Multiple Meson Production in Nucleon-Nucleon Interactions at Energies of 10^{12} ev*

MARCEL SCHEIN, D. M. HASKIN, E. LOHRMANN, AND M. W. TEUCHER
Department of Physics, University of Chicago, Chicago, Illinois
(Received July 13, 1959)

A nuclear interaction of type $0+20p$ observed in nuclear emulsion was analyzed by measuring the energies and angles of the secondary particles. The primary energy, as determined from the angular distribution of the tracks, is 2.7×10^{12} ev. This value is in agreement with an independent estimate obtained from the total energy dissipated. A secondary collision of type $0+20n$ was also analyzed in the same way. Its energy is 1.4×10^{12} ev, which is comparable to the primary energy. The inelasticity of the primary event is $0.54_{-0.19}^{+0.16}$.

The energy and angular distributions of the shower particles in the center-of-mass system (c.m. system) are given for both events. The shower particles show a correlation in the sense that those with the highest energies are emitted in the c.m. system under small angles with the shower axis.

The energy distribution of the mesons in the c.m. system is peaked toward low energies and shows a remarkably long tail at high energies extending up to 10 Bev. One of these particles is a π^0 meson, which carries off about 23% of the total energy.

The average value of the transverse momentum of the shower particles is 0.3 ± 0.05 Bev/c.

INTRODUCTION

A NUMBER of investigations have been carried out in order to obtain some of the experimental facts about multiple particle production in high-energy nuclear interactions ($E\sim10^{12}$ ev).[1–37] The most useful interactions to investigate are those representing the fundamental process in which a nucleon collides with one single unbound nucleon. Present experimental techniques do not allow us to observe this process unambiguously, since no criteria used in selecting such events from large samples of nuclear interactions observed in photographic emulsion are sufficient. Fortunately, a number of quantities can be observed, which are not very sensitive to this selection.

There are several[38–44] theoretical approaches to the problem of multiple particle production as well as phenomenological models,[36,45] but until now, none of them succeeded in explaining all of the experimental

* Supported by a joint program of the U. S. Atomic Energy Commission and the Office of Naval Research, and by the National Science Foundation.

[1] Lord, Fainberg, and Schein, Phys. Rev. **80**, 970 (1950); Nuovo cimento **7**, 774 (1950).
[2] Kaplon, Peters, and Bradt, Phys. Rev. **76**, 1735 (1949).
[3] Bradt, Kaplon, and Peters, Helv. Phys. Acta **23**, 24 (1950).
[4] Camerini, Fowler, Lock, and Muirhead, Phil. Mag. **41**, 413 (1950).
[5] E. Pickup and L. Voyvodic, Phys. Rev. **82**, 265 (1951).
[6] Daniel, Davies, Mulvey, and Perkins, Phil. Mag. **43**, 753 (1952).
[7] M. Teucher, Naturwissenschaften **39**, 68 (1952).
[8] Lal, Yash Pal, Peters, and Swami, Phys. Rev. **87**, 545 (1952).
[9] Kaplon, Peters, and Ritson, Phys. Rev. **85**, 900 (1952); **88**, 386 (1952); **86**, 932 (1952).
[10] K. Gottstein and M. W. Teucher, Z. Naturforsch. **8a**, 120 (1953).
[11] T. F. Hoang, J. phys. radium **14**, 395 (1953); **15**, 337 (1954).
[12] Dilworth, Goldsack, Hoang, and Scarsi, Nuovo cimento **10**, 1261 (1953).
[13] Castagnoli, Cortini, Franzinetti, Manfredini, and Moreno, Nuovo cimento **10**, 1539 (1953).
[14] R. G. Glasser and M. Schein, Phys. Rev. **90**, 218 (1953).
[15] Hopper, Biswas, and Darby, Phys. Rev. **84**, 457 (1951).
[16] Engler, Haber-Schaim, and Winkler, Nuovo cimento **12**, 930 (1954).
[17] Glasser, Haskin, Schein, and Lord, Phys. Rev. **99**, 1555 (1955).
[18] L. V. Lindern, Z. Naturforsch. **11a**, 340 (1956).
[19] E. Lohrmann, Z. Naturforsch. **11a**, 561 (1956).
[20] Hänni, Lang, Teucher, Winzeler, and Lohrmann, Nuovo cimento **4**, 1473 (1956).
[21] E. Lohrmann, Nuovo cimento **5**, 1074 (1957), contains also more references.
[22] Edwards, Losty, Perkins, Pinkau, and Reynolds, Phil. Mag. **3**, 327 (1958).
[23] Brisbout, Dahanayake, Engler, Fujimoto, and Perkins, Phil. Mag. **1**, 605 (1956).
[24] Debenedetti, Garelli, Tallone, and Vigone, Nuovo cimento **4**, 1142 (1956).
[25] Teucher, Haskin, and Schein, Phys. Rev. **111**, 1384 (1958).
[26] E. Lohrmann and M. W. Teucher, Phys. Rev. **112**, 587 (1958).
[27] M. Schein and D. M. Haskin, *Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics, 1957* (Interscience Publishers, New York, 1957).
[28] M. Schein and D. M. Haskin, Suppl. Nuovo cimento **8**, 710 (1958).
[29] Minakawa, Nishimura, Tsuzuki, Yamanouchi, Aizu, Hasegawa, Ishii, Tokunaga, Fujimoto, Hasegawa, Nishimura, Niu, Nishikawa, Imaeda, and Kazuno, Suppl. Nuovo cimento **11**, 125 (1959).
[30] Fenyves, Gombosi, and Suranyi, *1958 Annual International Conference on High-Energy Physics at CERN*, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958).
[31] Zhadanov, Zamchalova, Tretjakova, and Scerbakova Zhur. éksptl. i teoret. Fiz. **34**, 843 (1958) [translation: Soviet Phys. JETP **34(7)**, 582 (1958)].
[32] Gramenitzkij, Zhdanov, Zamcalova, Tretjakova, and Scerbakova, Suppl. Nuovo cimento **8**, 714 (1958).
[33] Zdanov, Zamcalova, Tretjakova, and Scerbakova, Suppl. Nuovo cimento **8**, 726 (1958).
[34] Boos, Vinitskii, Takibaev, and Chasnikov, J. Expt. Theoret. Phys. U.S.S.R. **34**, 622 (1958) [translation Soviet Phys. JETP **34(7)**, 430 (1958)].
[35] Ciok, Danysz, Gierula, Jurak, Miesowicz, Pernegr, Vrana, and Wolter, Nuovo cimento **6**, 1409 (1957).
[36] Ciok, Coghen, Gierula, Holynski, Jurak, Miesowicz, Saniewska, Stanisz, and Pernegr, Nuovo cimento **8**, 166 (1958).
[37] Teucher, Lohrmann, Haskin, and Schein, Phys. Rev. Letters **2**, 313 (1959).
[38] W. Heisenberg, Z. Physik **126**, 569 (1949); **133**, 65 (1952).
[39] E. Fermi, Progr. Theoret. Phys. (Kyoto) **5**, 570 (1950).
[40] E. Fermi, Phys. Rev. **81**, 683 (1951).
[41] S. S. Belenki and L. D. Landau, Suppl. Nuovo cimento **3**, 15 (1956).
[42] Lewis, Oppenheimer, and Wouthuysen, Phys. Rev. **73**, 127 (1948).
[43] S. Takagi, Progr. Theoret. Phys. (Kyoto) **7**, 123 (1952).
[44] W. L. Kraushaar and L. J. Marks, Phys. Rev. **93**, 326 (1954).
[45] G. Cocconi, Phys. Rev. **111**, 1699 (1958).

