Application of Sum Rules to Electron-Deuteron Scattering*

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A measurement of elastic and inelastic electron-deuteron scattering has been made at a momentum transfer of 206 Mev/c and an electron energy of 175 Mev for comparison with the Drell-Schwartz sum rules. The measured value of the non-energy-weighted sum rule is in good agreement with theory. The experimental result for the energy weighted sum rule is 30% larger than the value given for a pure Wigner potential and is consistent with a Rosenfeld two-body interaction if the analysis is restricted to central forces. The analysis is extended to include tensor forces for comparison with the prediction of the Gartenhaus potential. It is found that the gauge terms introduce major ambiguities when tensor forces are included.

I. INTRODUCTION

SUM rules are especially valuable in the analysis of inelastic electromagnetic interactions with nuclei because they can be used to extract information about the ground state of a nucleus without requiring knowledge about its excited states. Besides giving information about collective properties of a nuclear system, sum rules can also be used to probe some features of the nucleon-nucleon interaction that operates to bind the system. One such feature that may be investigated is the amount of exchange force in the interaction. Feenberg[1] and Siegert[2] first pointed out that the presence of charge exchange forces would modify the dipole sum rule for photonuclear reactions. The possible enhancement in dipole absorption from this effect has been calculated[3,4] and found to be large.

Recently Drell and Schwartz[5] investigated what could be learned from the construction of sum rules for the scattering of high-energy electrons by light nuclei. In particular they considered high-resolution studies since, in addition to permitting theoretical simplifications, high-resolution sum rules can utilize the precision possible in electron scattering experiments. They found that a sensitive measure of the amount of charge exchange force in an arbitrary two-body Hamiltonian for central forces is provided by a sum rule for the energy-weighted electrodisintegration cross section at a fixed three-momentum transfer. The sum rule is applicable for energies around 150 Mev and momentum transfers around 200 Mev/c. The energy-weighted cross section is

$$\sigma^E=\int_{|\mathbf{q}|=\text{constant}}(E_{\text{el}}-E)\frac{d\sigma}{dE}(\theta,E)\frac{dE}{\sigma_0(\theta,E_0)f^2}.$$

E_0 is the incoming electron energy, E the scattered energy, E_{el} the energy of elastic scattering for a momentum transfer $|\mathbf{q}|$, and $(d\sigma/dE)(E,\theta)$ is the measured cross section for electrons scattered through an angle θ, emerging with an energy E. The quantity f is the proton form factor[6] corresponding to a four-momentum transfer $(\Delta E^2-|\mathbf{q}|^2)^{\frac{1}{2}}$ and

$$\sigma_0(\theta,E_0)\equiv\frac{e^4}{4E_0^2}\frac{\cos^2(\theta/2)}{\sin^4(\theta/2)}\left(\frac{1}{1+[(E-E_0\cos\theta)/AM]}\right)$$

where M is the nucleon mass and A is the mass number of the target nucleus. The measured cross section is thus weighted with the excitation energy given to the struck nucleus, i.e., with the energy of each final nuclear state relative to its center of mass. The sum rule for σ_E calculated by Drell and Schwartz will be given specifically for electron-deuteron scattering since this is the process studied in the present experiment.

$$\begin{aligned}\sigma_E=\frac{q^2}{4M}\bigg\{1+\frac{\langle T\rangle}{M}+\tfrac{3}{4}(\mu_p-\mu_N)^2Y\frac{q^2}{2M^2}(1+\tfrac{1}{3}\langle e^{i\mathbf{q}\cdot\boldsymbol{\rho}}\rangle)\bigg\}\\ +2\langle(V_a+V_b)(1-e^{i\mathbf{q}\cdot\boldsymbol{\rho}})\rangle\\ +\frac{q^2}{4M}Y(\mu_p-\mu_N)^2\bigg\langle\frac{2V_a}{M}(3-e^{i\mathbf{q}\cdot\boldsymbol{\rho}})+\frac{4V_b}{M}(1-e^{i\mathbf{q}\cdot\boldsymbol{\rho}})\\ -\frac{2V_c}{M}(1+e^{i\mathbf{q}\cdot\boldsymbol{\rho}})\bigg\rangle,\end{aligned}$$

where the symbol $\langle\ \rangle$ denotes ground-state expectation value and $\langle T\rangle$ is the average kinetic energy in the ground state. The quantities μ_N and μ_p are the magnetic moments of the neutron and proton and

$$Y=\tfrac{1}{3}[2\sec^2(\theta/2)-1].$$

The terms V_a, V_b, V_c are defined by the potential V of a general two-body Hamiltonian in the following way: $V=V(r)+\tau_1\cdot\tau_2V_a(r)+\tau_1\cdot\tau_2\sigma_1\cdot\sigma_2V_b(r)+\sigma_1\cdot\sigma_2V_c(r)$. Tensor force terms are not included in this analysis. The

