.
[19] E. Silverstein, Suppl. Nuovo cimento **10**, 41 (1959); P. E. Schlein, Phys. Rev. Letters **2**, 220 (1959). Schlein's value $Q=1.5$ for ${}_\Lambda He$ includes only those nonmesonic modes which give **two** charged prongs. The true value corresponding to $\Lambda+p\to n+p$ processes could well be somewhat larger than this.

decay, its branching ratio for two-body decay is correspondingly more sensitive to p/s. Using the calculations of Leon[20] for $B_\Lambda=0.25$ Mev, the fraction of ${}_\Lambda H^3 \to \pi^- + He^3$ events among π^- events of ${}_\Lambda H^3$ decay are predicted as

$$\text{(a) } J=\tfrac{1}{2}: \quad (0.71s^2+0.10p^2)/(1.35s^2+0.915p^2),$$

$$\text{(b) } J=\tfrac{3}{2}: \quad 0.403p^2/(0.915s^2+1.14p^2). \tag{4.2}$$

The present empirical ratio 4/14 has statistics too poor for comparison with (4.2) to be useful; also, the B_Λ value for ${}_\Lambda H^3$ has a large uncertainty, which involves a correspondingly large uncertainty in the sticking probability since this is roughly proportional to $(B_\Lambda)^{\frac{1}{2}}$.

Observations on the π^0 and π^- decay rates for ${}_\Lambda He^4$ decay will allow some comparison of (s_0,p_0) with (s,p). If $J=0$ holds, the two-body mode ${}_\Lambda He^4 \to \pi^0 + He^4$ will have frequency $1.48s_0^2/(0.43s^2+0.35p^2)$ relative to the $\pi^- + p + He^3$ modes. If the $\Delta T=\frac{1}{2}$ rule holds for the Λ-decay interaction, this ratio has value about unity for $s\sim p$, so that this mode may then be expected to be rather prominent.[21] For $J=1$, the corresponding expression is $1.04p_0^2/(0.43s^2+0.35p^2)$. The total rate for all π^0 modes of ${}_\Lambda He^4$ decay, obtained from the π^0 expressions analogous to (3.17), also depends on the spin J of this hypernucleus. The π^0/π^- ratios obtained from the above calculations are as follows:

$$\begin{aligned} &\text{(a) } J=0; \quad R(\pi^0)/R(\pi^-)=(1.96s_0^2+0.35p_0^2)/(0.40s^2+0.32p^2),\\ &\text{(b) } J=1; \quad R(\pi^0)/R(\pi^-)=(0.43s_0^2+1.40p_0^2)/(0.40s^2+0.32p^2). \end{aligned} \tag{4.3}$$

Assuming again $s/s_0=p/p_0=-\sqrt{2}$, as the $\Delta T=\frac{1}{2}$ rule predicts, these ratios take quite large values for either spin J, their values lying between $\sim$1.3 and $\sim$2.2 for p/s ratios allowed by the asymmetry parameter for p/s decay and by the ${}_\Lambda H^4$ branching ratio. The largeness of these π^0/π^- ratios arises partly from the strong Pauli suppression of the π^- mode, and partly from the enhancement of the two-body π^0 mode by the Pauli principle and by the large release of kinetic energy in this mode. The processes of ${}_\Lambda He^4$ decay may now be studied, free from the confusion with ${}_\Lambda He^5$ decay events which is a source of difficulty in the emulsion work,[3] by observations in helium bubble chambers, following the reaction[4] $K^- + He^4 \to {}_\Lambda He^4 + \pi^-$; the preliminary evidence[4] does suggest a large π^0/π^- ratio in ${}_\Lambda He^4$ decay modes. Observations on these ${}_\Lambda He^4$ decay events will also allow, in due course, a determination of the total lifetime for ${}_\Lambda He^4$ and of the branching ratio for its π^- mode. From these, the partial lifetime of the π^- mode can then be obtained, for comparison with the estimate given in Sec. 3(c). From our present discussion. it appears that the total lifetime of ${}_\Lambda He^4$ will be somewhat shorter than the free Λ-decay lifetime τ_Λ, although its precise value will depend on the unknown rate for the nonmesonic modes which lead to at most one energetic outgoing charged particle. With $p\sim s$ and the $\Delta T=\frac{1}{2}$ rule, the pionic modes have a decay probability of about $0.6/\tau_\Lambda$ (depending a little on the value of J); the branching ratio 1.5 for the known nonmesonic modes[19] (relative to the π^- modes) adds a further $0.4/\tau_\Lambda$ to this, so that the ${}_\Lambda He^4$ decay probability is at least τ_Λ^{-1}.

