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Cross Sections for the Excitation of the Metastable 2s State of Atomic Hydrogen by Electron Collision*

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The function for excitation of the 2s state of atomic hydrogen by electron impact has been measured from threshold to 45 ev by an atomic beam method. The absolute value of the total cross section has been determined by two independent methods which are in agreement. In one method the excitation function was normalized to the Born approximation at the higher energies. The mechanism of cascade from higher p states was found to play a significant role in population of the metastable 2s level. The other method proceeded by determining the metastable detection efficiency in terms of the known efficiency for Lyman-α photons. The yield for ejection of electrons from an untreated platinum surface by H(2s) is 0.065 ± 0.025. The total cross section reaches a maximum value of $(0.35\pm0.05)\pi a_0^2$ at 11.7 ev. The exchange cross section was also measured by the atomic beam method. The incident atoms were polarized in a Stern-Gerlach experiment; the metastable atoms were analyzed by the selective quenching action of a magnetic field of 575 gauss. The ratio of the exchange to total cross section is 0.45 ± 0.05 near threshold. At higher energies, this ratio approaches zero.

The cross section for production of metastable atoms by direct bombardment of molecular hydrogen is $0.03\pi a_0^2$. This value is considered correct to within a factor of two.

I. GENERAL DISCUSSION AND INTRODUCTION

THE excitation of the 2s state of hydrogen represents one of the simplest cases of scattering of slow electrons by atoms. Yet it involves the general features of rearrangements and electron exchange which play an important role in collisions with complex atoms. For these reasons, a great many attempts have been made to calculate cross sections for the excitation of the 2s state.[1] Prior to the present experiment, no quantitative experimental measurements had been made of this important quantity.[2] Lamb[3] and his co-workers carried out their historic investigations of the fine structure of the $n=2$ states of H by exciting atoms to the metastable 2s level by electron bombardment. However, they made no systematic attempt to measure the cross section for this process. As a preliminary estimate they assumed the cross section to be one-half the value given by the Born approximation. However, their observed detector current corresponded to only one electron for each 80 metastable atoms predicted by their estimate.

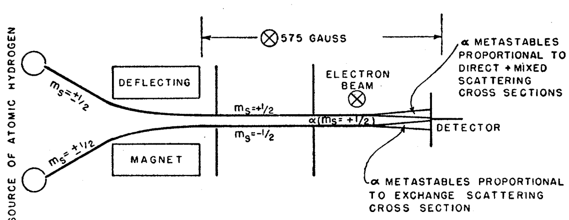

FIG. 1. Plan of the experiment.

In the case of intercombination excitations for light atoms, the total and exchange cross sections are identical. Excitation functions of this type have been measured.[4] In the more general case it is sometimes possible to measure separately *both* cross sections. Table I shows the well-known relationships for electron scattering by a hydrogenic atom. It can be seen that the total cross section can be resolved into three partial cross sections; exchange, direct, and mixed. Four

TABLE I. Electron scattering by a hydrogenic atom.

	Component of electron spin along magnetic field				
	Before collision		After collision		
Case	Atomic electron	Incident electron	Atomic electron	Scattered electron	Partial cross section
A	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\|f-g\|^2/2$ (mixed)
	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\|f\|^2/2$ (direct)
	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\|g\|^2/2$ (exchange)
B	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\|g\|^2/2$ (exchange)
	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\|f\|^2/2$ (direct)
	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\|f-g\|^2/2$ (mixed)

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[1] For a summary, see the article by H. S. W. Massey, Revs. Modern Phys. **28**, 199 (1956).

[2] Preliminary results of our measurements of both exchange and total cross sections were given at the Conference on Physics of Electronic and Atomic Collisions, New York University, January 1958 (unpublished). Recently, at the May, 1959 Washington meeting of the American Physical Society, Fite and co-workers reported preliminary measurements of the total cross section for excitation of 2s atoms: Stebbings, Brackmann, Fite, and Hummer, Bull. Am. Phys. Soc. **4**, 263 (1959).

[3] The articles on the fine structures of hydrogen were written as a series, and inthe text will bereferred to as HI, HII, HIII, and HVI. These are: HI, W. E. Lamb, Jr., and R. C. Retherford, Phys. Rev. **79**, 549 (1950); HII, W. E. Lamb, Jr., and R. C. Retherford, Phys. Rev. **81**, 222 (1951); HIII, W. E. Lamb, Jr., Phys. Rev. **85**, 259 (1952); HVI, Dayhoff, Triebwasser, and Lamb, Phys. Rev. **89**, 106 (1953).

[4] See, for example, H. S. W. Massey and E. H. S. Burhop, *Electronic and Ionic Impact Phenomona* (Clarendon Press, Oxford, 1952), second edition.

possible operations can be performed to obtain additional information from a scattering experiment. Two of them are polarization of the incident electron or struck atom: two are analysis of the scattered electron or atom after collision. It can be seen that performance of any two of these operations results in measurement of one partial cross section in addition to the total cross section. Performance of any three operations results in measurement of all three cross sections.

To date all experiments have measured, in effect, at most the polarization of the atom before and after collision. There have been measurements of elastic electron-atom[5] or atom-atom[6] exchange cross sections in the case of alkali metals.