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[1] E. Feenberg, Phys. Rev. **49**, 328 (1936).
[2] A. J. F. Siegert, Phys. Rev. **52**, 787 (1937).
[3] J. S. Levinger and H. A. Bethe, Phys. Rev. **78**, 115 (1950).
[4] M. L. Rustgi and J. S. Levinger, Phys. Rev. **106**, 530 (1957).
[5] S. D. Drell and C. L. Schwartz, Phys. Rev. **112**, 568 (1958).

[6] f represents both the electric (F_{1p}) and magnetic (F_{2p}) form factors. These are equal within the accuracy of current measurements (see references 8 and 9). Also used here is the experimental result of R. Hofstadter and M. R. Yearian, Phys. Rev. **110**, 552 (1958), that $F_{2p}\simeq F_{2N}$.

quantity σ_E thus depends sensitively on the terms V_a, V_b, and V_c which can be related to exchange forces in the interaction. The purpose of this experiment is to get information about these potential terms by applying the above sum rule to the electrodisintegration of the deuteron.

One other high-resolution sum rule was constructed for electron scattering experiments. It is similar to the one above except that it does not weight the cross section by the excitation energy and thus includes the elastic scattering cross section. The calculated result for this is

$$\sigma_c=1+\frac{q^2}{4M^2}+\left[\frac{1}{3}\frac{\langle T\rangle}{M}+\frac{q^2}{4M^2}(\mu_p{}^2+\mu_N{}^2)+\frac{2}{3}\frac{q^2}{4M^2}\mu_p\mu_N\langle e^{iq\cdot\rho}\rangle\right]\times[2\sec^2(\theta/2)-1].$$

While this sum rule contains little information about nuclear properties, it does permit a direct experimental test of the formalism. This is certainly a requirement to be met before the more complicated sum rule for σ_E can be used to extract information from the data. Consequently, in this experiment the sum rule for σ_c is also tested.

Also considered here are the modifications introduced into the sum rule for σ_E by the inclusion of tensor forces.

II. EXPERIMENTAL METHOD

The experiment was carried out with a 175-Mev electron beam from the Stanford Mark III linear accelerator. The measurement was made in the end station with the use of the 36-in. double-focusing spectrometer to magnetically analyze the scattered electrons. The experimental apparatus used has been fully described on several occasions.[7,8]

A 0.217-inch CD_2 target of density 1.06 g/cm^3 was used for the measurement. The carbon background was determined by using a 0.151-inch carbon target having 1.46 the carbon content of the CD_2 target. The two targets were roughly matched both in thickness in radiation lengths and in average energy loss. The measured relative carbon content of the two targets was checked by comparing the counting rates of the carbon elastic peaks from both.

The latter procedure insured that the slight difference in multiple scattering in the two targets did not affect the results and that the difference in average energy loss was negligible.

The incoming beam had an energy spread of 1% and the momentum acceptance of the 36-in. spectrometer was set to 1%; the over-all energy resolution was about 1.5%. The electrons were detected with a Lucite Čerenkov counter 5.0 inches long with an input diameter of 2.75 inches and an output diameter of 3.75 inches viewed by a 5-in. photomultiplier. The counting equipment is the same as that described in references 7 and 8.

Scattered electrons having energies from 164 Mev to 78 Mev were detected in this experiment, and consequently it was necessary to make sure that the counter had a constant efficiency in this energy range. A decrease in efficiency for lower energies could result from electrons undergoing multiple scattering out of the Čerenkov counter and spreading the pulse-height distribution to a point substantially below the discriminator setting. This was checked by comparing the pulse-height distributions at the two limits of the energy range. No measurable difference (less than 1% of all detected electrons) was found in the number of true counts lost, indicating a constant counter efficiency.

