

# THE PHYSICAL REVIEW

*A journal of experimental and theoretical physics established by E. L. Nichols in 1893*

SECOND SERIES, VOL. 116, No. 5

DECEMBER 1, 1959

## Incoherent Microwave Radiation from Plasmas\*

G. BEKEFI, JAY L. HIRSHFIELD, AND SANBORN C. BROWN

*Department of Physics and Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts*  
(Received July 6, 1959)

A study is made of the incoherent radiation from an isotropic, quiescent plasma of a low degree of ionization. Three cases are treated theoretically: the transparent plasma, the semiopaque plasma, and the opaque plasma. Radiation from positive columns of dc glow discharges in helium and hydrogen for the three cases treated theoretically are studied experimentally at 3000 Mc/sec, and good agreement is obtained between theory and experiment.

### PLASMA AS A THERMAL EMITTER

KIRCHHOFF'S radiation law relates the emission to the absorption properties of a hot body and thus links the radiated power to its physical state. Two forms of this law are of special interest in the regions of microwave and radio frequencies. One is the emission from a body in free space, the other from a body enclosed in a waveguide.

Let an isotropic radiator at a uniform temperature  $T^\circ\text{K}$  emit a power of  $\mathcal{P}$  (watts) in the radian frequency interval between  $\omega$  and  $\omega+d\omega$  (wavelengths between  $\lambda$  and  $\lambda+d\lambda$ ) into a unit solid angle in the direction of a distant observer. The observer is in the Fraunhofer region of the emitter, at a distance from the body that is several times greater than the ratio of the square of the characteristic size of the emitter to the wavelength. If a plane polarized test wave is sent out from the position of the observer and a fraction  $A'$  of the incident energy is absorbed by the body,  $\mathcal{P}$  (for one degree of polarization) and  $A'$  are related<sup>1,2</sup> by

$$\begin{aligned}\mathcal{P} &= \frac{kT}{2\pi\lambda^2} A' S d\omega \\ &= \frac{kT}{2\pi\lambda^2} S d\omega \left\{ \int \frac{1}{2} \text{Re}(\mathbf{J} \cdot \mathbf{E}^*) dV / \right. \\ &\quad \left. \int_S \frac{1}{2} \text{Re}(\mathbf{E}_i \times \mathbf{H}_i^*) dS \right\}. \quad (1)\end{aligned}$$

Here  $k$  is Boltzmann's constant, and  $S$  is the projection of the cross-section area of the emitter upon a plane normal to the direction of propagation. The quantity in curly brackets,  $A'$ , represents the absorption coefficient, which is defined as the ratio of the power absorbed by a body of volume  $V$  to the power incident upon it;  $\mathbf{J}$  and  $\mathbf{E}$  are the alternating current density and the electric field strength, respectively, at a point within the emitter; and  $\mathbf{E}_i$  and  $\mathbf{H}_i$  are the unperturbed electric- and magnetic-field components of the incident test wave. When the radiator is situated in a waveguide, Eq. (1) takes the form<sup>2</sup>

$$P = kT A d\omega / 2\pi, \quad (2)$$

where  $P$  is the total power flowing down the waveguide, and  $\mathbf{E}_i$ ,  $\mathbf{H}_i$  are the field strengths of the propagating mode supported by the infinite, perfectly conducting waveguide. The attenuation coefficient  $A$  differs from  $A'$  in that the integration in the denominator of Eq. (1) is carried out over the complete cross section of the waveguide rather than over the cross-sectional area,  $S$ . If more than one mode propagates down the waveguide,  $A$  represents the sum of the attenuation coeffi-

\* This research was supported in part by the U. S. Army (Signal Corps), the U. S. Air Force (Office of Scientific Research, Air Research and Development Command), and the U. S. Navy (Office of Naval Research); and in part by the U. S. Atomic Energy Commission.

<sup>1</sup> M. L. Levin, *J. Exptl. Theoret. Phys. U.S.S.R.* **31**, 302 (1956) [translation: *Soviet Phys. JETP* **4**, 225 (1957)]; *Doklady Akad. Nauk S.S.S.R.* **102**, 53 (1955).