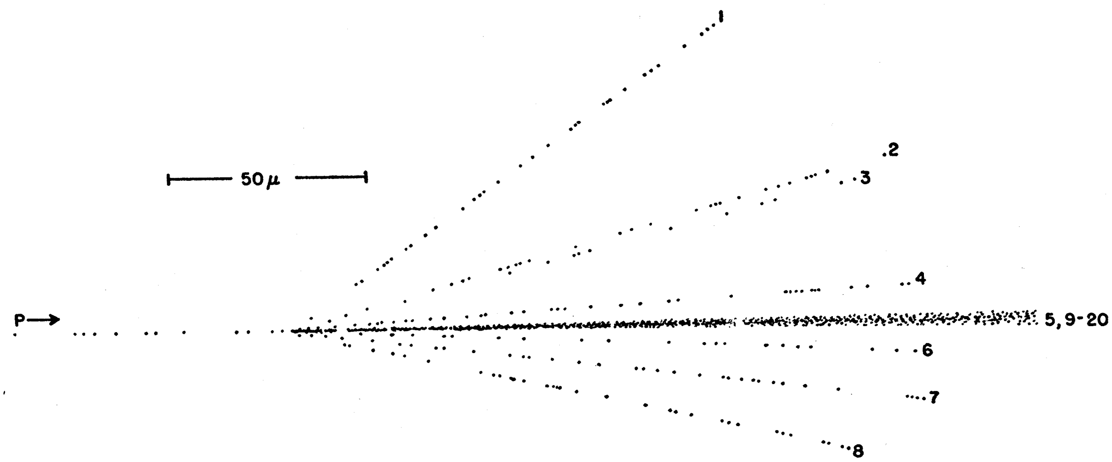

FIG. 1. Projection drawing of the 0+20*p* event.

facts, for instance, the energy and angular distribution of the produced particles, the mass spectrum, the multiplicity, and the elasticity.

In order to obtain more experimental information about some of the topics just mentioned, it seemed desirable to analyze two high-energy nuclear interactions in great detail because they seem to come closest to the fundamental process of a nucleon-nucleon collision, and offer favorable possibilities for carrying out reliable measurements. A brief account of some of the results has already been published.[25,27,28]

1. EXPERIMENTAL PROCEDURES

A stack of nuclear emulsion consisting of 100 Ilford G-5 pellicles, 30×15 cm, 600 μ thick, was exposed to the cosmic radiation on a Skyhook balloon flight over Texas. The flight remained at an altitude of 104 000 feet for 8 hours.

The stack was scanned for energetic electron-photon cascades which were traced backwards. One of these was found to be originated by a high-energy nuclear interaction of type 0+20*p* (Fig. 1). The incident proton of this event enters the top of the stack with a zenith angle of 2.5° and a dip angle of 1.2° with respect to the plane of the emulsion. It traverses 14 cm of emulsion before interacting. The tracks in the central cone of the shower remain for more than 2 cm inside this particular pellicle. The central cone can be followed for 16 cm inside the stack.

All of the 20 tracks of the event were followed until they produced a secondary interaction or left the stack. In addition, the total volume of a cone (opening angle approximately 10^{-2} radian) around the extrapolated line of flight of the primary proton was carefully scanned for neutral secondary interactions. By this method an event of type 0+20*n* (Fig. 2) was found at a distance of 5.7 cm from the primary event.

The neutral interaction is located within the uncertainty of measurement on the extrapolated line of flight of the primary proton. The axis of the meson shower of the secondary event is parallel to this line of flight. Figure 3 shows a projection drawing of the primary interaction and all observed secondary collisions. Figures 4(a) and 4(b) show the angular distributions of

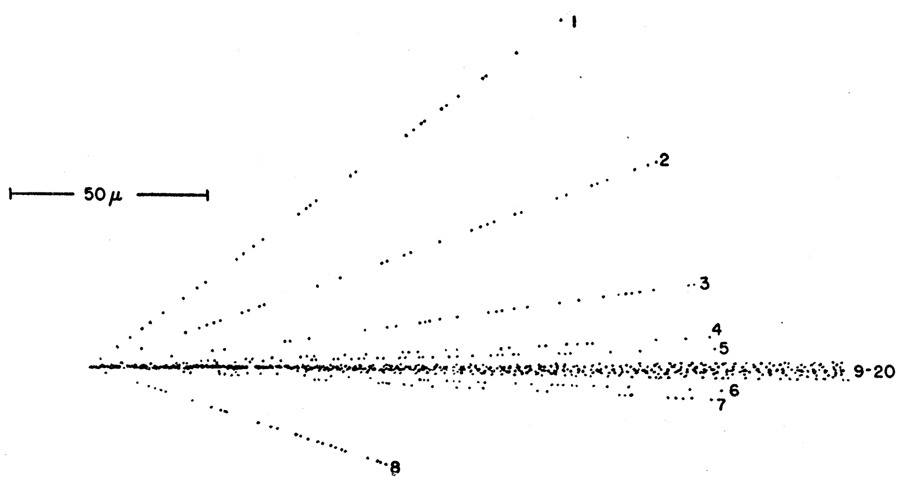

FIG. 2. Projection drawing of the 0+20*n* event.