Since the sum rule for σ_E was constructed for a constant momentum transfer, this constraint had to be maintained in measuring the spectrum of inelastically scattered electrons. The angle of scattering is related to the momentum transfer $|\mathbf{q}|$ by the expression

$$\cos\theta=(E_0{}^2+E^2-\mathbf{q}^2)/2E_0E,$$

which is plotted in Fig. 1. Thus for a constant beam energy E_0, $|\mathbf{q}|$ can be kept constant by varying the scattering angle θ as a function of the scattered energy E. It can be seen that $|\mathbf{q}|$ can also be kept constant by varying E_0 as a function of E. This would involve changing the accelerator energy for each measured point. Because of the normalization procedures used in this experiment the latter method was not thought to be as satisfactory. The former method relies on the fact that this sum rule is approximately independent of angle. The weighting factor $1/\sigma_0(E_0,\theta)$ which appears in the definition of σ_E takes out the major angular dependence. An angular dependence that is not removed by this factor comes from terms resulting from the inter-

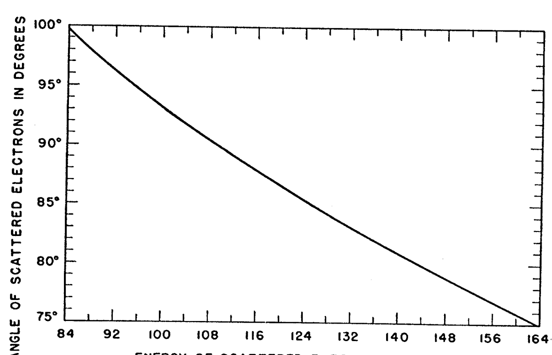

FIG. 1. The electron scattering angle as a function of the energy of the scattered electrons for a constant momentum transfer of 206 Mev/c. The energy of the incident electrons is 175 Mev.

[7] R. Hofstadter, Revs. Modern Phys. **28**, 214 (1956).
[8] E. E. Chambers and R. Hofstadter, Phys. Rev. **103**, 1454 (1956).

action of the electron with the nucleon magnetic moments. These bring in the characteristic $2\sec^2(\theta/2)-1$. However these terms are not the dominant ones for the range of angles of this measurement and only contribute a small angular dependence. In the numerical evaluation of σ_E for comparison with the measured value the mean value of $2\sec^2(\theta/2)-1$ is used. An evaluation of σ_E has been made (for the models considered in the analysis of the results) at the limits of the angular region 75° to 90°, which contains the major part of the energy weighted cross section. The values differ from the mean value by only about $\pm 5\%$.

The momentum transfer at which the measurement was made was 206 Mev/c. For an incoming energy of 175 Mev this momentum transfer corresponds to elastic scattering from the deuteron at 75°. The elastic scattering energy defines zero excitation energy at this $|\mathbf{q}|$. Because of this, the deuteron elastic peak was first measured in each run. The inelastic continuum was then measured, the spectrometer angle being varied for each successive point as a function of the excitation energy. Following the measurement of each point with the CD_2 target, a carbon background point was taken at the same spectrometer setting. These counting rates were normalized to measurements of the proton elastic peak at 90° made during each run with the use of a 0.217-inch CH_2 target. From the known electron-proton cross sections,[9] the above counting rates have been reduced to cross sections.

III. CORRECTIONS TO DATA

In Fig. 2 the experimental results for elastic and inelastic scattering are shown. Figure 3 gives the results for only inelastic scattering. Both of these curves include corrections for the dispersion of the spectrometer. The measured points shown represent roughly half the data used to evaluate σ_E and σ_c. In order to determine the inelastic scattering spectrum, the radiative tail of

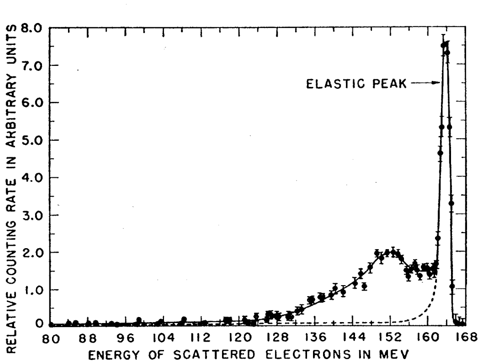

FIG. 2. The measured energy distribution of electrons scattered by the deuteron at an incoming energy of 175 Mev and a momentum transfer of 206 Mev/c. The dashed curve represents the calculated contribution to the inelastic spectrum from large-angle bremsstrahlung and from the radiative straggling of elastically scattered electrons.

