a lifetime in the region 10^{-4} to 10^{-1} second, and mass greater then 60 electron masses, is one per 700 μ mesons stopping (i.e., $\leqslant 0.14\%$).

Of the 28 events which could be actual decays, no case was seen of a μ meson decay on the second trace. Such a decay would indicate the existence of a millisecond-lifetime particle which decays into a light meson. The intensity for this type event was thus found to be $\leqslant 0.03\%$.

The distribution of decay times is seen in Fig. 3. Here the double traces are grouped according to the type of stopping event. The distribution is consistent with a random decay time.

In the vicinity of each double trace a sample of single traces was recorded as to ρ, β, or f type; the ratio $\rho/\beta=1.1$ for these tracks was then compared with the same ratio for double events ($\rho/\beta=1.0$). The comparison is a satisfactory internal consistency check of the data and again indicates the double traces are random coincidences.

The results of this experiment are summarized in Table II.

ACKNOWLEDGMENT

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K^+-Nucleon Scattering in the Tamm-Dancoff Approximation

Grzegorz Białkowski and Andrzej Jurewicz
Institute for Theoretical Physics, Warsaw University, Warsaw, Poland
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Calculations of K^+-nucleon scattering phase shifts are reported. A simple model of Yukawa interactions between K mesons and baryons (containing no derivatives) is adopted. The method used is a three-dimensional Tamm-Dancoff approximation but with no simplifications as far as recoil effects are concerned. This method is particularly convenient for K^+-nucleon scattering, since the principle of associated production excludes graphs leading to nonrenormalizable effects.

The resulting integral equation has been solved for the $T=1$, $S_{\frac{1}{2}}$, $P_{\frac{1}{2}}$, and $P_{\frac{3}{2}}$ states in the energy region up to 1.26 Bev. The coupling constant is the only parameter to be determined from experimental data. This was chosen to fit the experimental value of the total cross section at about 350 Mev. It turned out that the best agreement with experiment was obtained with $(G^2+\mathcal{G}^2)/4\pi=12.5$. This value is much greater than the corresponding one adopted in previous papers.

SEVERAL theoretical papers concerning the scattering of K^+ mesons by nucleons have been published.[1] The results obtained are rather inconsistent with experimental data. The authors have not been able to get the isotropy of the angular distribution (for the K^+, p scattering) together with the constancy of the total cross section in the energy region up to 200 Mev.

This paper presents an attempt to calculate K^+-nucleon phase shifts, but with no simplifications as far as the magnitude of the coupling constant and recoil effects are concerned. We have adopted a simple model of Yukawa interactions between K^+ mesons and baryons. A d'Espagnat-Prentki[2] interaction Hamiltonian has been taken:

$$H'=G\int\bar{\psi}_\Lambda\gamma_5\psi_N\varphi^*_K dv+\mathcal{G}\int\bar{\psi}_\Sigma{}^\alpha\gamma_5\psi_N\tau_\alpha\varphi^*_K dv+\text{H.c.}$$

No attempt has been made to include interactions with pions, in particular three- or four-boson interactions. The method used is the three dimensional Tamm-Dancoff approximation, similar to the method adopted by Dyson *et al.*[3] to the scattering of pions by nucleons. This method is particularly convenient for K^+-nucleon scattering calculations since the principle of associated production excludes graphs leading to nonrenormalizable effects [graphs (a) and (b) in Dyson's paper]. The only divergences are connected with self-energy processes arising in the kernel of the integral equation [(α), (β), and (γ) in Dyson's paper]. These terms have been rejected with simultaneous change of all constants to renormalized ones, in agreement with Dyson's idea of noncovariant renormalization.[4]

Confining ourselves to processes involving 3 particles (1 hyperon and 2 mesons or 2 nucleons) in intermediate states, we get in the center-of-mass system a 3-dimensional integral equation for the scattering amplitude $f(\mathbf{p})$. The angular variables can then be separated exactly, a one-dimensional integral equation resulting.

