Imaginary Part of the Optical Potential*

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The imaginary part of the optical potential has been investigated for low-energy incoming neutrons, by means of the nucleon-nucleon cross sections in nuclear matter. The cross sections have been calculated under the assumption that pair correlations for low excited states of nuclear matter are the same as those formed in the ground state. The dependence of the effective mass on the single-particle momentum has been taken into consideration using an empirical solution which reproduces the present assumptions for the single-particle spectrum. The results have been applied to the nuclear surface in the Thomas-Fermi approximation. The maximum in the imaginary potential was found to be at the surface outside of the half-density radius. For low incident energies it is about 1.5×10^{-13} cm beyond this radius.

THE reasonable success of the independent-pair approximation[1] for the calculation of the properties of nuclear matter suggests an application of the same method to the calculation of the imaginary part of the potential in the optical model of the nucleus. It is the magnitude which determines the "absorption" of a nuclear particle propagating within nuclear matter with an energy higher than the Fermi limit. This absorption is equivalent to the removal of the particle from the configuration space of the one-particle problem described by its motion in the optical potential. In the approximation considered here it is equivalent to a collision with another particle within the nuclear matter.

The imaginary potential $-iW$ which would describe this absorption is given by

$$W=\tfrac{1}{2}v_\alpha\rho\langle\sigma\rangle, \tag{1}$$

where v_α is the velocity of the particle absorbed, and $\langle\sigma\rangle$ is its average collision cross section with the particles in the nuclear matter of density ρ. Hence the problem reduces to the calculation of $\langle\sigma\rangle$.

In this note we try to estimate the value of the imaginary potential with simple considerations which are not very accurate, but which are probably accurate enough to bring out the essential features. It must be borne in mind that the approximations inherent in the fundamental assumptions do not warrant exact evaluations. Very similar considerations were carried out by Verlet and Gavoret.[2] Their approach differs from ours only in the treatment of the nucleon-nucleon forces. They make use of a separable potential which fits the scattering data at low energy. The separability of the potential makes it possible to calculate exactly the influence of nuclear matter on the scattering. It is questionable, however, whether this advantage outweighs the uncertainties introduced by the unphysical character of a separable potential. A treatment of the same problem has also been reported by Shaw,[3] and Harada and Oda.[4]

The collision cross section is different from its value for a collision of two isolated particles because of three reasons:

1. Certain final states of the collisions are excluded because of the Pauli principle.
2. The effective mass of the particles is different from their actual mass.
3. The interaction acts differently for a pair within a Fermi gas than for an isolated pair.

The effect of point (1) has been calculated by Lane and Wandel[5] and Clementel and Villi.[6] We will refer to their results in which the points (2) and (3) have been left out, as the "final states" approach. Point (2) can be easily taken into account if the effective mass is known as a function of the momentum. We will use the following empirical dependence which reproduces present assumptions for a particle with momentum k_α (k_F is the Fermi momentum):

$$\frac{m}{m^*}=\frac{1}{\nu}=1+\frac{Ak_F^3}{1+Bk_\alpha^2}, \tag{2}$$

with $A=0.48\text{f}^3$, $B=1.53\text{f}^2$ (f is a fermi$=1\times10^{-13}$ cm). Point (3) implies that the scattering is governed by the Bethe-Goldstone equation rather than by the ordinary two-particle Schrödinger equation. A collision of two particles with initial momentum $\mathbf{k}_\alpha$ and $\mathbf{k}_\beta$ is described by

$$(\nabla^2+k^2)\psi_{\alpha\beta}(\mathbf{r})=\tfrac{1}{2}\nu\int F(\mathbf{r}-\mathbf{r}')v(r')\psi_{\alpha\beta}(\mathbf{r}')d\mathbf{r}'. \tag{3}$$

Here $k=|\mathbf{k}_\alpha-\mathbf{k}_\beta|$ is the magnitude of the relative momentum, $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$ is the vector between the particles, $v(r)$ is the interaction potential, and $F(\mathbf{r})$ is

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[1] Gomes, Walecka, and Weisskopf, Ann. Phys. 3, 241 (1958).
[2] L. Verlet and J. Gavoret, Nuovo cimento 3, 505 (1958).
[3] G. L. Shaw, Bull. Am. Phys. Soc. 4, 49 (1959).
[4] K. Harada and N. Oda, Progr. Theoret. Phys. (Kyoto) 21, 260 (1959).
[5] A. M. Lane and C. F. Wandel, Phys. Rev. 98, 1524 (1955).
[6] E. Clementel and C. Villi, Nuovo cimento 2, 176 (1955).