Measurements of inelastic scattering cross sections have certain advantages over observations of elastic events. Elastic scattering from residual gases often introduces troublesome background which must be differentiated from the signal in question. In the case of inelastic scattering, there are often more specific events, such as an emitted photon or metastable atom, which serve to distinguish the desired signal from interfering background.

In the case of elastic scattering, space charge effects place a lower limit on the electron energy available for measurements. For inelastic scattering, experiments can be performed much closer to threshold.

Measurement of the inelastic cross section for scattering of electrons by H is especially favorable in the case of excitation of the $2s$ state. $H(2s)$ is metastable and can eject electrons from a metal detector which is removed from the excitation region.[7] Secondly, $H(2s)$ can be quenched by electric fields.[7] This unique property enables $H(2s)$ to be distinguished from all other interfering photons and metastables. Finally in a magnetic field of approximately 575 gauss, a beam of $H(2s)$ is polarized; that is, the β atoms ($m_s=-\frac{1}{2}$) are quenched; the α atoms ($m_s=+\frac{1}{2}$) are still metastable and can reach the detector. We have used these properties to measure both the total and exchange cross sections for excitation of the metastable $2s$ state.

II. MEASUREMENTS

A. General Plan of the Experiment

A source of atomic H could be placed in one of two positions with respect to a deflecting magnet (Fig. 1). Each position corresponded to the selection of a beam of polarized $H(1s)$ atoms,[8] with magnetic quantum numbers $m_s=+\frac{1}{2}$ or $m_s=-\frac{1}{2}$, respectively. The polarized beam was bombarded by unpolarized electrons of controlled energy in a homogeneous magnetic field of 575 gauss. Because of the analyzing effect of the magnetic field, only atoms in the α state could reach the detector and produce a signal.

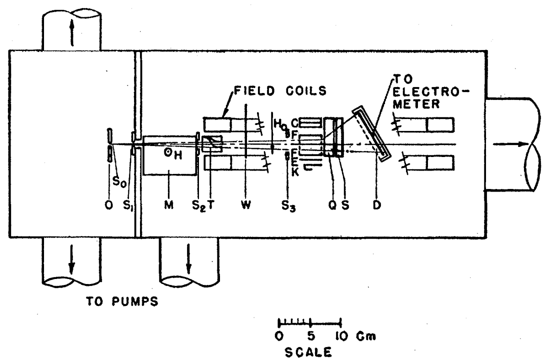

FIG. 2. Apparatus: O, tungsten oven; S_0, source slit; S_1, separating slit; M, deflecting magnet; H, direction of magnetic field; S_2, collimating slit; T, field rotator; W, movable stop wire; H_0, uniform magnetic field; S_3, exit slit; C, collector; F, field free box; E, control electrode; K, cathode; Q, electrostatic quenching electrodes; S, electrostatic shields; D, detector.

Thus (Table I), two cases arose. In case A, the detected signal $S(+)$, was proportional to the sum of the mixed and direct cross sections, which we have called[9] $(|f|^2+|f-g|^2)/2$; in case B, the signal $S(-)$ was proportional to the exchange cross section $|g|^2/2$. The sum of both signals was proportional to the *total* cross section, $\sigma_T(2s)$. Hence, the ratio of the exchange to total cross sections is given by the expression;

$$\frac{S(-)}{S(+)+S(-)}=\frac{|g|^2}{|f|^2+|f-g|^2+|g|^2}.$$

B. Apparatus and Method

The source of atoms (Fig. 2) was a tungsten oven, copied from Hendrie's[10] improvement of the original design of Lamb and Retherford.[11] Typical operating conditions were oven temperature 3000°K and gas pressure 2 mm. Under these conditions dissociation was 91% complete.[12] The atoms were polarized by a Permendur magnet with the usual two wire field,[13] with $H_{\max}=17.5$ kilogauss and a ratio of gradient to field of 2.5.

In order to have the electron and atomic beams coplanar it was necessary to rotate the magnetic field 90° in going from the deflecting magnet to the bombardment region. Also, to prevent accidental depolarization

[5] H. G. Dehmelt, Phys. Rev. **109**, 381 (1958); Franken, Sands, and Hobart, Phys. Rev. Letters **1**, 52 (1958); Rubin, Perel, and Bederson, Bull. Am. Phys. Soc. Ser. II, **4**, 234 (1959).

[6] Franken, Sands, and Hobart, Phys. Rev. Letters **1**, 52 (1958); R. Novick and E. H. Peters, Phys. Rev. Letters **1**, 54 (1958).

[7] See HI (reference 3).

[8] Strictly speaking, the effect of the hyperfine interaction of the nucleus should be taken into account. In our experiment the fields are high enough to provide almost complete decoupling so that this is a small effect. It is treated in the Appendix.

[9] In our experiment, partial, as opposed to differential, cross sections are measured. The partial cross sections actually are equal to the integrals over all solid angles of the squares of the amplitudes of the quantities; f, $|f-g|$, and g. Since no ambiguity arises, we consistently omit the integral sign in our expressions.

[10] J. M. Hendrie, J. Chem. Phys. **22**, 1503 (1954).

[11] See HI (reference 3) paragraph 21.