[1] C. Ceolin and L. Taffara, Nuovo cimento **5**, 435 (1957); D. Amati and B. Vitale, Nuovo cimento **6**, 261 (1957); C. Ceolin and L. Taffara, Nuovo cimento **6**, 425 (1957); C. Ceolin and L. Taffara, Padova-Venice Conference (1957).

[2] B. d'Espagnat and J. Prentki, Nuclear Phys. **1**, 35 (1956).

[3] F. J. Dyson *et al.*, Phys. Rev. **95**, 1644 (1954).

[4] F. J. Dyson, Phys. Rev. **91**. 421 (1953).

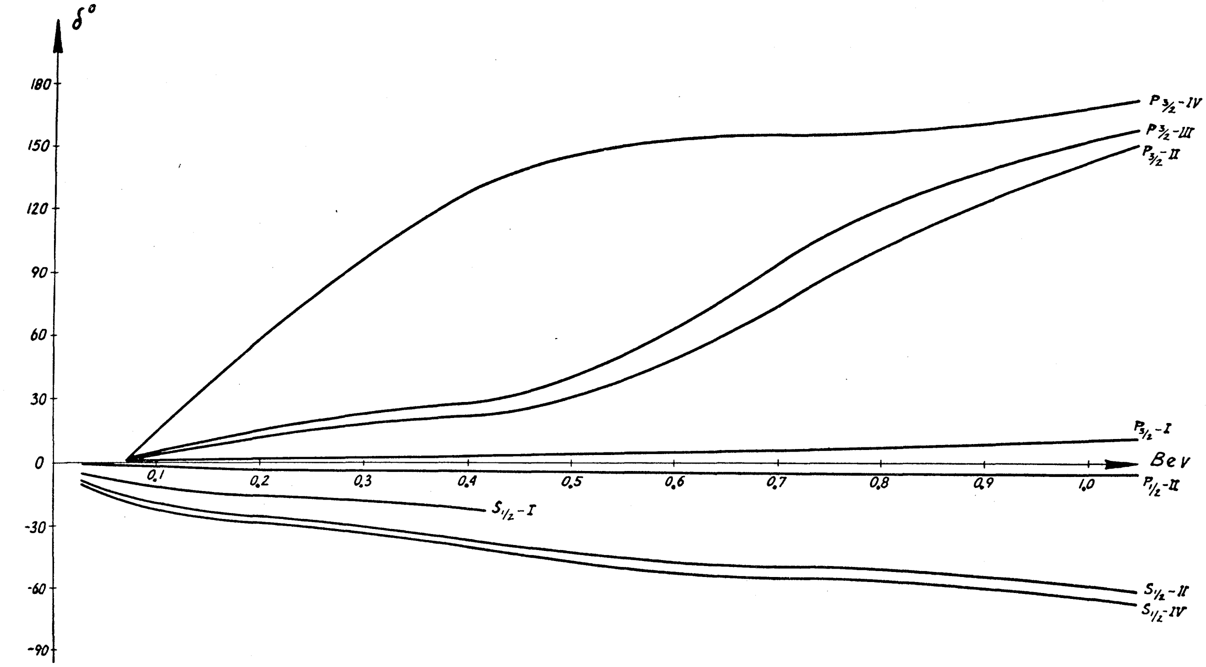

FIG. 1. Calculated $S_{\frac{1}{2}}$, $P_{\frac{1}{2}}$, and $P_{\frac{3}{2}}$ phase shifts for K^+ on p, against the kinetic energy of K in the lab system. I—$g^2=4$; II—$g^2=12.5$; III—$g^2=13.5$; IV—$g^2=17$.