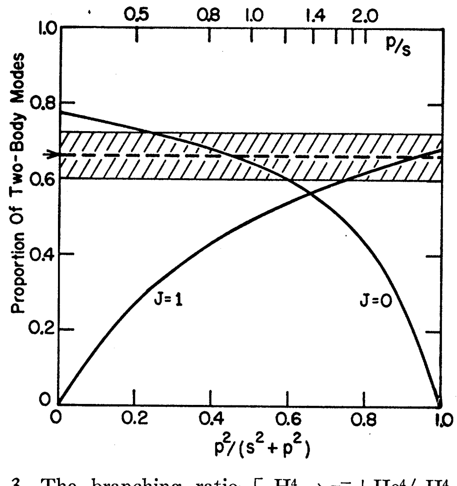

FIG. 3. The branching ratio $[{}_\Lambda H^4 \to \pi^- + He^4/ H^4 \to \pi^-$(all modes)$]$ is plotted as function of $p^2/(s^2+p^2)$ for the cases spin $J=0$ and $J=1$ for the ${}_\Lambda H^4$ hypernucleus, and compared with the current experimental ratio (shaded area) of 0.66 ± 0.06. We note that the up-down asymmetry in Λ decay limits $p^2/(s^2+p^2)$ to lie between 0.2 and 0.8.

For ${}_\Lambda H^4$ decay, the π^0/π^- ratio is modified in the opposite direction as a result of the Pauli principle. The values obtained here are

$$\begin{aligned} &\text{(a) } J=0: \quad R(\pi^0)/R(\pi^-)=(0.40s_0^2+0.32p_0^2)/(1.84s^2+0.35p^2),\\ &\text{(b) } J=1: \quad R(\pi^0)/R(\pi^-)=(0.40s_0^2+0.32p_0^2)/(0.43s^2+1.12p^2). \end{aligned} \tag{4.4}$$

The π^0/π^- ratio for ${}_\Lambda H^4$ is then predicted to be quite small, if the $\Delta T=\frac{1}{2}$ rule holds. With $J=0$, the ratio predicted runs from 0.12 to 0.20 for p/s between 0.45 and 1.5; with $J=1$, the ratio is between 0.17 and 0.21 as p/s runs from 2.2 to 1.2. With $p\sim s$, the pionic decay probability is $0.85/\tau_\Lambda$ for $J=0$, $0.65/\tau_\Lambda$ for $J=1$; the nonmesonic decay rate is not known for ${}_\Lambda H^4$. The observation of ${}_\Lambda H^4$ decay events following the reaction $K^- + He^4 \to {}_\Lambda H^4 + \pi^0$ may allow a determination of the total lifetime of ${}_\Lambda H^4$, as well as a determination of its nonmesonic/mesonic ratio.

For ${}_\Lambda He^5$ decay, the π^0/π^- ratio is $(s_0^2+0.85p_0^2)/(s^2+0.85p^2)$, a value $\sim$0.5 if the $\Delta T=\frac{1}{2}$ rule holds.

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[20] See reference 4. The ratios (4.2) are simply twice those of Leon, the correction being due to the inclusion of the additional factor $\sqrt{2}$ of Eq. (2.3) for the two-body matrix element. The same correction is necessary in the calculations of Picasso and Rosati for the two-body mode.

[21] One clear example of $He^4 \to \pi^0 + He^4$ has been established, in which a subsequent $\pi^0 \to \gamma + e^+ + e^-$ decay allowed a complete analysis of the event. See R. Levi-Setti and W. Slater, Phys. Rev. **111**, 1395 (1958).

Both pionic modes are strongly suppressed by the Pauli principle, the total pionic rate being $0.36/\tau_\Lambda$ if $s\sim p$. The known nonmesonic modes[19] bring the decay rate up to $0.7/\tau_\Lambda$, so it appears quite possible that the $_\Lambda He^5$ lifetime may be longer than that for the free Λ particle, even after allowance for other nonmesonic modes. Similarly, for $_\Lambda H^3$ decay, the $\Delta T=\frac{1}{2}$ rule predicts a π^0/π^- ratio of $\frac{1}{2}$. In this case, from the denominators of expressions (4.2), the total pionic decay probability is expected to lie within 15% of $1.1/\tau_\Lambda$ for all the various cases possible. To this decay rate must be added the unknown rate for nonmesonic modes of $_\Lambda H^3$ decay, a rate expected to be appreciably less than that for $_\Lambda He$ decay because of the lower B_Λ value for $_\Lambda H^3$. In general, the lifetime for $_\Lambda H^3$ decay may be expected to be somehat less (by a factor ~ 0.6) than that for free Λ decay.