[12] Wooley, Scott, and Brickwedde, J. Research Natl. Bur. Standards **41**, 379 (1948).

[13] Rabi, Kellogg, and Zacharias, Phys. Rev. **46**, 157 (1934).

of the atomic beam, some care had to be taken to prevent nonadiabatic transitions of the atoms. Experience of atomic beam workers indicates that these transitions do not occur to any measurable extent unless the magnetic field goes to zero at some point along the atomic beam. A magnet constructed with helical pole faces served to rotate the field (Fig. 3). It insured an adiabatic transition into the uniform field region.

The entire bombarding and detecting regions were in a uniform primary field of 575 gauss, produced by a magnet made of two oval coils of water cooled copper tubing. This field had the dual function of quenching $H(2s,\beta)$ and collimating the bombarding electrons in the plane of the atomic beam. A pair of coils, external to the apparatus, produced a small secondary uniform field perpendicular to the primary field for the purpose of controlling the direction of the electron beam.

The electron bombardment apparatus was designed to obtain a minimum loss of metastable atoms through recoil dilution and accidental quenching. In the latter respect we benefitted greatly from advances made by Heberle *et al.*[14] in the design of equipment for producing $H(2s)$. The bombarding electron beam was inside a field free box of $5\times5\times1$ cm.

In order to achieve accurate measurement of the total cross section, it was important for the electron beam current to be known correctly. This implied the necessity of quantitative collection of the electron beam. The difficulties of achieving this in kinetic vacuum systems are caused by the formation of insulating layers upon the collector surface which reflects incident electrons. These difficulties were obviated by the use of a copper collector heated to 350°C.[15] The collector was biased 45 v positive with respect to the field free box and was tilted 22.5° from parallelism with the electron beam (Fig. 4). Under these conditions there was quantitative electron collection.

A movable stop wire could be placed such that the atomic beam could not reach the bombardment region.

The detector was a platinum sheet, enclosed in a brass box. The side facing the electron gun was covered with a grid of 0.024-cm nichrome wires spaced 4 per cm. Between the detector and the electron gun was a pair

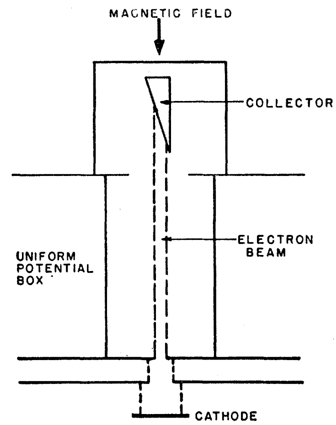

FIG. 4. Schematic view of electron gun. The atomic beam is perpendicular to the plane of the paper.

of parallel plates spaced 1.5 cm apart. By applying a potential difference of 45 volts between the plates, the assembly acted as an electrostatic quencher. Additional shielding electrodes prevented spurious signals arising from stray fields from the quenchers. The current to the detector arising from electrons ejected by the metastable atoms was measured by a conventional electrometer circuit. The detector accepted all atoms within a recoil angle $\leqq 22°$ (vertical), and $\pm 7°$ (horizontal). The recoil dilution factor (percent of metastable atoms failing to strike the detector) was calculated by assuming s-wave scattering.[16] For deuterium atoms, the calculated recoil dilution was negligible up to the ionization potential. At 20 volts it rose to 10%. At high energies where the Born approximation holds, it was about 10% and roughly independent of energy.

To insure that the apparatus was in fact operating as we anticipated, several test experiments were made. These will be discussed in the next section.

III. PROCEDURE

A. Preliminary Observations and Adjustments

Except for preliminary optical alignment of the apparatus, all adjustments were made by observation of the detector current itself. With the deflecting magnet turned off, the atomic H beam and electron beam were made coplanar by lateral displacement of the oven and by adjustments of the secondary magnetic field.

Typical observations are shown in Fig. 5. The total galvanometer deflection was diminished when a voltage was applied to the quenchers. We define this diminution as the *quenchable deflection.* The remaining galvanometer deflection is called the DC *background.* The *quenchable deflection* was diminished when the stop wire blocked off the atomic beam. We define this diminution as the *signal.* The remaining quenchable deflection is designated as the *background.* The signal arose from excita-

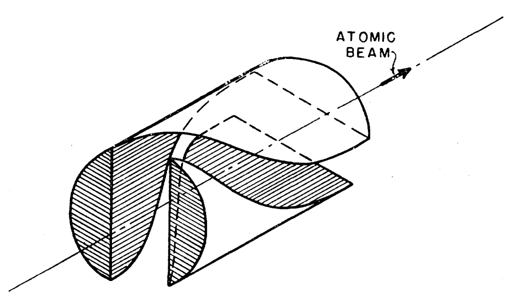

FIG. 3. Field rotator.

[14] Heberle, Reich, and Kusch, Phys. Rev. **101**, 612 (1956).
[15] A. E. Ennos, Brit. J. Appl. Phys. **5**, 27 (1954).

[16] HI (reference 3) Appendix III. It is necessary to multiply the velocity distributions used in this analysis by the factor $1/v$, to allow for the effect of the excitation process.

tion of atomic H in the collimated beam.[17] The background arose from metastable atoms formed from bombardment of molecular hydrogen gas in the chamber containing the electron gun.[18]

Before hydrogen was admitted to the oven, it was established that the quenchers had no effect on the DC background current in the detector. This insured that the relative potentials of the components of the detector and electron gun were chosen such that no ions or electrons reached the detector.