given by

$$F(\mathbf{r})=\int_\Gamma e^{i\mathbf{k}\cdot\mathbf{r}}\frac{d\mathbf{k}}{(2\pi)^3},$$

where the region of integration Γ includes only non-occupied momentum states for the pair:

$$|\mathbf{P}+\mathbf{k}|\geqslant k_F,\quad |\mathbf{P}-\mathbf{k}|\geqslant k_F,\quad \mathbf{P}=\tfrac{1}{2}(\mathbf{k}_\alpha+\mathbf{k}_\beta).$$

The solution of (3) can be written in the form

$$\psi_{\alpha\beta}(\mathbf{r})=e^{i\mathbf{k}\cdot\mathbf{r}}+f_{\alpha\beta}(\theta,\varphi)e^{ikr}/r,$$

and the scattering amplitude is given by

$$f_{\alpha\beta}=-\frac{1}{4\pi}\nu\eta(\mathbf{k}_f,\mathbf{p})\int e^{-i\mathbf{k}_f\cdot\mathbf{r}}\frac{v(r)}{2}\psi_{\alpha\beta}(\mathbf{r})d\mathbf{r},\qquad(4)$$

where $\mathbf{k}_f$ is the final momentum of magnitude k and direction θ, φ; furthermore we have

$$\eta(\mathbf{k}_f,\mathbf{p})=\begin{cases}1 \text{ for } |\mathbf{k}_f+\mathbf{p}|\geqslant k_F \text{ and } |\mathbf{k}_f-\mathbf{p}|\geqslant k_F.\\ 0 \text{ otherwise.}\end{cases}$$

Expression (4) differs from the scattering amplitude for the isolated case by the factor $\nu\eta$ and by the fact that $\psi_{\alpha\beta}(\mathbf{r})$ is different from the wave function in the isolated case. Let us call

$$S_{\alpha\beta}=\left|\frac{1}{2}\int e^{-i\mathbf{k}_f\cdot\mathbf{r}}v(r)\psi_{\alpha\beta}(\mathbf{r})d\mathbf{r}\right|^2.\qquad(5)$$

$S_{\alpha\beta}$ would be the differential scattering cross section $\sigma_{\alpha\beta}$ of an isolated pair, if $\psi_{\beta\alpha}$ were the solution of the ordinary two-particle Schrödinger equation. $\sigma_{\alpha\beta}$ is almost independent of the scattering angle at the momenta considered here. We therefore are justified in assuming that, here also, $S_{\alpha\beta}$ is independent of θ, φ. We then can calculate the average cross section $\langle\sigma\rangle$ appearing in (1):

$$\langle\sigma\rangle=\frac{1}{N}\sum_\beta\int|f_{\alpha\beta}|^2\frac{|\mathbf{k}_\alpha\cdot\mathbf{k}_\beta|}{k_\alpha}\sin\theta d\theta d\varphi$$

$$=\frac{3}{4\pi k_F^3}\nu^2\int d\mathbf{k}_\beta\frac{|\mathbf{k}_\alpha-\mathbf{k}_\beta|}{k_\alpha}\xi(\mathbf{k}_\alpha,\mathbf{k}_\beta)\langle S_{\alpha\beta}\rangle_{\text{spin}},\qquad(6)$$

where

$$\xi(\mathbf{k}_\alpha,\mathbf{k}_\beta)=\frac{1}{4\pi}\int\eta(\mathbf{k}_f,\mathbf{P})\frac{\delta(k_f^2-k^2)}{k^2}d\mathbf{k}_f,$$

and $\langle S_{\alpha\beta}\rangle_{\text{spin}}$ is the average of $S_{\alpha\beta}$ over the different spin pairs, isotopic and ordinary. We make use of a well known approximate relation, $\sigma_{pp}=\sigma_{nn}=\frac{1}{4}\sigma_{np}$ which gives us

$$\langle S_{\alpha\beta}\rangle_{\text{spin}}=\tfrac{5}{8}(S_{\alpha\beta})_{np},$$

where $(S_{\alpha\beta})_{np}$ is the cross section given by (5) between a neutron and a proton of momentum $\mathbf{k}_\alpha$ and $\mathbf{k}_\beta$, respectively.