Assuming the standing-wave boundary condition

$$f(p)=\delta(E-E(p)-\Omega(p))+P\frac{\varphi(p)}{E-E(p)-\Omega(p)},$$

(where P denotes the fact that the Cauchy principal value is to be taken), we get the following integral equation for the nonsingular amplitude $\varphi(p)$:

$$\varphi(p)=\varphi_0(p)+P\int_0^\infty dq[E-E(q)-\Omega(q)]^{-1}$$
$$\times[A_1(G^2/4\pi)L_\Lambda(p,q)+A_2(\mathcal{G}^2/4\pi)L_\Sigma(p,q)]\varphi(q);$$

$$L_\Lambda(p,q)=[E(p)E(q)\Omega(p)\Omega(q)]^{-\frac{1}{2}}$$
$$\times\Big\{[E(p)+m][(M-2m+E)N_\beta(b)]$$
$$+[E(p)+E(q)+\Omega(p)+\Omega(q)+M-2m-E]$$
$$\times N_\beta(d)+\frac{pq}{E(q)+m}\{(2m-M+E)N_\gamma(b)$$
$$+[E(p)+E(q)+\Omega(p)+\Omega(q)+2m-M-E]$$
$$\times N_\gamma(d)\}\Big\};$$

$$\varphi_0(p)=\Omega(p_0)E(p_0)(p_0E)^{-1}$$
$$\times[A_1(G^2/4\pi)L_\Lambda(p,p_0)+A_2(\mathcal{G}^2/4\pi)L_\Sigma(p,p_0)];$$

where m, $E(p)$; μ, $\Omega(p)$; M, $\mathcal{E}(p)$; $\mathfrak{M}$, $\epsilon(p)$ are the masses and relativistic energies of N, K, Λ, and Σ, respectively. E denotes the total energy of the system and $E(p_0)+\Omega(p_0)=E$, $b=E(p)+E(q)-E$, $d=\Omega(p)+\Omega(q)-E$. $A_1=-1$ and $A_2=3$ in the $T=0$ state; $A_1=+1$ and $A_2=1$ in the $T=1$ state. $\beta=0$, $\gamma=1$; $\beta=1$, $\gamma=0$; $\beta=1$, $\gamma=2$, in the $S_{\frac{1}{2}}$, $P_{\frac{1}{2}}$, and $P_{\frac{3}{2}}$ states, respectively.

$$N_0(b)=(2pq)^{-1}\ln\{[\mathcal{E}(p+q)+b]/[\mathcal{E}(p-q)+b]\};$$
$$N_1(b)=-(2pq)^{-1}(M^2+p^2+q^2-b^2)N_0(b)$$
$$+(2pq)^{-1}[1-2b[\mathcal{E}(p+q)+\mathcal{E}(p-q)]^{-1}];$$
$$N_2(b)=-3(4pq)^{-1}(M^2+p^2+q^2-b^2)N_1(b)$$
$$-2^{-1}N_0(b)+2b[\mathcal{E}(p+q)+\mathcal{E}(p-q)]^{-3};$$

$L_\Sigma(p,q)$ may be obtained from $L_\Lambda(p,q)$ by replacing M by $\mathfrak{M}$ everywhere.

The kernel of the integral equation is a singular one since it contains (a) the denominator $M(q)=E(q)+\Omega(q)-E$, and (b) logarithms in N functions. It should be noticed that the latter singularity appears for energies $E>M+2\mu$, which corresponds to the kinetic energy of K in laboratory system of about 1260 Mev. Thus, for energies up to 1260 Mev we have to deal with the singularity (a) only. Later, this singularity has been taken exactly into account.

Further, it should be mentioned that the kernel is quadratically integrable in the neighborhood of infinity.

The equation has been solved by seminumerical methods. If we represented the kernel of the equation in the form $K(p,q)=L(p,q)/M(q)$, then $L(p,q)$ would be approximated by a function $Q(x,y)=\alpha(x)\alpha(y)W(x,y)$ where $W(x,y)$ stands for a polynomial of degree 3 in both variables and α assures that $Q(x,y)$ behaves in the neighborhood of 0 and infinity as $L(p,q)$; $x=p/E(p)$, and $y=q/E(q)$. $\varphi_0(p)$ was approximated in a similar way.