B. Energy Calibration

The electron energy scale was calibrated by making a linear extrapolation of the signal to zero current near threshold. This was assumed to be 10.2 volts, the excitation energy of H(2s). The bombardment current was held constant; this necessitated a small correction for the change in space charge depression of potential.

C. Secondary Processes

Auxiliary experiments verified that the signal was proportional to bombarding current beyond the 100 μa used in the experiment. This served to exclude the possibility of secondary processes involving electron bombardment. The pressure in the chamber containing the electron gun could be varied by changing the opening of the separating slit S_1 (Fig. 2). The signal was found to be independent of the pressure over the range of pressures observed during the experiment (2×10^{-7}—10^{-6} mm) as read on an ionization gauge. Thus, there was no evidence for secondary processes in the main chamber. On the other hand, it was found that

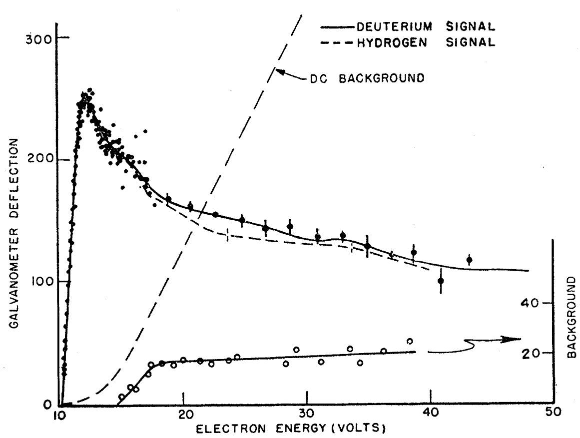

FIG. 5. Typical observations: The points represent single observations. The points with bars represent averages of several observations. The bars passing through the points show limits of $\pm$ one standard deviation of the mean.

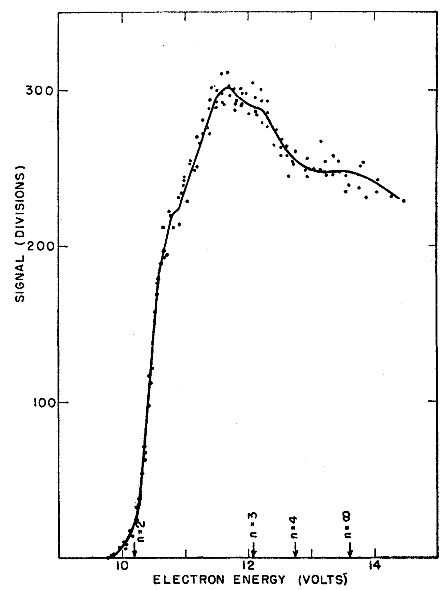

FIG. 6. Typical single observations of signal at low energies for both H and D. The results of each run are normalized to give the same area under the curve representing the excitation function.

the signal was *not* proportional to the source pressure under typical conditions of the experiment. This is attributed to collisions of the atomic beam in the vicinity of the source slit S_0, and not to secondary processes. Since the experiments were conducted at constant source pressure, relative cross sections were unaffected. However, absolute cross sections were obtained by extrapolating source pressure to zero.‡

D. Exchange Cross Section

After measurements of the total cross section were performed, the deflecting magnet current was turned on for the purpose of producing polarized beams of H(1s). The oven displacement was chosen large enough to insure a negligible amount of H(1s) of the wrong polarization. Also, the oven was placed at a position corresponding to maximum intensity to minimize errors caused by irregularities and misalignments of the apparatus. Estimated asymmetries in the deflection pattern caused an error in the exchange cross section of 1% or less of the total cross section. The effective electron beam width was 0.029 cm, effective atomic beam width, $2a=0.026$ cm, displacement of the oven was 0.019 cm, and deflection of the atom with the most probable velocity, $S_\alpha=0.039$ cm.

In the absence of a magnetic field the decay lengths of α and β states become equal. By varying the primary

[17] The signal arising from molecular hydrogen in the atomic beam was negligibly small.

[18] Since no background was observed below the threshold of 14.7 ev for the process $e^-+H_2\rightarrow e^-+H(2s)+H(1s)$, we conclude that, except for the directed beam, there was a negligible amount of atomic H in the apparatus.

‡ *Note added in proof.*—It is possible that extrapolation to zero oven pressure can introduce an error on account of thermal transpiration effects. Under these circumstances our absolute determination of the cross section by the direct calculation would be erroneous. However, we feel our determination of the absolute cross section rests primarily on the validity of the normalization to the Born Approximation over the higher energies.

magnetic field and extrapolating the ratio of the signals $S(-)/S(+)$ to zero field, we determined the asymmetry of the apparatus. The measured ratio was $S(-)/S(+) = 1.01 \pm 0.07$, which corresponds to a maximum standard error in the ratio of exchange to total cross section of $(0.25 \pm 1.8)\%$. This constitutes a negligible error, in agreement with our expectations.