[9] F. Bumiller and R. Hofstadter (to be published).

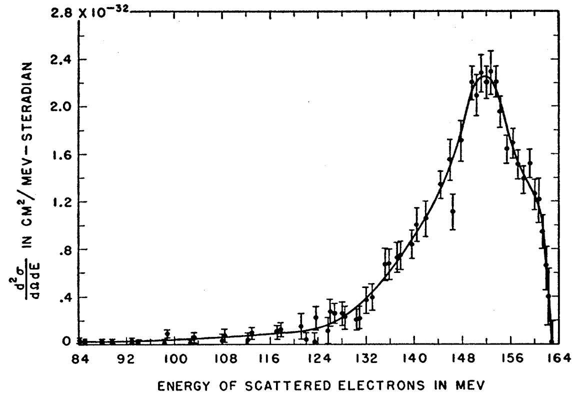

FIG. 3. The inelastic spectrum from electron-deuteron scattering at an incident energy of 175 Mev and a momentum transfer of 206 Mev/c. The data shown here have been corrected for the contributions from the radiative tail of the elastic peak and for the radiative broadening of the inelastic distribution.

the elastic peak had to be subtracted from the inelastic scattering data. This tail, shown as the dashed curve in Fig. 2, results from two processes. The first of these is the radiative degradation[10] of the electron energy in the target material before and after scattering. The second is photon emission during scattering. The differential correction for the radiative degradation is given in good approximation by

$$\frac{d^2\sigma}{d\Omega dE}=\tfrac{1}{2}b_1\left(\frac{E_{\rm el}-E}{E_{\rm el}}\right)^{b_1}\frac{1}{E_{\rm el}-E}\left[\frac{d\sigma}{d\Omega}(E_0)+\frac{d\sigma}{d\Omega}(E_1)\right],$$

where $b_1=t/\ln 2$, t=number of radiation lengths of the target; $E_{\rm el}$ and E are the elastic scattering energies resulting from incoming energies of E_0 and E_1 respectively; and $(d\sigma/d\Omega)(E_1)$ and $(d\sigma/d\Omega)(E_0)$ are the cross sections for electron-deuteron scattering at the energies E_1 and E_0. The correction resulting from photon emission during scattering is evaluated by using the Schiff calculation[11] of the differential cross section for large-angle bremsstrahlung. With a redefinition of the relevant energies in the expression in order to take into account the nuclear recoil energy, the correction is given by

$$\frac{d^2\sigma}{d\Omega dE}=\left[\frac{\alpha}{\pi}\left(1+\frac{E_1^2}{E_0^2}\right)\right]\left[\ln\frac{2E_0\sin(\theta/2)}{m}-\tfrac{1}{2}\right]\times\frac{1}{E_{\rm el}-E}\left[\frac{d\sigma}{d\Omega}(E_0)+\frac{d\sigma}{d\Omega}(E_1)\right].$$

Here α is the fine structure constant and m is the electron rest mass. In the energy region near the elastic peak the above corrections are folded into a peak shape representing the distribution of $E_{\rm el}$ corrected for radiation straggling and photon emission. Beyond a

[10] W. Heitler, *The Quantum Theory of Radiation* (Oxford University Press, London, 1954), third edition, p. 377.

[11] L. I. Schiff, Phys. Rev. 87, 750 (1952).

few peak widths this is not necessary and the center of the elastic peak defines E_{el}. Since the scattering angle was varied as a function of E in the measurement, the cross sections $d\sigma(E_1)/d\Omega$ and $d\sigma(E_0)/d\Omega$ are evaluated for a different angle at each point of the continuum. The cross sections used are taken from the results of McIntyre and Burleson.[12]

In order to fold out the broadening of the inelastic spectrum due to the aforementioned radiative effects, expressions similar to those shown above were used to predict the radiation tails resulting from each differential element of the spectrum. In this application the elastic sections in the expressions are replaced by inelastic cross sections with appropriate energy substitutions. In addition, because the inelastic cross sections have a weaker energy dependence, the approximation is made that photon emission before and after an inelastic interaction have roughly the same effect on the straggling. In carrying out the unfolding procedure the change in angle for each point was taken into account.