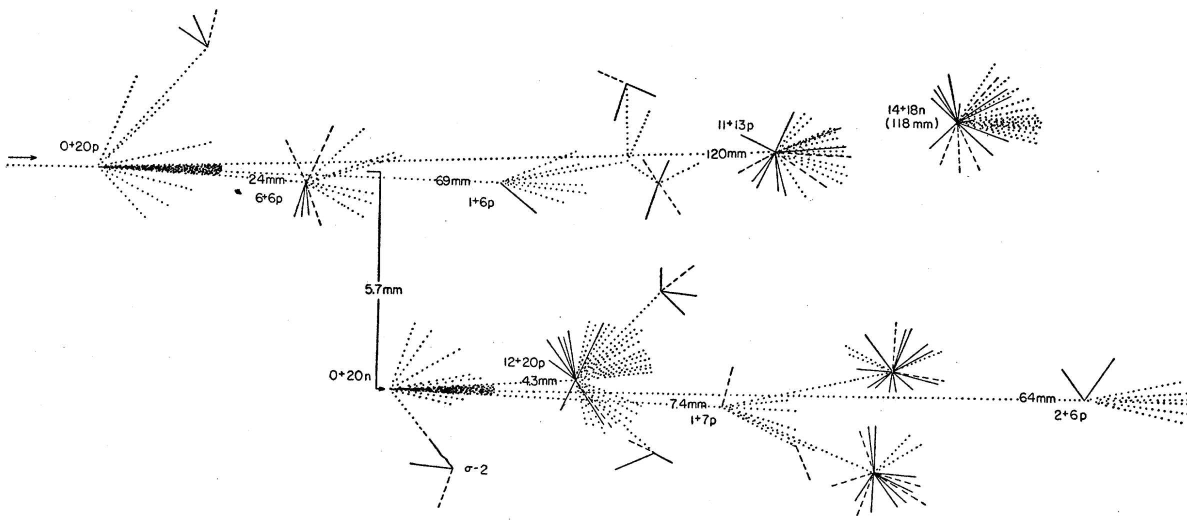

FIG. 3. Schematic drawing of the primary event and its secondary interactions.

the 0+20p and the 0+20n event in a diagram which will be discussed in Sec. 3.

Due to the favorable geometry of both events, rather extensive multiple scattering measurements were possible. The energy of the highly collimated tracks in the central cones were obtained by track to track scattering measurements[1] whereas the low-energy tracks in the outer cone were measured by the usual methods of multiple scattering.[46] The experimental results for all tracks are given in Tables I and II. In several cases, only lower limits could be established. This is due either to insufficient track length, to too high an energy of the particle, or to spurious scattering. In addition to the scattering data, Tables I and II also contain some information on the type of the secondary interactions. The energy of these interactions has been estimated using the Castagnoli formula, which will be discussed later in detail.

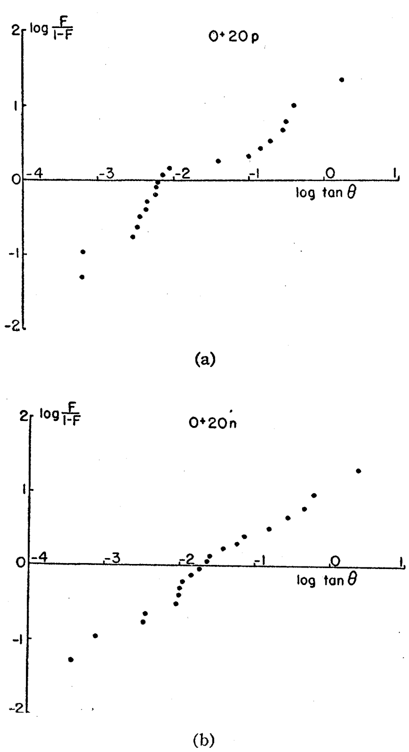

FIG. 4. (a) Angular distribution of the 0+20p event. (b) Angular distribution of the 0+20n event.

In case of track 10 in the 0+20n event, this method leads to a result which is obviously impossible. The energy thus obtained would not be sufficient to produce the rest masses of all the observed charged mesons plus the neutral mesons whose number is assumed to be one half of the number of the charged mesons. Therefore, we estimated the energy of this particle such that it could just produce the observed number of mesons.

In addition, it is possible to obtain some information about the π^0 mesons produced in the interactions. A scan of the forward cone of the primary event 0+20p up to 0.5 cascade units (1 C.U.=2.9 cm in nuclear emulsion) yielded four high-energy electron pairs which are probably due to γ rays from the decay of high-energy π^0 mesons. Three of these electron pairs originate rather energetic cascades in the stack. The lateral distribution of the electrons in these three cascades was measured and plotted at distances of 2.4 and 4 C.U. from the origin of the pairs. Using Pinkau's method[47] based on computations of Nishimura and Kamata,[48] we estimated the energies of the three γ rays. In addition, the Cudakov effect[49,50,28] was measured on those electron

[46] P. H. Fowler, Phil. Mag. **41**, 169 (1950).
[47] K. Pinkau, Phil. Mag. **2**, 1389 (1957).
[48] J. Nishimura and K. Kamata, Progr. Theoret. Phys. (Kyoto) **7**, 185 (1952); **5**, 889 (1950).
[49] D. H. Perkins, Phil. Mag. **46**, 1146 (1955).
[50] A. E. Cudakov, Compt. rend. acad. sci. U.R.S.S. **19**, 651 (1955).

pairs. Taking into account the kinematics of the π^0 decay, one can try to match the γ rays. This leads to two π^0 mesons of 700 Bev and 200 Bev, respectively [see Table I(a)]. The energies of all other π^0 mesons must be small compared with these two because they do not originate comparably energetic cascades. Their contribution to the measured electron densities at 2.4 and 4 C.U. is negligible.

We were not able to find any electron pairs of very high energy originating from the secondary neutral event $0+20n$. Low-energy pairs were observed but they do not produce dense cascades and, therefore, cannot be distinguished from the large cascades originating from the primary event.

2. TRANSVERSE MOMENTUM

The angles and energies of shower particles are correlated in such a way as to make the transverse

TABLE I. Angles, energies, and transverse momenta p_t of the secondaries of event $0+20_p$.