Successive trials of different values for S_α/a led to a lower limit for the % polarization of the incident atomic beam. If it is assumed that the atoms underwent only adiabatic transitions before electron bombardment occurred, the limit would be approximately 95%.

IV. RESULTS

A. Detector Yield

The metastable detector efficiency was found in terms of the known yield $(1.8 \pm 0.5)\%$[19,20] of an untreated platinum surface for Lyman-α photons. By increasing the electric field between the detector and the grid, substantially all the H(2s) atoms were quenched before reaching the detector. The signal then arose completely from Lyman-α photons which originated from the quenched atoms.[21] Utilizing the known geometry and photoelectric yield,[22] we obtained the metastable yield of an untreated platinum surface for H(2s) to be 0.065 ± 0.025.

B. Excitation and Polarization Functions

The excitation functions for H(2s) are shown in Fig. 5 and Fig. 6. Different runs were all normalized to the same area over suitable ranges. The high-energy and low-energy excitation functions were normalized to each other over the energy range 12.5 to 14.5 ev, since the shape of the excitation function there is relatively insensitive to energy spread of the electron beam. An oxide-coated cathode (energy resolution 0.2 ev) was used in the low-energy experiments (Fig. 6) to show any fine structure in the excitation functions near threshold. In the high-energy experiments (Fig. 5),

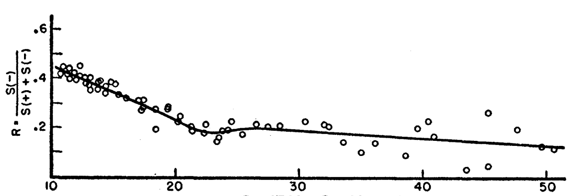

FIG. 7. Polarization measurements: each point represents a single observation.

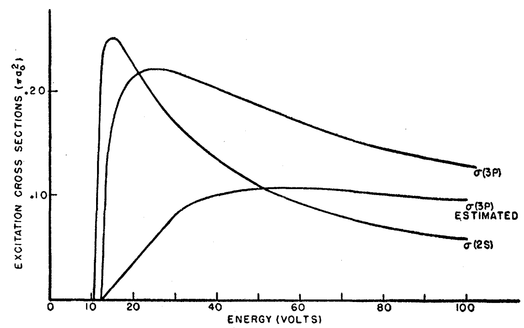

FIG. 8. Born approximation for the total cross sections for the excitation of the 2s and 3p states. The additional curve for $\sigma(3p)$ is our best estimate based on the results of Fite and Brackmann for $\sigma(2p)$, (reference 27).

a thoriated tungsten cathode (energy resolution 1.5 ev) was used to achieve greater stability in the signal. The excitation function rises sharply from threshold to a maximum at 11.7 ev. Figure 7 shows the results of the measurements of the polarization of the metastable atoms. The excited atoms are almost completely depolarized near threshold.

C. Total Cross Section

The excitation function was normalized to give absolute values for the total cross section in two ways. The first method was an ***absolute*** one, which utilized our determination of the yield of the detector, the geometry of the apparatus, and the known conditions in the source and electron gun.[23] This determination was carried out at an electron energy of 11.7 ev, which is low enough to insure that negligible recoil dilution occurred. However, corrections were applied for the estimated effects of quenching losses, the effect of the grid wires, incomplete detector electron collection, and incomplete action of the quenchers. The result for the maximum total cross section was $\sigma_T(2s) = (0.28 \pm 0.14)\,\pi a_0^2$, where the largest source of uncertainty was the metastable detection efficiency.

There always is the possibility of accidental loss of metastable atoms by stray fields from contaminated electrode surfaces, poor vacuum conditions, etc. Under these circumstances, our estimate would be erroneously low. However, we took all possible precautions to avoid accidental loss of metastable atoms. On the other hand, we feel that lower values of $\sigma_T(2s)$ are quite possibly incorrect.[24]

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[19] Walker, Wainfan, and Weissler, J. Appl. Phys. **26**, 1367 (1955).

[20] H. E. Hinteregger and K. Watanabe, J. Opt. Soc. Am. **43**, 604 (1953).

[21] The relevant equations for this type of calculation are given in HIII, (reference 3), Sec. 67.

[22] The value for the yield was chosen equal to the mean, and the uncertainty taken as the difference between the values given in references 19 and 20.

[23] The relevant equations for this type of calculation are given in HI (reference 3), Sec. 12.

[24] For example, one might try to estimate $\sigma_T(2s)$, from the work of Lamb *et al.* Although they used a W detector, they noted that the yield for W and Pt were approximately the same (reference 3, HVI). Therefore, from the data given in HI (reference 3) Sec. 26, and from our estimated yield for Pt, $\sigma_T(2s) \sim 0.03\pi a_0^2$. Presumably this value is low because of the large amount of accidental quenching which plagued early experimenters working with metastable H. Also, failure to extrapolate results to zero source pressure can give erroneously low results. Probably Lamb's results and some of our preliminary values (reference 2) were low for this reason.

The second method involved normalizing the excitation function to the Born approximation at high energies. At *low* electron energies ($E \leqq 12.1$ ev), the signal resulted only from excitation of the $2s$ state. At *higher* energies, population of the $2s$ state also resulted via cascade from higher states excited by electron bombardment. This mode of excitation affects the magnitude and shape of the excitation function and the polarization of the metastable atoms. In order to compare experimental results with theoretical calculations, it was necessary to make some correction for these cascade processes.

