Diffraction of Thermal Waves in Liquid Helium II by a Spherical Mirror*

J. FAJANS
Stevens Institute of Technology, Department of Physics, Hoboken, New Jersey

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Diffraction patterns of 3- and 13-kc/sec thermal waves in liquid helium II caused by a spherical mirror were measured. The main features of the patterns were developed from the Kirchoff diffraction formula combined with a standing wave distribution. Both the amplitude and phase of the wave at each point of space could be determined.

I. INTRODUCTION

A STANDING thermal wave pattern arising from multiple reflections between parallel plates was discovered by Peshkov.[1] Mercereau and Pellam[2] have demonstrated interference effects from a thermal wave antenna array. These experiments give strong validation to the point of view that a linear wave equation is appropriate to the description of thermal waves.

By showing that diffraction from a spherical mirror is governed by the Kirchhoff formula, the present work strengthens this conception and indicates that perhaps many more concepts and techniques from the body of geometrical and physical optics may be extended to thermal waves.

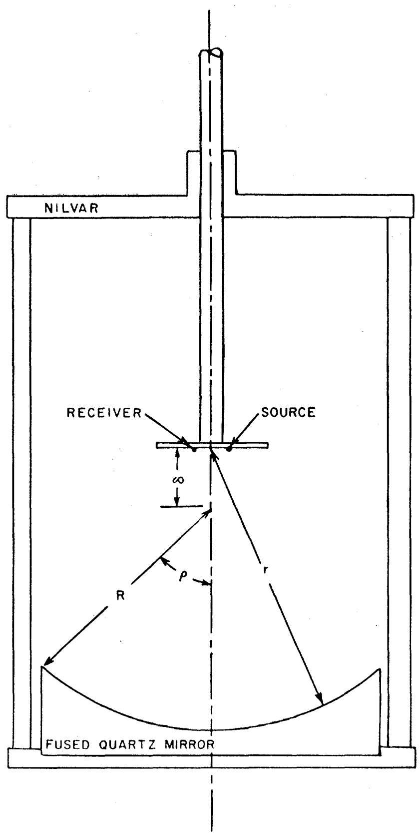

FIG. 1. The arrangement of the mirror and carbon transmitter and receiver.

The high wave number approximation to Kirchhoff's diffraction formula for the amplitude of a point linear wave source is

$$u_p=\frac{ie^{i\omega t}}{2\lambda}\int_S\int\frac{e^{-ik(r_p+r_q)}}{r_pr_q}[\cos(\mathbf{n}\cdot\mathbf{r}_p)-\cos(\mathbf{n}\cdot\mathbf{r}_q)]dS, \quad (1)$$

where dS is an element of the diffracting surface, $\mathbf{r}_p$ and $\mathbf{r}_q$ are the vectors from the surface element to the field and source points, $\mathbf{n}$ is the normal to the surface element, and ω is the thermal wave angular frequency.

The specific computation of the diffraction pattern, for points on the axis of the spherical mirror, Fig. 1, followed from

$$u_p=\text{const}\int_S\int\frac{\cos(2kr-\omega t)\cos(\mathbf{n}\cdot\mathbf{r})}{r^2}dS. \quad (2)$$

Source and field points were taken coincident on the axis, although actually they were on opposite sides of a $\frac{1}{4}$-in. diameter circle.

Ordinarily (e.g., with optical radiation), the time dependence is of no interest and an average factor is used; however, phase sensitivity of the receiver amplifier necessitated retention of the time in the integral. Only the part of the received signal in time phase with a reference voltage wave was measured. This made apparent a large amount of fine structure in the diffraction pattern which usually is not observed. Essentially, both the amplitude and phase of the wave at a point in space were measured.

Integration over the surface of the mirror, retaining terms to first order in δ/R, gave

$$u_p=N\cos(\omega t+\beta) \quad (3)$$

with

$$N=\frac{\sin[k\delta(1-\cos\rho)]}{k\delta(1-\cos\rho)},$$

$$\beta=k[2R+\delta(1+\cos\rho)].$$

It proved necessary to add the effects of multiple reflections to this "primary" diffraction to check the observed patterns, since the one-inch diameter disk

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[1] V. P. Peshkov, J. Phys. U.S.S.R. **8**, 131, 381 (1944).
[2] J. E. Mercereau and J. R. Pellam, Phys. Rev. **106**, 1113 (1957).

which was the mechanical support for the source and receiver acted as a plane mirror. When this disk was near the center of the sphere, geometrical losses of the wave energy were at a minimum and multiple reflections were strong.