<sup>2</sup> S. M. Rytov, *Theory of Electrical Fluctuations and Thermal Radiation* (Akademii Nauk S.S.S.R., Moscow, 1953).

cients of the individual modes. Equations (1) and (2) are restricted to that range of frequencies in which the Rayleigh-Jeans limit is applicable; but they have been generalized to cover the whole frequency range.<sup>2</sup> Furthermore, Eq. (1) has been extended to include the case of an observer stationed near the emitting body,<sup>1</sup> Eq. (2) has been extended to resonant waveguide structures,<sup>2</sup> and both have been generalized to radiation from anisotropic materials.<sup>2,3</sup>

The microwave radiation from a volume of ionized gas originates primarily from free-free collisions of electrons with the other constituents of the gas (bremsstrahlung). When the electrons have a Maxwellian distribution of velocities,  $T$  of Eqs. (1) and (2) is the electron temperature.<sup>4,5</sup> Although an attempt has been made to extend a form of (1) to a Druyvesteyn distribution of velocities,<sup>6</sup> the whole problem of non-Maxwellian distributions has not been resolved. For this reason, the temperatures determined from noise measurements are referred to as "radiation temperatures." In the cases that have been examined, these temperatures were found to agree approximately with probe measurements of the electron temperature.<sup>6-9</sup>

The power dissipated from the test wave has been considered Ohmic; this allows one to relate the current density  $\mathbf{J}$  in (1) and (2) to the complex radiofrequency conductivity<sup>10,11</sup>  $\sigma$  of the plasma by  $\mathbf{J} = \sigma \mathbf{E}$ . The conductivity results from the electron motion in the applied field, while the contribution from ions is generally small. In computing the rf conductivity for plasmas with a low degree of ionization (the case in our experiments), electron-atom collisions, the values of which are known fairly accurately for many gases,<sup>12</sup> need only be considered. In fully ionized plasmas, an effective collision frequency can be obtained from the dc conductivity.<sup>13,14</sup> Except for special limiting cases that will be considered below,  $\mathbf{E}$  must be found by solving the diffraction problem<sup>15,16</sup> appropriate to a plane test

wave impinging upon a lossy dielectric of dielectric coefficient  $K = 1 + (\sigma/j\omega\epsilon_0)$ .

## 1. Transparent Plasma

An isotropic plasma of finite size and low charge density presents little attenuation to a wave incident upon it. The real part of the dielectric coefficient tends to unity, and scattering from the plasma boundaries becomes negligible. This implies that in the limit as the ratio of plasma frequency squared to the radian frequency squared approaches zero [i.e.,  $(\omega_p/\omega)^2 = ne^2/m\epsilon_0\omega^2 \rightarrow 0$ ], the electric field  $\mathbf{E}$  approaches  $\mathbf{E}_i$  and  $\mathbf{J} \rightarrow \sigma \mathbf{E}_i$ . Therefore the power radiated from the plasma can be determined completely from the electron density  $n$ , from the electron temperature  $T$ , and from the collision frequency for momentum transfer  $\nu$  of the electrons with atoms or ions. In fact, when the plasma is transparent we find from (1) and (2) that the noise power increases linearly with the electron density and, if  $(\nu/\omega) \ll 1$ , it also increases proportionately with the collision frequency.

In the transparent limit the microwave power radiated into free space as calculated from (1), using the electrical conductivity, should agree with the emission calculated from free-free binary collisions after they are summed over all individual events. For the case of a slow electron colliding with a helium atom, the two calculations agree within 10%. In making this comparison, the rf conductivity<sup>10</sup> involves the known experimentally determined collision frequency for momentum transfer,<sup>12</sup> while the binary collision calculation<sup>17</sup> involves theoretical values for the screening radius and the effective charge of the atomic potential.<sup>18</sup> When electron-ion encounters are the predominant radiation mechanism, the agreement is less favorable. The difference lies in the form of the Coulomb logarithm,  $\log \Lambda$ . Use of the conductivity<sup>14</sup> involves a  $\Lambda$  that varies inversely as the square root of the electron density, and the binary collision calculation<sup>19</sup> yields a Coulomb logarithm,  $\log(\omega_p \Lambda / \sqrt{2}\omega)$ . Thus it is seen that this difference can be significant, since in the transparent limit  $\omega_p/\omega$  can be very small.

## 2. Semiopaque Plasma

We shall now calculate the emission from a finite, partially absorbing plasma as though the waves propagated in an infinite medium. If  $\alpha$  is the power-attenuation coefficient of this medium, the amplitude of the electric field of a plane wave that has traversed a distance  $z$  in the medium will be reduced by a factor  $\exp(-\frac{1}{2} \int \alpha dz)$ . We allow approximately for reflections from boundaries. Thus

<sup>17</sup> L. Nedelsky, Phys. Rev. **42**, 641 (1932).