Since none of the cross sections for excitation of higher states of H has been measured, we used the Born approximation[25] and the known branching ratios for radiative decay[26] to make at least order of magnitude estimates. We concluded that only np ($n \geqq 3$) states should make appreciable contributions.

The energy dependence of the cross sections for the excitation of the $2p$ state and for ionization of H have been made by Fite and Brackmann.[27] Both cross sections deviate considerably from the Born approximation at lower energies, but both curves have approximately the same shape. Also, the Born approximation predicts essentially identical shapes for the excitation of all np states. Therefore we feel it is reasonable to assume that the shape of *all* the higher p states are identical to that of the $2p$ state and that the absolute magnitudes of the cross sections are proportional to the square of the respective dipole matrix elements $\langle 1s|z|np\rangle$. Also, the branching ratios between the $1s$ and $2s$ states are approximately the same for all np states.[26] On the basis of these two assumptions we have obtained an approximate formula for the cross section for production of metastable H by all atomic processes: $\sigma_P(2s) = \sigma_T(2s) + 0.21\sigma(3p)$. The estimated curve for the $3p$ cross section is given in Fig. 8.[28]

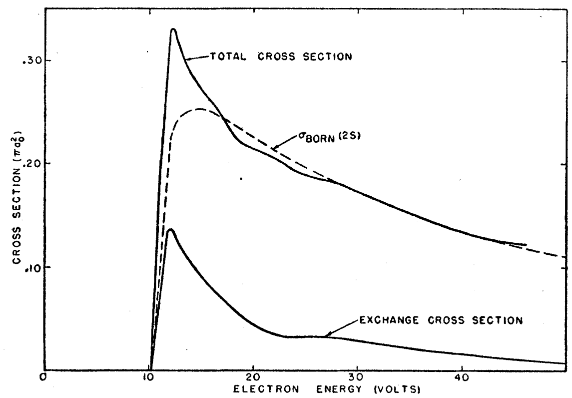

FIG. 10. Total and exchange cross sections for excitation of $H(2s)$ by electron impact. The estimated error for the total cross section is 15%. The estimated error for the exchange cross section is 8% ***relative to the total cross section.*** The errors are the root-mean-square sum of all systematic errors and three times the probable statistical error.

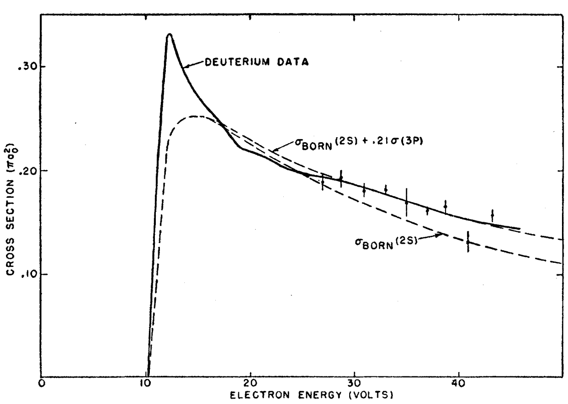

FIG. 9. Normalization of the excitation function to $\sigma_P(2s) = \sigma_{\text{Born}}(2s) + 0.21\sigma(3p)$, where $\sigma(3p)$ is our estimated cross section.

The normalization of the deuterium high-energy data to $\sigma_P(2s)$ is shown in Fig. 9. The fit is excellent; the standard deviation of the error of the normalization factor is 1.5%. The results for $\sigma_T(2s)$ are shown in Figs. 10 and 11. The maximum cross section is $(0.36 \pm 0.05)\pi a_0^2$, where the major source of error is caused by the uncertainty of $\pm 10\%$ introduced by recoil dilution at high energy.

The agreement between the two independent values for the total cross sections is within the experimental error. Because of this, we have considerable confidence in our final weighted value for the maximum cross section, $\sigma_T(2s) = (0.35 \pm 0.05)\pi a_0^2$.

D. Exchange Cross Section

The determination of the exchange cross section can be made from the polarization measurements (Fig. 7) and from the known total cross section. At low energies, where cascade phenomena play a negligible role, this follows straightforwardly from our discussion of Sec. II A. At higher energies, correction must be made for cascade from p states. This procedure is complicated by the fact that there are two cross sections for the excitation of p states, $\sigma(\pm)$ and $\sigma(0)$.[29] $\sigma(\pm)$ and $\sigma(0)$ represent the cross sections for exciting an electron of orbital magnetic quantum number ± 1 or 0, respec-

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[25] Formulas for the Born approximation to the cross sections are given in HI (reference 3). The numerical values for the cross sections for the $2p$ and $3p$ states given in Fig. 6, HI, are in error. Published values of the Born approximation frequently are in error near threshold because of the use of a high-energy approximation. This approximation gives too large a cross section.

[26] H. A. Bethe and E. E. Salpeter, *Quantum Mechanics of One and Two Electron Atoms* (Academic Press, Inc., New York, 1957), Table 15.

[27] W. L. Fite and R. T. Brackmann, Phys. Rev. **112**, 1141 and 1151 (1958).