To evaluate the contribution of these secondary reflections the following approximation proved effective:

$$u_p = Ne^{i(\omega t+\beta)}[(1+m+m^2+m^3+\cdots) - g(m+m^3+m^5+\cdots)], \quad (4)$$

with

$$m = fNe^{i\beta},$$

where $f<1$ and $g<1$ account for attenuating factors and were chosen for good fit to the data. Summation of the series gave

$$u_p = P\cos(\omega t+\gamma) - gQ\cos(\omega t+\Omega), \quad (5)$$

with

$$P = 2N(1+f^2N^2-2fN\cos\beta)^{\frac{1}{2}},$$
$$Q = 2fN^2/(1+f^4N^4-2f^2N^2\cos 2\beta)^{\frac{1}{2}},$$
$$\tan\gamma = \sin\beta/(\cos\beta - fN),$$
$$\tan\Omega = \sin 2\beta/(\cos 2\beta - f^2N^2).$$

II. EXPERIMENTAL

a. Thermal Wave Cell

A fused quartz spherical mirror with a radius of 1.99 in. and aperture diameter of 2.85 in. was mounted in a Nilvar frame. The transmitter and receiver were opposite 90° arcs of a $\frac{1}{4}$-in. diam carbon resistor circle on a bakelite disk which traversed the longitudinal axis of the mirror. Centering of the circle with respect to the axis of the mirror was maintained to 0.002 in.

b. Electronic Equipment

An effective bandpass of one cycle per second for the thermal waves was obtained with the system whose block diagram is given in Fig. 2. The filtered output of a Hewlett-Packard 200*c* oscillator drove the thermal transmitter. Frequency doubling took place in the generation of the thermal wave since the heating was the same on the positive and negative halves of the voltage wave. The oscillator frequency was also doubled electronically and bypassed to an intermediate stage of the amplifier, where it was added to the amplified input signal as a reference. By employing a reference larger than the random noise, linear rectification of the received thermal wave signal was effected. The reference voltage was the dominant signal to the rectifier with the noise and thermal wave riding "piggy back." Averaging of the noise to zero occurred, as well as averaging of the reference to a constant value. The "in phase" component of the thermal wave made a linear contribution to the rectified signal. This arrangement was merely an expedient adaptation of conventional phase detection circuits.

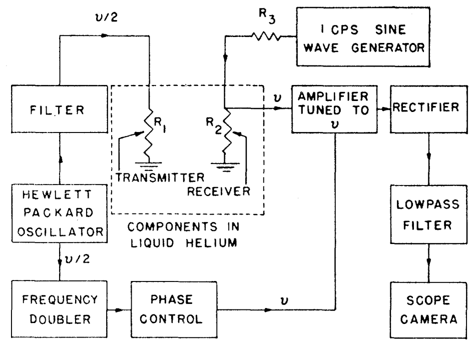

FIG. 2. Block diagram of the electronic system.

In addition, the bias voltage to the temperature-sensitive resistor R_2 was a one cycle per second sine wave, generated with a continuously rotating sine potentiometer. Thus, the thermal wave signal was modulated at the receiver at a one cycle per second rate but electrical pickup was not so modulated.

At the output of the rectifier, a one cycle per second signal existed which was fed to an oscilloscope with a slow sweep for photographic recording. The discrimination effected between one cycle per second and the thermal wave frequency was sufficient to completely eliminate electrical pickup, electrical cross talk, etc.

The signal entering the first amplifier stage was the thermal wave signal modulated at a one cycle per second rate.

$$V = [P\cos(\omega t+\gamma) - gQ\cos(\omega t+\Omega) + S\cos\omega t]\cos\omega' t, \quad (6)$$

with ω' representing the one-cps modulation and $S\cos\omega t$ representing a magnitude and phase-invariant portion of the thermal wave arising from direct source to receiver transmission. The large reference signal added after the first stage rendered this signal, after rectification and filtering, as

$$V = [P\cos\gamma - gQ\cos\Omega + S]\cos\omega' t. \quad (7)$$

This is the pattern which appeared on the oscilloscope.

c. Measuring Technique

Traverse of the source and receiver along the axis of the system at a rate of $\frac{1}{8}$ in. per minute with the reference voltage maintained at constant phase was accompanied by a slow oscilloscope sweep which crossed the tube face in two and a half minutes. An electronic switch made two channels available for simultaneous recording. Markers were used on one trace to indicate each $\frac{1}{32}$ in. of traverse in the helium bath, and the one cycle per second signal, whose envelope indicated the thermal wave diffraction pattern, was recorded by the second trace. Recording of an appreciable part of the thermal wave field required, in general, several oscilloscope sweep traversals.