We assume that the contribution to the cross section from angular momenta l higher than 0, are not different from the isolated case. We can calculate them by subtracting from the experimental cross section the $(l=0)$ part as given by the phases of Christian, Gammel, and Thaler.[7] The $(l=0)$ part is calculated by means of formula (3). The wave function $\psi_{\alpha\beta}$ is taken from the work of Gomes, Walecka, and Weisskopf.[1] In that paper only collisions of pairs are considered for which both partners are within the Fermi distribution. For the actual density of nuclear matter the s-part of this function can be approximated by

$$\varphi_0=(4\pi)^{\frac{1}{2}}\left(\frac{\sin kr}{kr}-\frac{\sin kc}{kc}\cdot\frac{(\pi/2)-\mathrm{Si}(1.10k_Fr)}{(\pi/2)-\mathrm{Si}(1.10k_Fc)}\right),\qquad(7)$$

where c is the core radius and "Si" is the sine integral. We use here the same function for pairs of which one partner is outside the Fermi distribution. This will be a reasonable approximation if its momentum is not too far from k_F. For the potential $v(r)$ we use a central potential with a core radius $c=0.4$ and an attractive exponential well which reproduces the singlet scattering length and effective range. The effect of the tensor force and of the singlet-triplet difference is neglected. It probably plays a smaller role here than in the isolated case just as in nuclear matter.[1] An approximate evaluation of expression (5) is shown in Fig. 1 as function of $4k^2=|\mathbf{k}_\alpha-\mathbf{k}_\beta|^2$ together with the scattering cross section $\sigma_{\alpha\beta}$ for an isolated pair. This allows us now to calculate the imaginary potential with the help of (6), for a given incident momentum $k_\alpha>k_F$. The relation of k_α with the incident energy ϵ of the entering particle is as follows. The kinetic energy of the particle inside the nucleus will be $\epsilon+B+E_F$, where E_F is the Fermi energy and B is the binding energy. Hence we find

$$(1/\nu)k_\alpha^2=(1/\nu)k_F^2+B+\epsilon,\qquad(8)$$

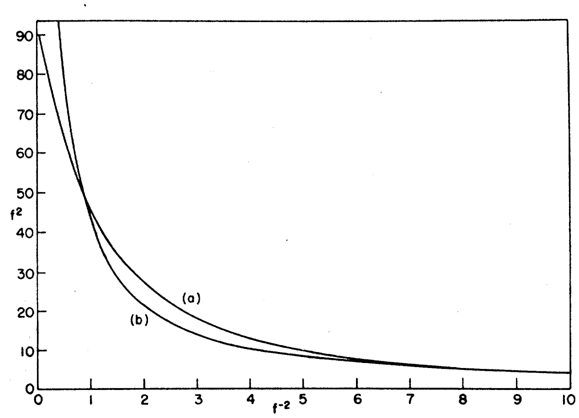

FIG. 1. Curve (a) is $(S_{\alpha\beta})_{np}$ as defined in Eq. (5) and (b) is $\sigma_{\alpha\beta}$, the scattering cross section for an isolated pair. The vertical scale is in units of f^2 ($1f=10^{-13}$ cm) and the horizontal scale is $|\mathbf{k}_\alpha-\mathbf{k}_\beta|^2=4k^2$ in f^{-2}.

[7] Gammel, Christian, and Thaler, Phys. Rev. **105**, 311 (1957).

TABLE I. The first column gives the excitation energy ϵ of the neutron in nuclear matter. The second column gives the value of the imaginary potential W calculated by our method. The third column gives the value of ν, the effective mass in units of nucleon mass, for the corresponding ϵ as given by Eq. (8). The fourth column gives W as calculated by the "final states" approach, and the fifth is the value calculated with our method but with $\nu=1$.

ϵ (Mev)	Independent-pair model W (Mev)	Independent-pair model ν	W by the "final states" approach (Mev)	Independent-pair model W (Mev) for $\nu=1$
1	1.06	0.77	1.88	2.11
7	2.34	0.78	4.03	4.59
14	4.62	0.80	6.88	7.82

where ν is the ratio of effective mass to normal mass. Equations (8) and (2) give a relation between ϵ and k_α.