[28] Dr. Fite has informed us that the published $2p$ cross section is in error from threshold to 25 ev. This in turn introduces an error in our estimate of $\sigma_T(2s)$. On the basis of preliminary results for the remeasurement of the $2p$ cross section the maximum error introduced in our estimate of $\sigma_T(2s)$ is 3%. This is negligible compared to our stated error.

[29] S. Khashaba and H. S. W. Massey, Proc. Phys. Soc. (London) **71**, 574 (1958).

tively. It can be shown[30] that

$$\frac{S(-)}{S(+)+S(-)}=\frac{|g|^2/2+K(0.21)\sigma(3p)}{\sigma_T(2s)+0.21\sigma(3p)},$$

where $K=4(1+X)/9(1+2X)$ and $X=\sigma(\pm)/\sigma(0)$.

Percival and Seaton[31] have shown that X is related to the polarization of radiation emitted in $H(np\rightarrow ms)$ transitions by the expression $P=3(1-X)/(7+11X)$, where P is the polarization of the emitted radiation. It is possible to do experiments which would give us the necessary information to allow for the effect of cascade on the observed results of the polarization experiments. In lieu of such experiments we have made an estimate of the effect of np states.

X is close to zero near threshold.[32] This is confirmed by the calculations of Khashaba and Massey[29] for the $2p$ state of H. Corroboration is added by the observation that electron excited Lyman-α radiation, $(2p\rightarrow 1s)$, is highly polarized at energies up to 100 ev.[27,33]

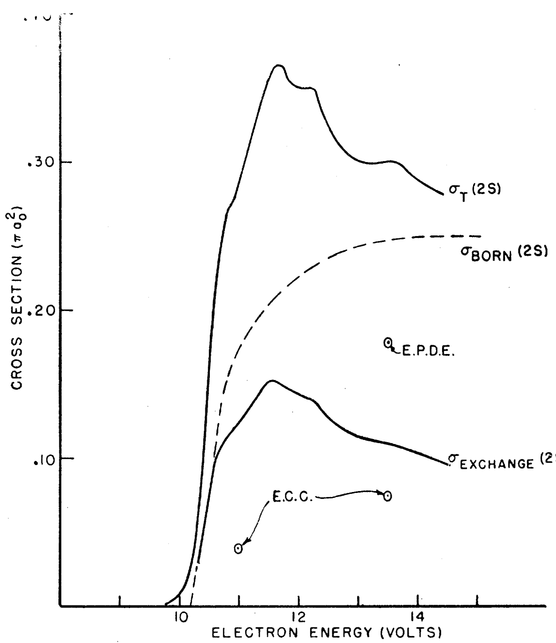

FIG. 11. Experimental total and exchange cross sections near threshold. The absolute errors for the cross sections are the same as in Fig. 10, but the error for the ratio of exchange to total cross section is 5%. Theoretical estimates of total cross sections: (E.P.D.E.), G. A. Erskine and H. S. W. Massey, Proc. Roy. Soc. (London) **212**, 521 (1952); (E.C.C.), reference 35.

We made the reasonable assumption that X is small for all np states up to 45 ev. We took K to be 0.4, which corresponds to small X. We also assumed the values for the cross sections for np and $2s$ discussed in the previous section. The results for the exchange cross section for excitation of H(2s) are shown in Figs. 10 and 11.

E. Formation of Metastable Hydrogen from Molecular Hydrogen

From the magnitude of the background signal (Fig. 5) and from the pressure of the molecular hydrogen in the apparatus, it was possible to make an estimate of the cross section for production of metastable H from H_2 molecules. It is $0.03\pi a_0^2$. This value is believed to be correct to within a factor of two.

V. DISCUSSION

A. Comparison with Theory and Other Experiments

The rapid rise of the H(2s) excitation function to a sharp peak near threshold is typical of forbidden transitions. The steep decline of the exchange cross-section curve at higher energies resembles the pure exchange singlet-triplet excitations in light atoms.[4]

Cross sections for the excitation of optically allowed $s\rightarrow p$ transitions in H and He agree with the Born approximation only at energies several times E_0, the threshold energy.[1,4,27] Near threshold, the Born approximation is greater than experimental results by about a factor of two.[27] Here, the distorted wave calculation[29] leads to a *lower* cross section than the Born approximation. This represents an improvement which is too small to be considered significant.[27]

The results of the present experiment form a striking contrast. Massey and Burhop conclude that the Born approximation agrees with experimental results for excitations of singlet s states in He down to energies as low as 1.5 E_0. Within experimental error, our results also agree with the Born approximation in this energy range (Fig. 10). Near threshold, the Born approximation *underestimates* the experimental cross section by about a factor of two. Other theoretical calculations (Fig. 10) fall still further below the experimental results.[34] Marriott's close coupling calculation[35] of $\sigma_T(2s)$ contains the least number of approximations of all computations. Yet it is lower than the experimental results by a factor of seven, in greater disagreement with our results than any other calculation.

In both cases of inelastic scattering of electrons from H, the best calculations are in poor agreement with experiment. On the other hand, calculations of the *elastic* scattering cross section of H are in good agree-

[30] Unpublished calculation by the authors. The expression for K given above was derived neglecting electron exchange. Since the cascade contribution is only important at the higher energies where the exchange amplitude is small, it is felt that the omission of exchange effects is tolerable. A calculation including exchange effects placed K within the limits of 2/9 and 2/3.