The above discussions have tacitly assumed that exactly the same pionic decay interactions are effective for a Λ hyperon bound in nuclear matter as for a free Λ hyperon. Actually it is possible for other weak pionic interactions of the Λ particle to become effective in the presence of nucleons. For example, on the basis of the four-fermion weak interaction $(\bar{\Lambda}p)(\bar{p}n)$, a decay interaction $\Lambda\rightarrow n+\pi^++\pi^-$ may be generated through the sequence

$$\Lambda\rightarrow n+(p+\bar{p})\rightarrow n+\pi^++\pi^-. \tag{4.5}$$

Although this interaction is energetically forbidden for a free Λ hyperon, it can lead to additional pion emission from a Λ hyperon in the neighborhood of another nucleon which can absorb one of the pions. The processes

$$\Lambda+p\rightarrow\begin{cases} n+n+\pi^+, \\ p+p+\pi^-, \end{cases} \qquad \Lambda+n\rightarrow n+p+\pi^-, \tag{4.6}$$

can take place in consequence of the interaction (4.5). The former could give rise to the identifiable decay process $_\Lambda He^4\rightarrow\pi^++n+H^3$, of which no example is known (compared with ~ 20 identified $_\Lambda He^4\rightarrow\pi^-+p+He^3$ events and a similar number of these events where the recoil He^3 momentum was too low to allow unique identification). No attempt will be made here to estimate theoretically the importance of this stimulated pionic decay process for the π^- decay of Λ hypernuclei. Since the processes (4.6) require close approach (to a distance $\lesssim\hbar/m_\pi c$) between the Λ particle and nucleon, their relative importance in the decay of light hypernuclei will be much less than for the larger and more strongly bound hypernuclei $A\geqslant 7$. Even here, at most one example of π^+ emission has been reported, compared with about 50 examples of π^- emission from heavy hypernuclei; Schneps[22] has described an event which can best be interpreted as $_\Lambda Li^7\rightarrow n+\pi^++He^6$. In view of the prominence and greater identifiability for two-body modes of the heavier hypernuclei, it is interesting to remark that the lightest two-body decay possible for π^+ emission from a hypernucleus is the mode $_\Lambda Be^8\rightarrow\pi^++Li^8$; no example of this has yet been seen, although six examples of $_\Lambda Li^8\rightarrow\pi^-+Be^{8*}$ are known.[23] Owing to the rarity of π^+ emission from Λ hypernuclei, and because of the light Λ binding in the light hypernuclei, the possibility of the stimulation of π^- emission in Λ decay by the presence of neighboring nucleons has not been included in the considerations of this paper.

APPENDIX. $\Delta T=\frac{1}{2}$ RULE AND THE DECAY MODES OF $_\Lambda H^4$ AND $_\Lambda He^4$

It is of interest to discuss briefly some limitations on the decay rates for particular $_\Lambda H^4$ and $_\Lambda He^4$ modes which would follow from the $\Delta T=\frac{1}{2}$ selection rule for Λ decay, together with charge independence for the interactions in the initial and final systems. These limitations do not depend on the assumptions which have been made in the calculations above, the neglect of secondary pion interactions and of the additional Λ-decay interactions allowed by the presence of nucleons. With $T=\frac{1}{2}$ for the $(_\Lambda H^4,_\Lambda He^4)$ doublet, the $\Delta T=\frac{1}{2}$ Λ-decay interaction leads only to final states with $T=0$ and $T=1$. For $_\Lambda H^4$ decay, only final states $T=1$ can be reached, since $T_3=-1$ holds after the Λ decay; the amplitudes for these states will be denoted by β. For $_\Lambda He^4$ decay, both $T=0$ and $T=1$ states are reached, since $T_3=0$ finally. The amplitudes for these $T=1$ states are now $-\beta/\sqrt{2}$, following the $\Delta T=\frac{1}{2}$ rule[24]; the amplitude for the $T=0$ state will be denoted by α.

For the two-body modes $\pi+He^4$, $T=1$ holds for the final state and the ratio of the decay rates $(_\Lambda He^4\rightarrow\pi^0+He^4)/(_\Lambda H^4\rightarrow\pi^-+He^4)$ is predicted to be $\frac{1}{2}$. This prediction of course neglects the effect of the π^-, π^0 mass difference; as we have seen above [following Eq. (2.5)], the larger energy release in $_\Lambda He^4$ decay will produce a significant deviation from this prediction, especially if $J=1$ holds for this doublet. The ratio $\frac{1}{2}$ is also the prediction for the ratio between the decay rates for the modes $\pi+H^2+H^2$ of $_\Lambda He^4$ and $_\Lambda H^4$ decay.

For the three-body pionic modes $\pi+N_1+N_3$, where N_1 and N_3 denote the doublets (n,p) and (H^3,He^3), two $T=1$ final states can be formed, according as the N_1 and N_3 isotopic spins sum to 0 or 1; the amplitudes for these two states will be denoted by β_0 and β_1. There

[22] J. Schneps, Phys. Rev. **112**, 1335 (1958). Note that there is good reason to believe that $_\Lambda Be^7$ is not stable against nucleon emission.