Table I shows the result of this calculation for nuclear matter of normal density ($\rho=1.94\times10^{38}$ nucleons cm^{-3}, $k_F=1.42\text{f}^{-1}$). The first column gives the energy ϵ, the second one the value of W as calculated by our method, and the third one the value of ν for the corresponding k_α as given by (8). The fourth column gives W as calculated by the "final states" approach, and the fifth is the value calculated with our method but with $\nu=1$. The values in the fifth column are larger than the ones in the fourth, because in the most important energy region $S_{\alpha\beta}$ is larger than the isolated cross section $\sigma_{\alpha\beta}$, as shown in Fig. 1. The difference between the second column and the fourth column comes mainly from the effective mass.

The small values of W reflect the fact that collisions are strongly repressed by the Pauli principle (5); this, in turn, is caused by the high Fermi momentum. We expect, therefore that the lower density at the nuclear surface will give rise to a higher absorption, in spite of the fact that the density ρ enters as a factor in the expression of W. In order to get a first orientation of this effect, we have calculated W as a function of the nuclear radius by first calculating W as a function of density and then substituting the well-known density distribution

$$\rho(r)=\rho(0)[1+\exp[(r-C)/a]]^{-1},$$

with $a\sim0.65$ and C being the half-density radius ($C=1.07\times10^{-13}A^{\frac{1}{3}}$ cm). This method can only serve as a crude approximation since our calculation of $W(\rho)$ is correct only for constant ρ. Hence it is applicable only if ρ does not change over distances d characteristic to the problem ($d\sim k_F^{-1}$). This is not fulfilled with the above $\rho(r)$.

The dependence of W on ρ can be found as follows: There is an explicit dependence of the integral (6) on k_F and Eq. (2) gives the dependence of r on k_F. The integral (5) also depends implicitly on the density because of the fact that the approximate expression (7) for $\psi_{\alpha\beta}$ only holds for densities close to the nuclear-matter density. For low ρ, $\psi_{\alpha\beta}$ goes over into the solution of the isolated problem. Hence $S_{\alpha\beta}$ should go over into $\sigma_{\alpha\beta}$ for $\rho\rightarrow0$. In order to obtain a crude orientation, we have calculated $S_{\alpha\beta}$ with expression (7) for densities ρ from the central density down to that density ρ^* for which we get $S_{\alpha\beta}(\rho^*)=\sigma_{\alpha\beta}$. From ρ^* to $\rho=0$, we simply have put $S_{\alpha\beta}(\rho)=\sigma_{\alpha\beta}$.

It is then simple to compute the imaginary potential as a function of the radius. The result is shown in Fig. 2. The curves show that there is a strong increase at the surface of the nucleus caused by an increase of the effective mass and a lessening of the effect of the Pauli principle. It is perhaps significant that the maximum of absorption lies outside the nuclear radius C which is the point where the density drops to one-half.

It is highly doubtful, however, whether our method of calculating W is applicable to the region where classically no particle would be allowed. This is the region in which the real part of the potential is less than the binding energy of the last nucleon (8 Mev). This region is outside a radius D, which is marked in Fig. 2 and was obtained from the potential as given by Ross, Mark, and Lawson.[8] The curves for W are entirely meaningless for $r>D$. In order to get some vague information about W in that region, we have also calculated W without taking into account the Pauli principle, by simply using the scattering cross sections for isolated pairs. The nucleons in the nucleus were assumed to be distributed with a Fermi distribution corresponding to the density ρ and the momentum of the incident particle was assumed to be given by (8). The resulting W is higher than the one calculated by the previous method, but it is of the same order for values $r>D$. We therefore believe that the falloff of W resulting from the previous calculation is not an unreasonable estimate, even for $r>D$.

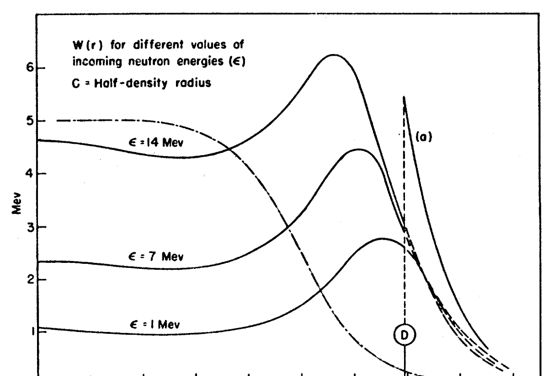

FIG. 2. The imaginary part W of the optical potential at the nuclear surface, for different values of incoming neutron energies (ϵ). The vertical scale is in Mev and the horizontal scale in f; C is the half-density radius. The dashed and dotted curve is the density function $\rho(r)$ in arbitrary vertical scale. D is the "classical" turning point. Curve (a) is W in the classically forbidden region calculated neglecting the exclusion principle.