Track No.	θ	$p\beta$ (Bev/c)	p_t (Bev/c)	$p\beta$ determined from
1	49.5°	0.53 ± 0.10	0.41	scattering
2	17.5°	1.0 ± 0.3	0.30	scattering
3	16.2°	1.1 ± 0.3	0.31	scattering
4	6.5°	$3.8_{-0.7}^{+2.0}$	0.43	scattering
5	5.7°	1.8 ± 0.5	0.18	scattering
6	2.3°	$4.9_{-0.9}^{+4.0}$	0.20	scattering
7	11.5°	0.52 ± 0.12	0.11	scattering
8	21.8°	0.30 ± 0.07	0.12	scattering
9	9.3×10^{-3} rad	>7	>0.065	scattering
10	3.7×10^{-3} rad	40 ± 20	0.15	scattering, interaction $11+13_p$
11	3.4×10^{-3} rad	>40	>0.14	scattering
12	5.9×10^{-3} rad	23 ± 7	0.13	scattering
13	5.8×10^{-3} rad	>65	>0.38	scattering
14	6.9×10^{-3} rad	>65	>0.45	scattering
15	0.63×10^{-3} rad	>70	>0.045	scattering
16	0.63×10^{-3} rad	>70	>0.045	scattering
17	3.0×10^{-3} rad	140_{-70}^{+200}	0.42	interaction $1+6_p$
18	4.5×10^{-3} rad	≥35	≥0.16	scattering
19	4.4×10^{-3} rad	$\gtrsim40$	$\gtrsim0.18$	scattering
20	6.1×10^{-3} rad	25 ± 15	0.15	scattering, interaction $6+6_p$

(a) Data of two high-energy π^0 mesons from event $0+20_p$

Particle No.	θ (rad)	E (Bev)	p_t (Bev/c)
π^0 1	0.9×10^{-3}	700	0.63
π^0 2	2.7×10^{-3}	200	0.54

momentum P_t approximately constant. This correlation, for example, can be seen quite easily in Figs. 6 and 7. This is to be expected from the theories of Heisenberg[28] and Landau.[41] The value of P_t is not changed by a relativistic transformation into another frame of reference, for instance by the transformations carried out in Sec. 4. It can, therefore, be discussed in a meaningful way without further assumptions regarding the primary energy.

The importance of the transverse momentum for the discussion of high-energy nuclear interactions was first pointed out by Nishimura[51] and by Milechin and Rosental.[52]

[51] Z. Koba, *Proceedings of the Sixth Annual Rochester Conference On High-Energy Nuclear Physics, 1956* (Interscience Publishers, New York, 1956), Vol. IV, p. 46.

[52] G. A. Milechin and I. L. Rosental, J. Exptl. Theoret. Phys. U.S.S.R. **33**, 197 (1957) [translation: Soviet Phys. JETP **33**(6), 154 (1957)].

TABLE II. Angles, energies, and transverse momenta p_t of the secondaries of event $0+20_n$.

Track No.	θ	$p\beta$ (Bev/c)	p_t (Bev/c)	$p\beta$ determined from
1	32°	0.66 ± 0.20	0.36	scattering
2	25°	0.55 ± 0.15	0.23	scattering
3	15°	0.23 ± 0.05	0.07	scattering
4	8.9°	>0.76	>0.12	scattering
5	3.2°	2.3 ± 1.0	0.13	scattering
6	2.3°	>5.8	>0.23	scattering
7	4.0°	$3.6_{-0.7}^{+3.0}$	0.25	scattering
8	51°	0.072 ± 0.002	0.09	range, σ star
9	10.9×10^{-3} rad	>4.2	>0.05	scattering
10	9.9×10^{-3} rad	50_{-20}^{+50}	0.50	interaction $12+20_p$
11	24.2×10^{-3} rad	4.2_{-2}^{+4}	0.10	scattering
12	3.5×10^{-3} rad	40_{-20}^{+40}	0.14	scattering
13	0.0×10^{-3} rad	250_{-170}^{+500}	···	interaction $2+6_p$
14	0.78×10^{-3} rad	60_{-30}^{+60}	0.05	scattering
15	3.4×10^{-3} rad	50_{-25}^{+50}	0.17	scattering
16	9.2×10^{-3} rad	>20	0.18	scattering
17	9.9×10^{-3} rad	20_{-10}^{+20}	0.20	scattering
18	14.3×10^{-3} rad	50_{-25}^{+50}	0.72	interaction $1+7_p$
19	18.3×10^{-3} rad	>15	0.27	scattering
20	25.5×10^{-3} rad	2.7 ± 0.5	0.07	scattering

On the two events, $0+20p$ and $0+20n$, the transverse momenta of the charged secondary particles can directly be determined. The transverse momentum is calculated from

$$P_t = P' \sin\Theta',$$

P' is the momentum, Θ' the angle of the secondary particle in the laboratory system as explained in Sec. 1.

The values of P_t for all the secondary particles of the two events are given in Tables I and II. For some cases, only a lower limit for P_t could be established. It will now be discussed, how this affects our conclusions regarding the average value of P_t.

Table III shows the directly measured average values of P_t ($\langle P_t\rangle$) for both events in forward cone and backward cone separately.

For the $0+20p$ event, $\langle P_t\rangle$ in the forward cone seems slightly lower than in the backward cone. This could be due to the fact that most of the energies of the backward cone tracks could be actually measured, whereas in the forward cone, for 7 out of the 11 tracks only a lower limit could be given. Assuming P_t to be the same in both forward and backward cones, the value of $\langle P_t\rangle$ in the forward cone cannot be increased by much more than about a factor of 1.2.

Another upper limit on $\langle P_t\rangle$ comes from arguments of energy balance, if the two events are accepted to be examples of a nucleon-nucleon collision. This will be discussed more completely in Secs. 4 and 5.

In this case, the primary energy of the event $0+20p$ can be determined by the angular distribution of the shower particles and by assuming symmetry in the c.m. system. A lower limit for the primary energy can be obtained by adding the energies of all secondary

TABLE III. Average transverse momentum $\langle P_t\rangle$.