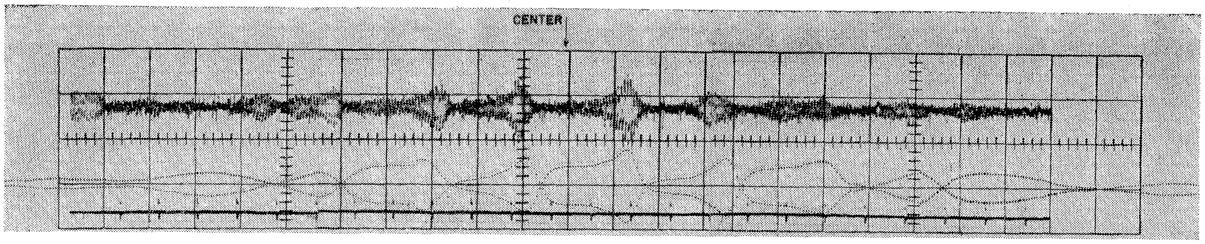

FIG. 3. Diffraction pattern along mirror axis for $\frac{3}{8}$ in. each side of center (from composite of three oscilloscopes traces). The dotted curve was computed from the amplitude factor of Eq. (7) with $f=0.65$, $g=0.45$, and $S=0.4$.

Results

Three oscilloscope traces in Fig. 3, matched to make a continuous pattern, represent the axial diffraction of a 3-kc/sec thermal wave (helium bath temperature, 1.79°K) in the region $\frac{3}{8}$ in. each side of the mirror center. The rapid oscillations were the one-cps signal discussed above, and the envelope of these oscillations was the amplitude factor in Eq. (7), which has been plotted for comparison as the dotted line in Fig. 3. The theoretical curve develops essentially all of the features of the experimental pattern. These were as follows:

1. The rapid spatial occurrence of zeros which was a consequence of measuring the phase of the wave. Alternate zeros formed sets with regular spacing. This distance was the same in both groups. (A shift in the phase of the amplifier reference voltage was followed by the appropriate shift in the location of the zeros.)

2. The amplitude gradually decreasing from the center of the mirror to the first zeros of the P function. This was roughly a $\sin x/x$ decay. The buildup to the first off-center maximum is visible at the extreme left.

3. Asymmetry about the maxima which was given almost quantitively by the equation.

4. Less prominent maxima and minima, qualitatively like those obtained experimentally. The writer believes that a closer rendition of these is within the framework of the formula.

When the temperature was varied, with the source and receiver at a fixed space point, a pattern with many zeros was developed, reflecting the variation in thermal wave velocity with temperature.

The features of the diffraction pattern for 13-kc/sec waves were similar. For 25-kc/sec waves only a few gross characteristics were discernible above the noise.

CONCLUSIONS

The diffraction of thermal waves by a spherical mirror has been measured in detail. The main features of the pattern have been shown to be given by the Kirchhoff formula.

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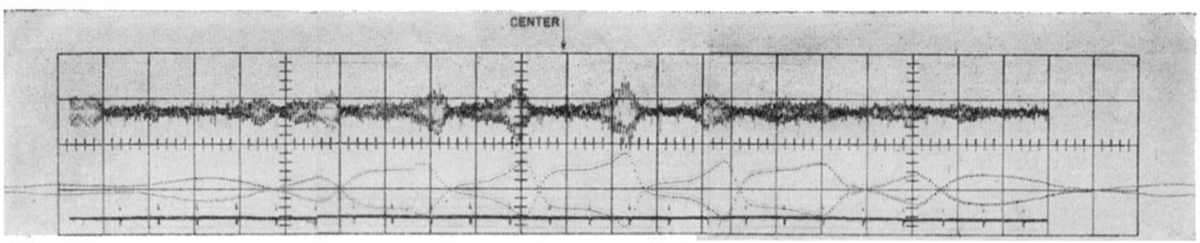

FIG. 3. Diffraction pattern along mirror axis for $\frac{3}{8}$ in. each side of center (from composite of three oscilloscopes traces). The dotted curve was computed from the amplitude factor of Eq. (7) with $f=0.65$, $g=0.45$, and $S=0.4$.