[8] Ross, Mark, and Lawson, Phys. Rev. **102**, 1613 (1956).

A preponderance of collisions outside the nuclear radius would have two consequences: It would mean that direct reactions are favored, since collisions in the surface would make compound nucleus formation less likely. Also, the Coulomb barrier for nuclear reactions is expected to corrrespond to a larger radius than C, in particular with respect to direct reactions.

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Excitation Function for the $Al^{27}(d,\alpha p)Na^{24}$ Reaction Between 0 and 28.1 Mev

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The excitation function for the $Al^{27}(d,\alpha p)Na^{24}$ reaction has been measured between 0 and 28.1 Mev. The external beam facilities of the Buenos Aires 71-in. synchrocyclotron were used together with the stacked-foil technique. Between 19 and 28.1 Mev present results are more precise than those previously obtained, showing that the maximum of the cross section is located at 24.25 Mev with a value of 51.4 mb.

1. INTRODUCTION

The excitation function for the $Al^{27}(d,\alpha p)$ reaction as a function of energy was measured previously between 0 and 190 Mev by Batzel *et al.*[1] using the external beam of the Berkeley 60-in. cyclotron (from 0 to 19 Mev) and the beam of the Berkeley 184-in. synchrocyclotron. Their measurements exhibit a sharp peak between 20 and 25 Mev. They used the stacked-foil technique and they point out that the peak is inherently ill-defined due to range straggling of the 190-Mev deuterons degraded by the absorber.

2. EXPERIMENT

We have used the recently available deflected deuteron beam facilities[2] of the Buenos Aires 71-in. synchrocyclotron, illustrated in Fig. 1, and the stacked-foil technique to measure the above-mentioned excitation function between 0 and 28.1 Mev. The aluminum foils were some 4.75 mg/cm^2 thick, cut in 4×4 cm pieces, weighed individually to 0.1 mg. The error in area was less than 0.5% and the error in thickness was therefore less than 0.8%. The aluminum was found to be at least 99.7% pure through spectrographic analysis.

The foils were stacked and aligned carefully, within 0.1 mm, in order to avoid geometry errors. They were compressed between two metal disks, one of them being perforated to permit the passage of the beam. The stack was irradiated at a distance of 20 feet from the machine (see Fig. 1), just in front of the scattering chamber. The pipe was pumped by the vacuum system of the machine. The deuteron beam was adequately focused by two pairs of alternating gradient quadrupole lenses. The beam spot is of the order of 1 cm^2. Beam current was 0.05 μA. Neutron background was low because there is no need of collimators or slits along the

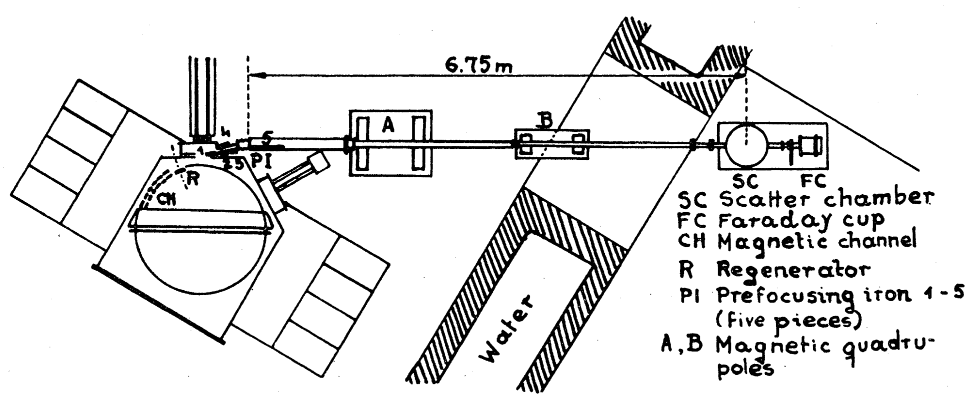

Fig. 1. External beam facilities of Buenos Aires 71-in. synchrocyclotron.

[1] Batzel, Crane, and O'Kelley, Phys. Rev. **91**, 939 (1953).