Interaction	$\langle P_t\rangle$ forward cone	$\langle P_t\rangle$ backward cone
$0+20p$	≥0.20 Bev/c	0.24 Bev/c
$0+20n$	≥0.28 Bev/c	≥0.17 Bev/c

particles. This sum includes the energy of the 0+20n event (calculated from its angular distribution), the energy of the soft cascade, and the energy of the charged secondary particles. The lower limits obtained by the scattering measurements were taken as the true values of the energies. This lower limit of the primary energy agrees well with the value deduced from the angular distribution, which in turn, indicates, that the values of the energies of the shower particles cannot be raised a great deal above the lower limits given. Otherwise the energy contained in the secondary particles would be higher than the primary energy. However, increasing the energies in the forward cone by a factor of 1.2, as mentioned above, would leave our conclusions unchanged because of the various experimental errors. We conclude therefore, that the most probable value of $\langle P_t \rangle$ for the event 0+20p is about 0.24 Bev/c.

A similar argument can be used for the 0+20n event.

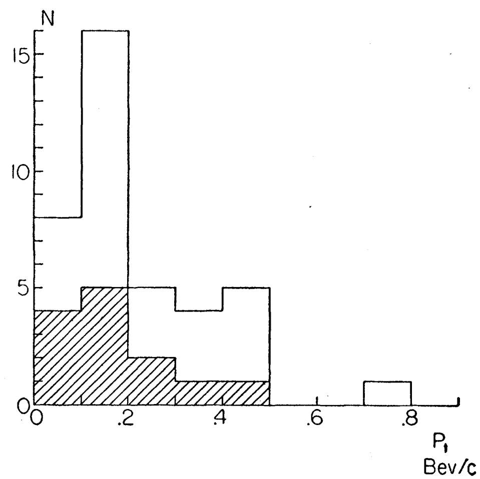

FIG. 5. Distribution of transverse momentum for 0+20p and 0+20n events. The shaded area represents measurements when only lower limits were obtained.

The average energy of the secondary particles cannot be increased by a factor greater than 1.6, otherwise their total energy would be higher than the primary energy determined from the angular distribution. This gives an upper limit of 0.35 Bev/c for $\langle P_t \rangle$. Furthermore, for this event, only five measurements out of a total of twenty are lower limits; therefore, their influence on $\langle P_t \rangle$ is expected to be small.

The upper and lower limits of $\langle P_t \rangle$ given for the secondaries of both events is between 0.25 Bev/c and 0.35 Bev/c. The results regarding $\langle P_t \rangle$ agree for both events within experimental error. Hence we can combine the data for both events in order to plot the distribution of P_t. It is shown in Fig. 5. The values, which represent lower limits only, are shaded. The distribution is peaked at a momentum of about 0.2 Bev/c. No values of $P_t > 0.8$ Bev/c were observed.

This result on $\langle P_t \rangle$ is compared with the values given by other authors in Table IV. They agree with our

TABLE IV. Average transverse momentum $\langle P_t \rangle$.

Reference	$\langle P_t \rangle$ Bev/c	Primary energy of events used (Tev)[a]
This work	0.30±0.05	1.4 −3.0
Fenyves *et al.*[b]	0.34	0.05−1.0
Edwards *et al.*[c]	0.41±0.13	∼5
Debenedetti *et al.*[d]	0.36	4
Glasser *et al.*[e]	>0.13	20
Lohrmann[f]	≥0.24	0.15

[a] 1 Tev $=10^{12}$ ev. [b] See reference 30. [c] See reference 22. [d] See reference 24. [e] See reference 17. [f] See reference 19.

results within the limits of error. The over-all results indicate that the value of $\langle P_t \rangle$ observed in these collisions is between 0.30 and 0.35 Bev/c, irrespective of primary energy. This value is furthermore confirmed by the determination of the transverse momentum of π^0 mesons by the Japanese group.[29] After correction for their acceptance criteria of electromagnetic cascades, they find a value of $\langle P_t \rangle = 0.325 \pm 0.02$ Bev/c. The Bristol[22] data on π^0 mesons are not in contradiction with this, since they are less accurate.

On the other hand, $\langle P_t \rangle$ as determined from the angular distribution of secondary interactions[22,20] is higher than the directly obtained value of 0.30−0.35 Bev/c adopted here by almost a factor of 2. This discrepancy could be explained by the fact that the energy of such events can be overestimated by a factor of 2 by using one of the standard methods involving the angular distribution of the shower particles (see Sec. 3).

The low value of about 0.30 Bev/c for the average transverse momentum will remove some of the difficulties arising in connection with the "two-center model" of multiple meson production. In particular, values of the inelasticity greater than unity, which were obtained by Cocconi[45] in his analysis of some of the showers, should disappear.

Finally the transverse momentum of the two high-energy π^0 mesons originating from the 0+20p event has to be discussed. The data are shown in Table I(a). The values of P_t are rather high. However, it has to be noted that the angles of these π^0 mesons, with respect to the line of flight of the primary, could not be measured accurately. Therefore, their direction of flight was determined with respect to the *primary* direction of the 0+20n event. This procedure should lead to an overestimate of P_t, because the true center is probably between the 0+20n event and the π^0 mesons.

These two π^0 mesons carry away an appreciable fraction of the total energy going into meson production. No attempt was made to determine the transverse momenta of the other π^0 mesons, which have lower energies. Therefore the value of P_t found for the two high-energy π^0 mesons cannot be combined with the results given above on charged secondaries.

3. ESTIMATE OF THE PRIMARY ENERGY

A direct measurement of the energy of the primary particle causing the 0+20p event is impossible. It is possible, of course, to add up the energies of all secondary charged particles. This will definitely lead to a lower limit for the primary energy. In addition, we have the energy found in electron-photon cascades associated with the event. But one has always to take into account that neutral particles may escape detection completely and that for some of the charged particles in the forward cone only lower limits of their energy can be established. Such a "calorimetric" method will, therefore, in general lead to an underestimate of the primary energy to a degree which is difficult to calculate, if one cannot make sure that most of the neutral particles were actually detected (see Sec. 5). A first estimate, therefore, has to be based on the kinematics of the $0+20p$ event. This method requires the assumption of a model for the fundamental process and some additional assumptions, which will be discussed briefly.

First we assume that the $0+20p$ event is due to a collision of a primary proton with a single proton target. This is suggested by the even multiplicity of the event. The absence of "heavy" prongs, as is well known, is not sufficient evidence for the assumption that the target is actually a free proton. In nuclear emulsion, about 10 to 15% of all nuclear interactions of primary protons are "clean," i.e., they do not show heavy prongs due to the evaporation of charged particles from a heavy nucleus. From the composition of the emulsion and assuming reasonable cross sections, one can, however, expect that roughly 6% of all nuclear interactions will occur on free proton targets. The rest is explainable by collisions between primary protons and peripheral protons of heavy nuclei without any visible excitation of the nucleus.