The phase shift δ is connected to the scattering amplitude:

$$\varphi(p_0)=-(1/\pi)\tan\delta.$$

Up to now the equation has been solved for 8 energies in the $T=1$, $S_{\frac{1}{2}}$, $P_{\frac{1}{2}}$, and $P_{\frac{3}{2}}$ states. In this case we put $M=\mathfrak{M}$. The resulting phase shifts for any energy and state depend on the constant $g^2=(G^2+\mathcal{G}^2)/4\pi$. (See Fig. 1.) The latter was chosen to agree with experimental total cross section at about 350 Mev. It turned out that we get the best agreement with experiment for $g^2=12.5$. (See Fig. 2.) Notice that this value is much greater than the corresponding one adopted in previous papers.

It might be interesting to mention that by calculating phase shifts in the Born approximation [taking into account $\varphi_0(p)$ only] we have obtained at low energies good agreement with experiment with $g^2\sim4$.

For $g^2=12.5$ the results are as follows: The $S_{\frac{1}{2}}$ and $P_{\frac{1}{2}}$ phase shifts are negative, whereas the $P_{\frac{3}{2}}$ one is positive —thus the interaction is repulsive $(S_{\frac{1}{2}},P_{\frac{1}{2}})$ or attractive $(P_{\frac{3}{2}})$ respectively. In the low-energy region up to 200 Mev, the $S_{\frac{1}{2}}$ state strongly predominates. The $P_{\frac{3}{2}}$ resonance for $g^2=12.5$ occurs at about 750 Mev and its magnitude is about 40 mb. The position of the resonance strongly depends on the magnitude of the coupling constant. For energies above the resonance the values of the total cross section are slightly below the experimental data of Burrowes *et al.*[5] This fact can be explained by contributions from higher orbital states.

A more extensive paper on this subject, including results for the $T=0$ state, will be published later.

Note added in proof.—The foregoing results were obtained with the masses of Λ and Σ both numerically

[5] H. C. Burrowes *et al.*, Phys. Rev. Letters **2**, 119 (1959).

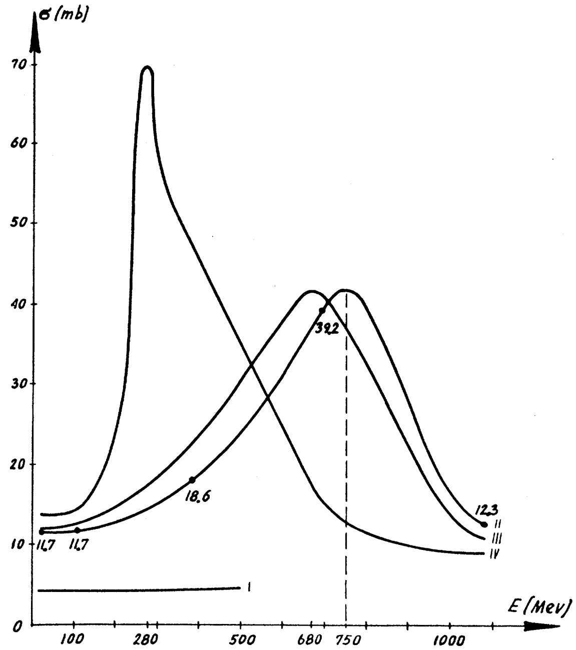

FIG. 2. Calculated total cross sections for K^+ on p.

equal to the mass of Λ. Since in this approximation charge exchange scattering vanishes with $G^2=\mathfrak{G}^2$ we have taken in our recent calculations $\mathfrak{M}=1.067M$ in agreement with experiment. With this $\mathfrak{M}$ value assumed the $S_{\frac{1}{2}}$ (K^+,p) phase shifts are but slightly smaller than previously, whereas the $P_{\frac{3}{2}}$ ones lower considerably. The resonance runs to higher energies (about 1 Bev).

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