Broadening of the inelastic spectrum also results from the spread in the momentum acceptance of the spectrometer, the spread in acceptance angle of the spectrometer defined by the setting of the entrance slits, and the spread in energy of the incoming beam. The correction to σ_E from this effect has been calculated to be less than 1% and has thus been omitted.

The measurement of the inelastic spectrum was carried out up to an excitation energy of 86 Mev and could not be meaningfully extended, because beyond this point the CD_2 counting rate was statistically indistinguishable from that of the carbon background. It should be noted that beyond 70 Mev most of the CD_2-C counting rate results from the radiation tail of the spectrum at lower excitation energies. The contribution from the unmeasured region beyond 86 Mev can be estimated from the Jankus[13] calculation of the spectrum of electrons inelastically scattered from the deuteron and is found to give an addition of 4% to σ_E. However, since the calculation does not include the effects of exchange forces, this is probably too small a contribution. As a rough estimate, one can say that this part of the spectrum probably has a fractional enhancement comparable to that of the measured part. Thus the contribution given by the Jankus theory is multiplied by the factor by which the measured σ_E exceeds the value of σ_E calculated from only the nonexchange terms. The resulting increase in the experimental value of σ_E from the unmeasured part of the spectrum is then 5.2%. Though this estimate is rough it leads to a small uncertainty in the final result. A few words should be said in this context about the consistency of the Jankus calculation with the electron sum-rule theory. The Jankus formalism leads to a sum rule for σ_c nearly identical to that of Drell and Schwartz since there is only a very small contribution to σ_c from charge exchange forces. The one other difference between the two is that the Jankus sum rule is for constant scattering angle with the momentum transfer being approximated as constant across the spectrum for closure. These effects make the two differ by less than 3%. Since the measured value of σ_c is in agreement with either of these sum rules, this gives some confidence to the use of the Jankus theory as a basis for extrapolation.[14] The resulting percentage corrections to σ_E and σ_c from the above effects are given in Table I.

IV. RESULTS

The experimental value of the sum rule for σ_c is in good agreement with theory to within the accuracy of the measurement: the experimental value is $(1.39\pm0.060)\pm0.019$ as compared to a theoretical value of 1.34. The statistical error is ±0.060 (one standard deviation) and there is an additional uncertainty of ±0.019 resulting from the radiative corrections and the extrapolation of the continuum beyond an excitation energy of 86 Mev.

The measurement of the energy-weighted sum rule gives a value of 20.6 Mev. The statistical error is ±1.1 Mev and there is a ±1.1-Mev error resulting from the estimated uncertainties in the corrections to the data. Though the proton cross section at 90° has been used to normalize the counting rates, the quoted error of this cross section does not contribute to the errors in the sum-rule measurements in first order. The reason for this is that both sum rules are inversely weighted with the experimentally determined proton form factor causing the error from the proton cross section to cancel out.

V. DISCUSSION OF RESULTS

A. Analysis in Terms of a Central Potential

As a first step in the analysis of the results, a comparison can be made with the predicted value of σ_E for a pure Wigner potential. This corresponds to the omission of terms proportional to V_a, V_b, and V_c and gives a result of 15.7 Mev. The measured value is a factor 1.31 larger than this result. In the approximation of a central potential, exchange forces are required to increase the theoretical value; however, this conclusion will be modified when tensor forces are included in the analysis.

It is perhaps most useful to analyze this experiment in terms of specific models for the interaction. Drell

[12] J. A. McIntyre and G. R. Burleson, Phys. Rev. 112, 2077 (1958).

[13] V. Z. Jankus, Phys. Rev. 102, 1586 (1956).