The secondary event $0+20n$ is assumed to be a collision between a neutral particle with a single neutron on the periphery of a heavy nucleus as suggested again by the even multiplicity of the event. It is furthermore possible to assume that the primary proton undergoes an inelastic charge exchange collision in the $0+20p$ event, continuing as a neutron which subsequently produces the $0+20n$ event.

These assumptions cannot be proved *a priori*. However, it can be shown that they lead to an analysis of both events, consistent with experiment.

It can generally be assumed that in nucleon-nucleon collisions the emission of particles in the c.m. system is symmetrical with respect to a plane perpendicular to the line of flight of the primary nucleon. On this basis, the so-called median angle formula is used to determine the primary energy

$$E_p/Mc^2=2\gamma_c^2-1\approx 2/\Theta_{\frac{1}{2}}^2 \qquad (1)$$

[E_p = primary energy, Mc^2 = rest mass of the proton, $\Theta_{\frac{1}{2}}$ = half the opening angle in the laboratory system of a cone around the extrapolated line of flight of the primary particle, which contains half of the charged particles produced in the collision, $\gamma_c=(1-\beta_c^2)^{-\frac{1}{2}}$, β_c = velocity of the c.m. system].

This formula does not lead to an accurate value of the primary energy for reasons discussed by several authors. It uses only part of the information contained in the angular distribution. Also, it is valid only under the approximation

$$\beta/\beta_c=1 \qquad (2)$$

(β = velocity of produced particles in the c.m. system). Therefore any energy distribution toward lower energies in the c.m. system is neglected. The situation is further complicated by any correlation between the energy of a particle and its angle of emission in the c.m. system. The experimental facts obtained so far show such a correlation.

The method developed by Castagnoli *et al.*[13] gives a somewhat better estimate than that of formula (1). They obtained the expression

$$\ln\gamma_c=-\frac{1}{n}\sum_{i=1}^{n}\ln\tan\Theta_i'+\frac{1}{n}\sum_{i=1}^{n}U(\cos\Theta_i,\beta_i) \qquad (3)$$

(n is the number of charged mesons produced in the collision; Θ_i' and Θ_i are the angle of the ith particle in the laboratory and in the c.m. system, respectively). In the spectrum-independent approximation (2) mentioned above the second term in (3) vanishes, but in general, the behavior of the function U has to be taken from the various theories on multiple meson production or from experiment. Castagnoli *et al.* quote results for the Fermi and the Heisenberg theory indicating that the second term in (3) is always ≥ 0 and not necessarily a constant. The spectrum-independent approximation, therefore, always tends to overestimate γ_c. The original method first used by Duller and Walker[53] is essentially the same as the one by Castagnoli with the additional assumption of *isotropy* of the shower particles in the center-of-mass system. They obtained

$$\frac{F(\Theta_i')}{1-F(\Theta_i')}=\gamma_c^2\tan^2\Theta_i', \qquad (4)$$

where $F(\Theta_i')$ is the fraction of particles emitted in the laboratory system within a cone of half opening angle Θ_i'. In order to determine γ_c, one has to plot $\log[F/(1-F)]$ *versus* $\log\tan\Theta_i'$. Under the assumptions mentioned above, the experimental points should be located on a straight line of slope 2. Due to fluctuations, this will, in general, not be completely true. Therefore one has to try to fit best a straight line through the experimental points. The intersection of this line with the line $\log[F/(1-F)]=0$ yields the value of $\log\gamma_c$. It has been shown[45] that under different assumptions, the experimental points of the Duller-Walker plot will be found either on straight lines having a slope different from 2 or on some curves. All these deviations can serve as indications of the actual angular distribution in the

[53] N. M. Duller and W. D. Walker, Phys. Rev. 93, 215 (1954).

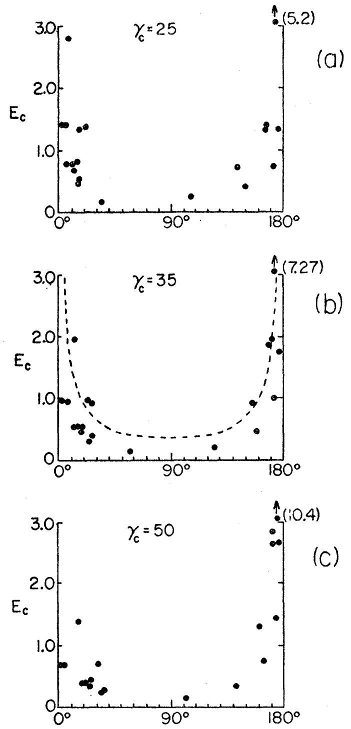

FIG. 6. Transformation to c.m. system of 0+20p event. In (a), $\gamma_c=50$, $E=5$ Tev. In (b), $\gamma_c=35$, $E=2.7$ Tev. The dashed line corresponds to a constant value of the transverse momentum $P_t=2m_\pi c$. In (c), $\gamma_c=25$, $E=1.2$ Tev.

c.m. system. By inspection of such plots, several authors[36,45] have come to the conclusion that in nucleon-nucleon collisions, the secondary particles might be emitted from two centers of emission of hitherto unspecified nature ("balls of fire") which move in opposite directions in the c.m. system. The methods of estimating the primary energy in such cases have been extensively discussed by Cocconi.[45]

In addition, there can be found various other approaches[11,12,14,18] to the problem in the literature which in most cases are actually only slight modifications of the methods discussed above. In order to determine the primary energy of our events, we follow Castagnoli's procedure including the second term in (3):

$$\frac{1}{n}\sum_{i=1}^{n} U_i=\ln C. \qquad (5)$$

The value of C was taken from experimental data[54] to be

$$C=0.7.$$

In the case of the 0+20n event, track No. 13 had to be relocated because its angle Θ_i' in the laboratory system is equal to 0 within the limits of measurement, i.e., it is parallel to the energetic cascades of $\pi_1{}^0$, of event 0+20p. We fitted a straight line in the Duller-Walker plot [Fig. 4(a), 4(b)] through the points of the 0+20n event and relocated track No. 13 in such a way that it is now exactly on this line.

[54] Jain, Lohrmann, and Teucher Phys. Rev. **115**, 643 (1959).