<sup>18</sup> W. P. Allis and P. M. Morse, Z. Physik **70**, 567 (1931).

<sup>19</sup> H. DeWitt, University of California Radiation Laboratory Report UCRL-5377, October, 1958 (unpublished).

<sup>3</sup> F. V. Bunkin, JETP **5**, 277 (1957); **5**, 665 (1957).

<sup>4</sup> J. E. Allen and W. R. Hindmarsh, Atomic Energy Research Establishment, Harwell, Report AERE-GP/R-1761, 1955 (unpublished).

<sup>5</sup> A. N. Dellis, Atomic Energy Research Establishment, Harwell, Report AERE-GP/R-2265, 1957 (unpublished).

<sup>6</sup> L. W. Davis and E. Cowcher, Australian J. Phys. **8**, 108 (1955).

<sup>7</sup> K. S. Knol, Philips Research Repts. **6**, 288 (1951).

<sup>8</sup> E. W. Collings, J. Appl. Phys. **29**, 1215 (1958).

<sup>9</sup> M. A. Easley and W. W. Mumford, J. Appl. Phys. **22**, 846 (1951).

<sup>10</sup> W. P. Allis, *Handbuch der Physik* (Springer-Verlag, Berlin, 1956), Vol. 21, pp. 383-444.

<sup>11</sup> I. B. Bernstein, Phys. Rev. **109**, 10 (1958).

<sup>12</sup> S. C. Brown and W. P. Allis, Technical Report 283 (fourth edition), Research Laboratory of Electronics, Massachusetts Institute of Technology, June 9, 1958 (unpublished).

<sup>13</sup> L. Spitzer, Jr. and R. Härm, Phys. Rev. **89**, 977 (1953).

<sup>14</sup> L. Spitzer, Jr., *Physics of Fully Ionized Gases* (Interscience Publishers, Inc., New York, 1956).

<sup>15</sup> H. C. Van de Hulst, *Light Scattering by Small Particles* (John Wiley and Sons, Inc., New York, 1957).

<sup>16</sup> N. Marcuvitz, *Waveguide Handbook*, Radiation Laboratory Series (McGraw-Hill Book Company, Inc., New York, 1951), Vol. 10.

$$\mathbf{E} = \mathbf{E}_i(1-\Gamma)^{\frac{1}{2}} \exp\left[-\frac{1}{2} \int_0^z \alpha dz\right], \quad (3)$$

where  $\Gamma$  is the power-reflection coefficient. Inserting (3) into (1), performing the integration along the direction of propagation  $z$ , and noting that for small attenuation  $\alpha = \text{Re}(\sigma/\epsilon_0 c)$ , we find that the power radiated varies as  $\int_S [1 - \exp(-\int \alpha dz)] dS$ . This is the result generally derived from the equation of transfer of radiation along a ray trajectory traversing an ionized medium.<sup>15,20</sup> Equation (3) and results based upon it are geometrical-optics approximations to the exact solution that are strictly valid only when the body is large compared with the wavelength (negligible diffraction effects), when the reflection and attenuation coefficients are small, and when the spatial inhomogeneities vary slowly as compared with  $\lambda$ .