[14] Because the theory of the sum rule for σ_E is applicable for only relatively small momentum transfers ($q\sim10^{13}$ cm^{-1}), the result is not sensitive to the structure of the nucleon potential at very small distances (<0.5 fermi). Aside from the presence of specific potential terms that contribute to σ_E, one would thus expect no major deviations from the Jankus result for the continuum at the momentum transfer of this experiment; the region of the potential that can be expected to give rise to deviations cannot be probed in this measurement.

and Schwartz have evaluated σ_E for three deuteron models. Model I is a Hulthén ground state with a Rosenfeld[15] two-body interaction of Yukawa spatial dependence:

$$V_{\rm I}=\tau_1\cdot\tau_2(0.1+0.23\sigma_1\cdot\sigma_2)(e^{-\mu r}/\mu r)V_0,$$

with $\mu^{-1}=1.4$ fermis (1 fermi $\equiv 10^{-13}$ cm) and $V_0=40$ Mev.

In Model II the Rosenfeld interaction is given a Gaussian shape:

$$0.7\exp(-0.46\mu^2r^2).$$

Model III consists of the central part of the Gartenhaus[16] potential for the deuteron. In Table II the values of σ_E calculated from these models are compared with the experimental result. The measured value is consistent with a Rosenfeld interaction but is too large to agree with the central part of the Gartenhaus potential. However, because the Gartenhaus tensor interaction is so large the effects of this term should be included. Consequently, the evaluation of σ_E for Model III made by Drell and Schwartz has been extended to include the tensor force.[17]

B. Inclusion of Tensor Force

The analysis with the inclusion of the tensor interaction is made ambiguous by the gauge current terms. These terms, which have been discussed extensively by Sachs,[18] arise from the presence of charge exchange forces and cannot be uniquely defined. They give about a 1% contribution to σ_E for Models II and III and about a 15% contribution to Model I which comes from the singularity at the origin; this contribution is discounted as coming from an unrealistic part of the potential. However, with the inclusion of the Gartenhaus tensor interaction the gauge terms increase σ_E to roughly a factor of three greater than its potential independent value, giving a result on the order of 2.5

TABLE I. Percentage corrections applied to the measured sum rules and the estimated uncertainties in the corrections.

	σ_E	σ_e
Radiative tail from elastic peak	$-10\ \pm1.5\%$	$+1.7\pm0.2\%$[a]
Radiation broadening of inelastic spectrum	$-\ 7\ \pm1.4\%$	$+5.5\pm1.0\%$
Extrapolation of continuum beyond an excitation energy of 86 Mev	$+\ 5.2\pm2.6\%$[b]	$+0.4\pm0.2\%$

[a] For excitation energies >86 Mev.
[b] This estimate of the uncertainty allows the correction to range between 0.4 and 2.0 times the value predicted by the Jankus theory.

TABLE II. Theoretical values of σ_E for various central n-p potentials. The experimental value of σ_E is 20.6 Mev with a statistical error of ±1.1 Mev (one standard deviation) and an additional uncertainty of ±1.1 Mev.

Potential	Calculated value of σ
Central Wigner	15.7 Mev
Rosenfeld interaction with a Yukawa shape	19.6 Mev
Rosenfeld interaction with a Gaussian shape	19.6 Mev
Central part of the Gartenhaus potential	14.4 Mev

times the experimental value. Drell[19] has pointed out that this large effect may indicate that tensor forces in a nuclear system interacting with the electromagnetic field may require additional gauge currents. For example, on the basis of a pseudoscalar meson theory the part of the nucleon-nucleon potential that arises from the one-meson exchange term is proportional to

$$\tau_1\cdot\tau_2\sigma_1\cdot\nabla\sigma_2\cdot\nabla(e^{-\mu r_{12}}/r_{12}).$$

According to the simplest gauge-invariant prescription, $\mathbf{p}\rightarrow\mathbf{p}-(e\mathbf{A}/c)$, there arise, in connection with this part of the potential, gauge terms of major importance which are specifically associated with the tensor interaction. These tensor-force gauge currents could possibly provide the large cancellations necessary for agreement with experiment.

Despite this ambiguity, it is of some interest to investigate the effect of the tensor force on the other terms of σ_E. In the evaluation for a central potential, the major contributions are made by the charge-charge and moment-moment terms (following the terminology of reference 5). These terms have been calculated for a Gartenhaus tensor interaction, using the Gartenhaus wave function. The formal results are given in the Appendix. The contribution to σ_E from the charge-charge term is 3.9 Mev and that from the moment-moment term is 7.5 Mev. The total result for the Gartenhaus central and tensor interactions, with the exclusion of the effects of currents, is thus 25.8 Mev. This gives some idea of the required sign and magnitude of the contribution from the current terms for agreement with experiment.

