Formation of Positronium in an Electron Gas

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It is shown to be unlikely that a positronium atom, described by a hydrogenic wave function, can exist in an electron gas of a density corresponding to that found in metals.

IN a recent paper with this title, Bég and Stehle have calculated the rate at which positrons, placed in an electron gas, should capture electrons to form positronium.[1] They have assumed that the resulting positronium "atom" is described by a hydrogenic ground-state wave function and that its binding energy is 6.8 ev, which is characteristic of positronium in free space. Arguments can be given, however, which suggest that it is unlikely that the positronium "atom" can exist in an electron gas of a density corresponding to that found in metals. This problem is of particular importance since, according to presently accepted theories, measurement of the angular correlation of the γ rays from positron annihilation in a metal gives information from which the electron momentum distribution can be deduced.[2] This possibility probably would not exist if annihilation took place in an atomic system with a hydrogenic wave function.

The essential point is that, according to recent theories of the properties of an electron gas, the Coulomb interaction of a positron and an electron is screened by the polarization of the medium.[3] This is manifested in that a distribution of charge whose Fourier transform is $\rho(\mathbf{k},\omega)$ produces a potential whose transform $\phi(\mathbf{k},\omega)$ is given by

$$\phi(\mathbf{k},\omega)=\frac{4\pi}{k^2}\frac{\rho(\mathbf{k},\omega)}{\epsilon(k,\omega)},$$

where $\epsilon(k,\omega)$ is the dielectric coefficient of the gas. If the positron is represented as a static point charge, we find approximately a potential energy of interaction

$$V(r)=-(e^2/r)\exp(-\lambda r),$$

where λ depends on the density of the gas.[4]

We can now investigate the question whether a bound-state solution with the required properties exists for this potential.[5] To do this, we consider the Schrödinger equation for an s state

$$\frac{d^2R}{dr^2}+\frac{2\mu}{\hbar^2}\left[E+\frac{e^2}{r}\exp(-\lambda r)\right]R=0,$$

where μ is the reduced mass of the system and $R=r\psi$. The substitution $x=\lambda r$ allows the equation to be put in the standard form

$$\frac{d^2R}{dx^2}+\left(a+\frac{be^{-x}}{x}\right)R=0,$$

in which $a=2\mu E/\lambda^2\hbar^2$, and $b=2\mu e^2/\lambda\hbar^2$. The properties of solutions of this equation have been investigated by several authors; in particular it can be shown that no bound-state solution exists unless $b\geqslant 1.68$.[6] For the problem at hand, this condition is that $\lambda\leqslant 0.595$ (in atomic units).

It is likely, however, that a more stringent requirement exists. It seems probable on physical grounds that the positronium "atom" could not exist in a metal unless its binding energy, E, were approximately equal to the Fermi measured with respect to the vacuum (not the bottom of the band). Otherwise, the positronium should be able to ionize spontaneously. We shall investigate the consequences of requiring $|E|\geqslant 0.2$ Rydberg, which should be a rather conservative figure.[7] From reference 6 we find this requires $b\geqslant 5$ or $\lambda\leqslant 0.2$.

From the relation between the screening constant λ and the density of the gas, the maximum possible density for which the required solution exists can be determined. Unfortunately, an exact expression for the dielectric coefficient is not known. The approximate theory of reference 3 yields $\lambda=1.56r_s^{-\frac{1}{2}}$ (in atomic units), while Bég and Stehle employ a much smaller value taken from the theory of Bohm and Pines,[8] $\lambda=0.67r_s^{-\frac{1}{2}}$. We will consider both choices as possibly representing extreme values. For the larger value of λ, we find that the smallest value of r_s for which a bound state can exist is $r_s=6.9$ (Bohr units). By way of comparison, the metal of lowest density, cesium, has (at room temperature) $r_s=5.8$. Using the smaller value of λ, we find a bound state may exist for reasonable r_s, but it will not have $|E|\geqslant 0.2$ unless $r_s\geqslant 11.2$.

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[1] M. A. B. Bég and P. M. Stehle, Phys. Rev. **113**, 1545 (1959).

[2] R. A. Ferrell, Revs. Modern Phys. **28**, 308 (1956).

[3] J. J. Quinn and R. A. Ferrell, Phys. Rev. **112**, 812 (1958).

[4] It is perhaps more appropriate to use a frequency (as well as wave number) dependent dielectric coefficient [Eq. (9) of reference 3]. However, in first approximation, the same screening of the interaction is obtained as in the case of a static point charge.

[5] A somewhat similar study has been made by H. Kanazawa, Progr. Theoret. Phys. (Kyoto) **20**, 400 (1958).

[6] L. Hulthén and T. Laurikainen, Revs. Modern Phys. **23**, 1 (1951).

[7] It is assumed here that the binding of the positron in the field of a real crystal is weak compared to that of an electron. Although a more detailed investigation of this point might be desirable, it seems quite plausible since the potential experienced by a positron in any atomic cell corresponds, in the approximation of a self-consistent field, to a repulsive force. Allowance is made for a weak binding of the positron by choosing a rather small $|E|$.

[8] D. Pines, in *Solid State Physics*, edited by F. Seitz and D. Turnbull (Academic Press, Inc., New York, 1955), Vol. 1, p. 367.