Thus we obtained the following values for the primary energies:

$$0+20p,\quad 2.7\times10^{12}\text{ ev};$$
$$0+20n,\quad 1.4\times10^{12}\text{ ev}.$$

4. ENERGY AND ANGULAR DISTRIBUTION IN THE CENTER-OF-MASS SYSTEM

After having estimated the primary energy of the two events, we can now obtain the transformations into the c.m. system. The primary energies of 2.7 and 1.4 Tev correspond to approximate values for γ_c of $\gamma_c=35$ and $\gamma_c=25$, respectively. In order to show how sensitively the energy and angular distribution in the c.m. system depends upon the correct value of γ_c, both events are also transformed for two additional values of γ_c, which are approximately 1.4 times larger and 1.4 times smaller than the values determined in the preceding section. The results of this procedure are shown in Figs. 6 and 7. They indicate that the transformation is not very sensitive upon the choice of γ_c. This is particularly true for the energy distribution. All transformations were carried out under the assumption that all secondary charged particles are π mesons because present experimental evidence indicates that the fraction of particles of nonpionic mass among the shower particles is small. In transforming the backward cone tracks, no approximations were used. One sees quite clearly that in both events, the high-energy particles are emitted preferencially in the forward and backward direction. It can further be noticed that there are particles with angles

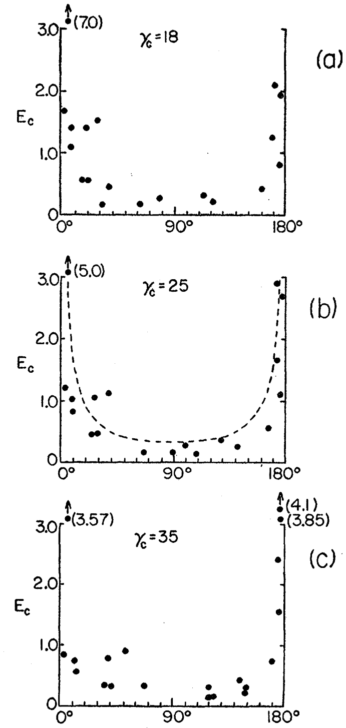

FIG. 7. Transformation to c.m. system of 0+20n event. In (a), $\gamma_c=35$, $E=2.7$ Tev. In (b), $\gamma_c=25$, $E=1.2$ Tev. The dashed line corresponds to a constant value of the transverse momentum $P_t=2m_\pi c$. In (c), $\gamma_c=18$, $E=0.6$ Tev.

close to 0 or 180 degrees having energies which exceed the average energies by rather large factors. In the $0+20p$ event ($\gamma_c=35$), one charged particle in the backward cone has an energy of 7.3 Bev, which has a predominant influence on the average energy of the charged particles in the backward cone, which comes out to be 2 Bev. The average energy in the forward cone, however, is only 0.76 Bev. This value is, first of all, only a lower limit, because for several tracks in the forward cone, only lower limits of the energy in the laboratory system could be established. Secondly, the existence of neutral mesons must be taken into account. One of these (π_1^0) would have an energy of 10 Bev in the c.m. system. This particle would, of course, have a considerable influence upon the average energy in the forward cone if the average energy could be calculated for both charged and neutral mesons. In the $0+20n$ event ($\gamma_c=25$), the situation seems to be reversed. There are no π^0 mesons of extraordinarily high energy in the forward cone, but there is one charged particle having an energy of 5 Bev, which increases appreciably the average energy in the forward cone. In the backward cone, one does not find high-energy charged particles in the c.m. system. However, there might be high-energy neutral pions, which, of course, are difficult to detect in the laboratory system because of their low energy.

In Fig. 8 the combined energy distribution of the mesons is given for the two events. There is a pronounced accumulation of particles at the lower end of the spectrum. The average energies for the $0+20p$ and $0+20n$ event are 1.25 and 1.1 Bev, respectively. These values are lower limits only, because of the lower limits for several particles in the laboratory system. But, by taking into account the neutral π mesons too, it can be shown from momentum balance in the c.m. system of the $0+20p$ event that this lower limit cannot be far from the true value. This follows directly from the fact that there is not too much energy in the c.m. system available in order to increase the energy of the particles where only lower limits were measured. In the $0+20n$ event, only a few such particles occur anyway. We, therefore, believe that the average energy of the mesons in the c.m. system of both events is approximately 1.4 Bev. This will be further supported by the energy balance presented in Sec. 5 and is not in agreement with Fermi's[40] theory, which would predict a considerably higher value. But it is also in disagreement with Heisenberg's[38] theory, which predicts about 0.5 Bev for primary energies of 2×10^{12} ev. Landau's[41] theory can be adjusted to agree with the experimental value of the average energy. However, one has to keep in mind that the obtained average energies are very sensitive with respect to the existence of a few particles in the high-energy tail of the distribution. A rather large fraction of the particles (lower limit 30%, upper limit 60%) can be found at energies below 1 Bev. But there are a few particles with energies exceeding the average value by five to ten times.

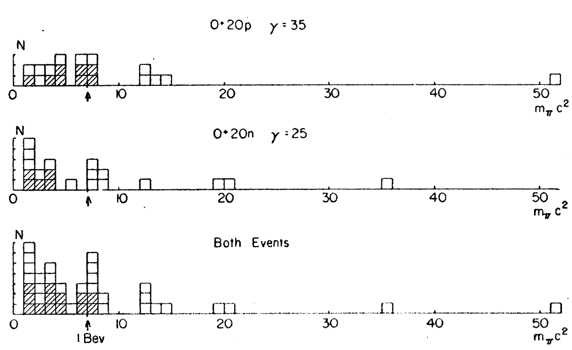

FIG. 8. Energy distribution in c.m. system of $0+20p$ and $0+20n$ events. The abscissa is in units of the pion rest mass.

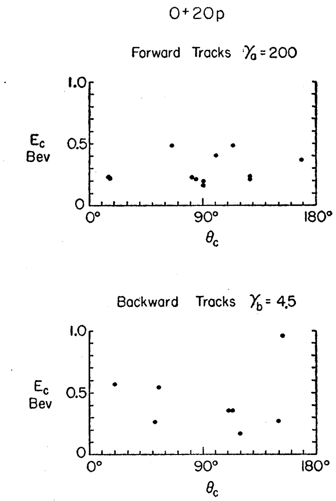

FIG. 9. Transformation of $0+20p$ event using the two-center model.