[31] I. C. Percival and M. J. Seaton, Phil. Trans. Roy. Soc. (London) **251**, 113 (1958).

[32] H. A. Bethe, *Handbuch der Physik*, edited by S. Flügge (Verlag Julius Springer, Berlin, 1933), second edition, Vol. 24, Part 1, p. 508.

[33] However, some caution should be applied, since these results exceed the theoretical limit of $P=3/7$. However, the discrepancies are within the author's quoted errors.

[34] The Born-Oppenheimer approximation generally falls too far above experimental results to be considered seriously.

[35] R. Marriott, Proc. Phys. Soc. (London) **72**, 121 (1958).

ment with experiment.[36] This may be related to the exceptionally large separation of the ground state of H from other states.

Massey[1] has indicated that close coupling between the degenerate 2*s* and 2*p* levels of H may be important, and that improvement may be obtained by taking this coupling into account. This represents going from a two-state to a three-state approximation.[1] It is possible that even an *n* state approximation, where *n* is a very large number, may fail to give satisfactory results. The eigenfunctions of present-day scattering calculations are built of products of one-electron wave functions. In the case of variational calculations of atomic energy levels, it was found that such wave functions did not give satisfactory agreement with experiment; it was necessary to introduce the interelectronic distance as an explicit variable.[37]

Massey and Moiseiwitsch have pointed out the importance of resonance effects for electron scattering near the threshold for excitation of the ^3S state of He.[38] Baranger and Gerjuoy[39] found that the excitation function of the ^3S state fits a one-level Wigner resonance formula. They have proposed the possibility of extending the resonance approach to other cases of electron scattering.[40] Dehmelt[5] also has used the Wigner approach in discussion of elastic scattering of electrons by Na atoms. The excitation function for the 2*s* state of H seems too complex to be fitted by a one-level formula, but it is possible that a resonance theory might be used to explain the results near threshold.

B. Conclusion

The present investigation of the excitation of the forbidden transition $H(1s \rightarrow 2s)$ and the corresponding results of Fite and Brackmann on the allowed transition $(1s \rightarrow 2p)$ represent a considerable body of experimental knowledge on the subject of inelastic collisions of electrons with the H atom. In both cases, the best approximate calculations show no significant improvement over the agreement of the fast-electron, Born approximation with experiment. This represents a largely unsolved problem in the theory of collisions of slow electrons with atoms.

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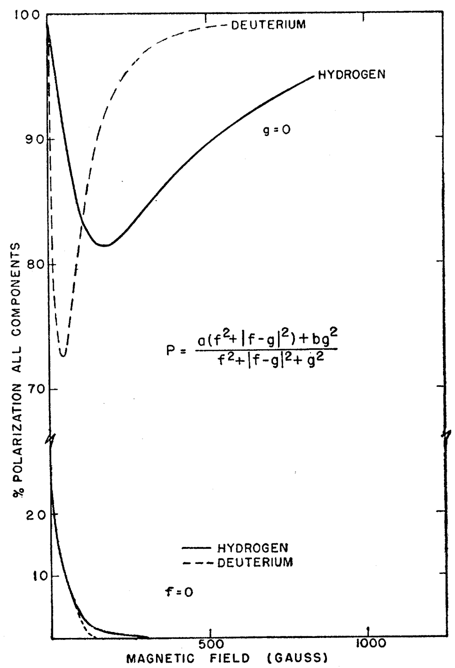

FIG. 12. Effect of nuclear hyperfine interaction on percentage polarization of the beam of $H(2s)$.

APPENDIX. EFFECT OF HYPERFINE INTERACTION ON POLARIZATION OF METASTABLE ATOMS

The theory given in Sec. II-A is valid only for magnetic fields large enough to decouple the electronic spin **S** from the nuclear spin **I**. We have computed the effect of the nuclear hyperfine interaction on the polarization of the excited atoms. Because of the smallness of all spin dependent interactions with respect to electrostatic forces, the approximation of sudden perturbation theory was assumed to hold for the collision.[41] The spin functions of the incident and scattered electrons were assumed to be diagonal in the $\mathbf{S}\cdot\mathbf{H}$ interaction; the eigenfunctions of the atom in the ground and excited states were found by diagonalizing the Breit-Rabi equation. The calculations then were averaged over all states having the same high field value of m_s. The results are shown graphically (Fig. 12).

[36] Brackmann, Fite, and Neynaber, Phys. Rev. **112**, 1157 (1958).
[37] E. A. Hylleraas, Z. Physik **65**, 209 (1930).
[38] H. S. W. Massey and B. L. Moiseiwitsch, Proc. Roy. Soc. (London) **A227**, 38 (1954).
[39] E. Baranger and E. Gerjuoy, Phys. Rev. **106**, 1182 (1957).
[40] E. Baranger and E. Gerjuoy, Proc. Phys. Soc. (London) **72**, 326 (1958).
[41] D. Bohm, *Quantum Theory* (Prentice Hall, Inc., Englewood Cliffs, New Jersey, 1951), p. 507.