[23] Ammar, Levi-Setti, Limentani, Schlein, Slater and Steinberg, Nuovo cimento (to be published).

[24] The derivation of this relation for the $(_\Lambda H^4,_\Lambda He^4)$ doublet follows exactly the discussion of the relation between K^0 and K^+ decay for the (K^0,K^+) doublet given by R. H. Dalitz, Proc. Phys. Soc. (London) **A69**, 527 (1956).

is only one final $T=0$ state, with amplitude α. The amplitudes for all of the ${}_\Lambda H^4$ and ${}_\Lambda He^4$ three-body modes may now be written down, as follows:

$$\begin{array}{lll} {}_\Lambda H^4 \to \pi^- pt, & \beta_0/\sqrt{2}+\beta_1/2; & \\ {}_\Lambda H^4 \to \pi^- n He^3, & -\beta_0/\sqrt{2}+\beta_1/2; & \\ {}_\Lambda H^4 \to \pi^0 nt, & \beta_1/\sqrt{2}; & \\ {}_\Lambda He^4 \to \pi^- p He^3, & \alpha/\sqrt{3}-\beta_1/2; & \text{(A1)} \\ {}_\Lambda He^4 \to \pi^0 n He^3, & -\alpha/6^{\frac{1}{2}}+\beta_0/2; & \\ {}_\Lambda He^4 \to \pi^0 pt, & -\alpha/6^{\frac{1}{2}}-\beta_0/2; & \\ {}_\Lambda He^4 \to \pi^+ nt, & \alpha/\sqrt{3}+\beta_1/2. & \end{array}$$

Since the π^+ mode of ${}_\Lambda He^4$ is known to be rare (no examples known, compared with about 40 examples of the π^- mode), we denote

$$\alpha/\sqrt{3}+\beta_1/2=\delta, \tag{A2}$$

where δ is small relative to β_0 and β_1. In fact, the present data indicate that $|\delta/\beta_1|=(\pi^+nt/\pi^-p\mathrm{He}^3)^{\frac{1}{2}}<0.15$.

With the approximation $\delta\equiv 0$, the ratios $r({}_\Lambda\mathrm{He}^4)=(\pi^0pt/\pi^0n\mathrm{He}^3)$ and $r({}_\Lambda\mathrm{H}^4)=(\pi^-n\mathrm{He}^3/\pi^-pt)$ are predicted to be equal. More generally, with small δ, we have

$$r({}_\Lambda\mathrm{He}^4)/r({}_\Lambda\mathrm{H}^4)\simeq 1-4\,\mathrm{Re}\left(\frac{\delta}{\beta_1-\beta_0/2}-\frac{\delta}{\beta_1+\beta_0/2}\right). \tag{A3}$$

Finally the (π^0/π^-) ratios for the three-body modes are

$$\frac{\pi^0}{\pi^-}({}_\Lambda\mathrm{He}^4)=\frac{(|\alpha|^2/3+|\beta_0|^2/2)}{|(\alpha/\sqrt{3}-\beta_1/2)|^2},$$

$$\frac{\pi^0}{\pi^-}({}_\Lambda\mathrm{H}^4)=\frac{\frac{1}{2}|\beta_1|^2}{(|\beta_0|^2+\frac{1}{2}|\beta_1|^2)}.$$

Substituting (A2), it follows that

$$\frac{\pi^0}{\pi^-}({}_\Lambda\mathrm{He}^4)\times\frac{\pi^0}{\pi^-}({}_\Lambda\mathrm{H}^4) \simeq\frac{1}{4}\left(1+\frac{2\,\mathrm{Re}(\beta_1^*\delta)}{|\beta_1|^2}\,\frac{2|\beta_0|^2-|\beta_1|^2}{2|\beta_0|^2+|\beta_1|^2}\right). \tag{A4}$$

This differs from $\frac{1}{4}$ by no more than 20%, with the above limit on δ/β_1. If the two-body modes are included in the (π^0/π^-) ratios, the result (A4) is modified only by the addition of a certain positive constant to both numerator and denominator of the last factor.

Finally, it may be noted that the partial lifetime for the pionic three-body modes for ${}_\Lambda\mathrm{He}^4$ decay is always less than twice that for the corresponding ${}_\Lambda\mathrm{H}^4$ modes, the precise value depending on the contribution from the final $T=0$ states for ${}_\Lambda\mathrm{He}^4$ decay. In fact this statement may be generalized to include the nonmesonic modes; if the $\Delta T=\frac{1}{2}$ rule holds for these decays, the total ${}_\Lambda\mathrm{H}^4$ lifetime is necessarily longer than one-half of the total lifetime for ${}_\Lambda\mathrm{He}^4$ decay.