The two events were plotted according to Cocconi's[45] version of the two-center model. The Duller-Walker plots for both events are broken into two branches. This yields the following values for γ_a and γ_b:

	γ_a	γ_b
$0+20p$	200	4.5
$0+20n$	100	5

The transformations show that the emission of particles from the assumed centers could probably be considered reasonably isotropic and that the average energy of the charged particles would then be 0.4 Bev (see Figs. 9 and 10). It should, however, be noticed that in the forward branch of the neutral event, one charged meson has an energy of 1.25 Bev. If, in the case of the $0+20p$ event, π_1^0 is included, it would carry about 1.75 Bev, which considerably exceeds the average value. From this evidence, it must be concluded that this model is certainly not able to remove the high-energy tail of the distribution. It only shifts the spectrum towards lower energies. The present limits of measurement do not allow very precise statements about the angular distribution for this model. They also do not indicate any obvious discrepancy concerning the two-center model.

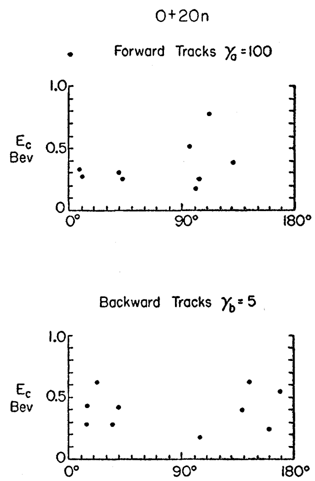

FIG. 10. Transformation of the 0+20n event using the two-center model.

5. ENERGY BALANCE AND INELASTICITY

The total energy dissipated in the primary collision can be estimated by means of energy measurements on the shower particles and on the soft cascade. The total energy contained in the individual components is shown in Table V. The values for the shower particles in the first column of Table V are based on scattering measurements; lower limits obtained by this method were assumed to represent true values. The estimate of the soft cascade was based on three high-energy electromagnetic cascades discussed in Sec. 1. No further high-energy cascades were observed. Therefore, the energy contained in any other π^0 mesons was neglected. The lower limit on the energy of the neutral (0+20n) jet was also derived from scattering measurements on the secondaries, adding 50% for the contribution by π^0 mesons. In this way, one obtains obviously a lower limit for the primary energy of the 0+20p event. The high energy of the 0+20n secondary shower suggests that this event is possibly produced by the particle which initiated the first jet and retained an appreciable fraction of its original energy. One may then assume that no major fraction of the energy of the primary particle escaped detection. In the second column of Table V, the total dissipated energy was therefore estimated as well as possible. This should give an approximate value for the primary energy. For the shower particles, this was done by raising the total energy by a factor of 1.2 on the basis of symmetry arguments between forward and backward cones. This was discussed in Sec. 2. A better value for the energy of the neutral jet 0+20n can be obtained from the angular distribution and symmetry arguments as discussed in Sec. 3. Assuming that no major fraction of the energy escaped detection, one arrives at a value of 3.1 Tev for the energy of the primary interaction. This is in very satisfactory agreement with the energy estimate of 2.7 Tev obtained from the angular distribution and symmetry arguments (Sec. 3). This agreement shows that a consistent picture can be obtained by assuming that both interactions do represent nucleon-nucleon collisions.

The inelasticity of the primary event can also be estimated. The inelasticity η is defined as the fraction of the total available energy, which is dissipated in the form of secondary particles produced in the collision. As was pointed out before, it is probable that the 0+20n secondary event was produced by the primary particle of the first shower, as it continued after the collision. The inelasticity can then be taken from Table V. In the laboratory system, one gets

$$\eta_L = 1670/3070 = 0.54.$$

The corresponding value in the c.m. system is

$$\eta_c = 0.57.$$

An appreciable part of the figures used to derive the inelasticity η comes from direct energy measurements and thus η does not depend very strongly on additional assumptions about the nucleon-nucleon character of the collisions.

It is interesting to note, that in the 0+20p event, a great fraction of the total primary energy ($\sim$23%) is carried off by a single energetic π^0 meson. This illustrates again that a single energetic particle can have great influence on the energy balance and on the average values of the energy dissipation. The high-energy π^0 meson also accounts for the fact that the energy contained in the soft cascade is about as high as the energy contained in the charged shower particles. Ordinarily one would expect that the soft cascade contributes only about $\frac{1}{2}$ of the energy of the charged shower particles.

Due to the various assumptions discussed above, it is difficult to estimate reliable limits of error for η. Using the lower limits of Table V, one gets an upper limit for $\eta \approx 0.70$. A lower limit for η can be obtained if one keeps the interpretation of the two events, 0+20p and 0+20n, as nucleon-nucleon collisions and assumes an uncertainty of about a factor of 2 for the determination of energies by means of the angular distribution of the shower particles. This leads to a lower limit of about $\eta = 0.35$. The final value is therefore

$$\eta_L = 0.54_{-0.19}^{+0.16}.$$

TABLE V. Energy balance of the 0+20p event.

	Total energy, lower limit (Bev)	Most probable value (Bev)
Shower particles	640	770
Soft cascades	900	900
0+20n event	870	1400
Total	2410	3070

For the neutral $0+20n$ event, the estimate of the inelasticity is less reliable. One gets the following result:

$$\eta = 0.70_{-0.40}^{+0.30}.$$

This is based on scattering measurements of the secondary particles, adding 50% energy for the π^0 mesons. The limits of error take into account a possible error of the primary energy of a factor of 2. They are rather high, because for this event, it is necessary to rely exclusively on the angular distribution of the shower particles to determine the primary energy.

The discussion concerning the value of η for the primary $0+20p$ event shows clearly the advantage gained by a thorough analysis of one event. In particular, it is possible to check the various assumptions involved and to estimate limits of error. It should, however, be noted that the two interactions described do not necessarily represent an unbiased example of a nucleon-nucleon interactions at energies of a few Tev. There might be some form of selection, because the events were found by scanning for high-energy electromagnetic cascades. Furthermore, the number of shower particles of the primary event must not be too small, otherwise, a complete analysis becomes more difficult and less reliable. It is possible that such a bias might in particular tend to select events with relatively high values of the inelasticity η. The important problem of determining the distribution of η in a reliable way is as yet unsolved and needs further studies.