One other effect of the tensor interaction should be mentioned. For models consisting entirely of central forces, only exchange potentials can increase σ_E above its potential-independent value. However, as will be seen from the results in the Appendix, a non-$(\tau_1\cdot\tau_2)$ tensor interaction can make a significant contribution to σ_E, this contribution resulting from the moment-moment term. Thus a large measured σ_E can indicate, but not definitely establish, a large fraction of exchange forces even if gauge terms are excluded.

In conclusion, this experiment provides a measured value of σ_E 30% larger than that given for a pure

[15] L. Rosenfeld, *Nuclear Forces* (North-Holland Publishing Company, Amsterdam, 1948), p. 234.
[16] S. Gartenhaus, Phys. Rev. **100**, 900 (1955).
[17] The inclusion of tensor forces has little effect on the predicted value of σ_e because the Hamiltonian does not enter directly into the calculation.
[18] R. G. Sachs, Phys. Rev. **74**, 433 (1948).
[19] S. Drell (private communication).

Wigner potential. However, a better understanding of gauge currents is required before this result can be used to test any specific potential model which includes noncentral forces.

ACKNOWLEDGMENTS

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APPENDIX

The calculation of the contributions to σ_E from the tensor forces has been made using the standard representation of the deuteron wave function,

$$\psi_d=\left[u(r)+\frac{w}{\sqrt{8}}(r)S_{12}\right]\chi_1^m,$$

where

$$S_{12}=\frac{3(\sigma_1\cdot r)(\sigma_2\cdot r)}{r^2}-\sigma_1\cdot\sigma_2.$$

First the results for the potential $V(r)S_{12}$ will be given. The major contribution X comes from the moment-moment term and is

$$X=\frac{b^2}{4M^2}\frac{[\mu_p{}^2+\mu_N{}^2]}{3}\left\{-\frac{64}{\sqrt{8}}\int_0^\infty V(r)uwdr+8\int_0^\infty V(r)w^2dr\right\}+\frac{b^2}{4M^2}\frac{\mu_p\mu_N}{3}\left\{+20\int_0^\infty V(r)\frac{\sin qr}{qr}u^2dr\right.$$
$$\left.-\frac{64}{\sqrt{8}}\int_0^\infty V(r)\frac{\sin qr}{qr}uwdr+28\int_0^\infty V(r)\frac{\sin qr}{qr}w^2dr-12\int_0^\infty V(r)N(r)u^2dr-20\int_0^\infty V(r)N(r)w^2dr\right\},$$

where

$$N(r)=\frac{1}{(qr)^2}\left[\frac{\sin qr}{qr}-\cos qr\right],$$

$$b^2=q^2[2\sec^2(\theta/2)-1].$$

For a potential of the form $\tau_1\cdot\tau_2V(r)S_{12}$, large contributions come from both the charge-charge and moment-moment terms. The contribution from the charge-charge term is

$$4\left\{+(8)^{\frac{1}{2}}\int_0^\infty V(r)\left(1-\frac{\sin qr}{qr}\right)uwdr\right.$$
$$\left.-\int_0^\infty V(r)\left(1-\frac{\sin qr}{qr}\right)w^2dr\right\}.$$

The contribution from the moment-moment term is

$$+X+\frac{b^2}{4M^2}(K+\tfrac{1}{2})^2$$
$$\times\left\{+\frac{128}{\sqrt{8}}\int_0^\infty V(r)uwdr-16\int_0^\infty V(r)w^2dr\right.$$
$$-\frac{32}{3}\int_0^\infty V(r)\frac{\sin qr}{qr}u^2dr+\frac{256}{3\sqrt{8}}\int_0^\infty V(r)\frac{\sin qr}{qr}uwdr$$
$$-\frac{64}{3}\int_0^\infty V(r)\frac{\sin qr}{qr}w^2dr+32\int_0^\infty V(r)N(r)u^2dr$$
$$\left.-\frac{128}{\sqrt{8}}\int_0^\infty V(r)N(r)uwdr+48\int_0^\infty V(r)N(r)w^2dr\right\},$$

where K is the nucleon anomalous magnetic moment.