The approach that has been outlined is also applicable to plasmas in waveguides. For instance, when a homogeneous plasma slab fills the cross section of a waveguide, (2) and (3) show that the normalized noise power  $P/P_m$ , defined as  $P[kT(1-\Gamma)d\omega/2\pi]^{-1}$ , equals  $1 - \exp(-\alpha l)$ , where  $l$  is the slab thickness, and  $\alpha = \text{Im}(4\pi/\lambda)[1 + \sigma/j\omega\epsilon_0 - (\lambda/\lambda_c)^2]^{\frac{1}{2}}$  with  $\lambda_c$  the cutoff wavelength of the propagating mode. Figure 1(b) shows a plot of  $P/P_m$  versus  $\alpha l$ . In contrast to the geometrical-optics solution, in which  $P/P_m$  is only a function of  $\alpha l$ , the exact solution, obtained from (2), depends explicitly on the collision frequency  $\nu$  and on the slab thickness. The curves of Fig. 1(c), (d), and (e) are plots of an exact calculation for the same plasma slab, for three thicknesses,  $l/\lambda_g$  (with  $\lambda_g$  the waveguide wavelength), and for three collision frequencies,  $\nu/\omega$ . The examples shown are for a slab in a rectangular waveguide operating in the  $TE_{01}$  mode at a frequency of  $3 \times 10^9$  cps. When the plasma slab is sufficiently thick [Fig. 1(c)], very good agreement is obtained with the geometrical-optics solution [Fig. 1(b)], even though the collision frequency is small and hence reflections from the plasma are large. The small undulations observed in Fig. 1(c) are the result of internal reflections. If, simultaneously, the slab thickness is decreased and the collision frequency increased [Fig. 1(d)], agreement with the geometrical-optics solution is good. However, when the slab is very thin [Fig. 1(e)] and the collision frequency small, large oscillations occur, and then the exact solution bears no resemblance to the approximate solution. In the limit of small attenuations (not shown in Fig. 1) the geometrical-optics and boundary-value computations reduce to the exact transparent limit discussed in Sec. 1.

When the plasma is inhomogeneous and not in the form of a slab, only geometrical-optics solutions are available. We have chosen for our example a plasma cylinder of radius  $R$  with its axis perpendicular to the

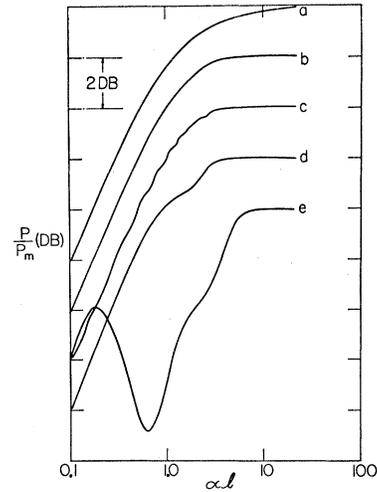


FIG. 1. Radiation as a function of the attenuation coefficient of a plasma in a waveguide: (a) represents the geometrical-optics calculation for an inhomogeneous cylinder of radius  $R$  with  $l=0.678R$ ; (b), the geometrical-optics calculation for a slab of thickness  $l$ ; (c), (d), and (e), exact calculations for the slab. In (c),  $l/\lambda_g=7.2$ ,  $\nu/\omega=0.0292$ ; in (d),  $l/\lambda_g=1.0$ ,  $\nu/\omega=0.292$ ; in (e),  $l/\lambda_g=0.072$ ,  $\nu/\omega=0.0559$ . (For clarity, curves are displaced vertically 2 db relative to each other.)

direction of propagation and to the electric field of the  $TE_{01}$  mode of a rectangular waveguide, and with a diameter equal to the narrow dimension of the waveguide. The attenuation coefficient is assumed to vary in the radial direction  $r$  as the zero-order Bessel function and to vanish at  $r=R$ . (This implies that for low attenuation coefficients, the electron density also varies as a Bessel function.) A plot of  $P/P_m$  versus  $\alpha l$  is shown in Fig. 1(a). Here  $\alpha$  denotes the axial attenuation coefficient, and  $l$  a characteristic length that, in this geometry, is given by  $l=0.678R$ .

### 3. Opaque Plasma

A perfectly opaque plasma absorbs all the power incident upon it in a given frequency interval, so that  $A=A'=1$  in Eqs. (1) and (2). Plasmas produced in the laboratory rarely approach this opaque condition because their fairly well-defined boundaries give rise to appreciable reflections. However, when the plasma is enclosed in a waveguide structure, special measures can be taken to achieve this limit. Reflections can be reduced by impedance-matching devices placed close to the radiator,<sup>7,8</sup> and if the optical depth  $\int \alpha dz$  is not sufficiently large, it can be increased artificially by placing the plasma in a resonant structure.<sup>6</sup> Such procedures mask the physical processes that go on inside the plasma, and hence they have been avoided in the present measurements.

The power-reflection coefficient  $\Gamma$ , in our experiments, ranged between 0.02 for transparent plasmas and 0.5 for dense plasmas. In presenting our measurements we have normalized the noise power  $P$  emitted to the power  $P/(1-\Gamma)$  that would be emitted in the

<sup>20</sup> R. v. d. R. Woolley and D. W. N. Stibbs, *The Outer Layers of a Star* (Clarendon Press, Oxford, 1953).

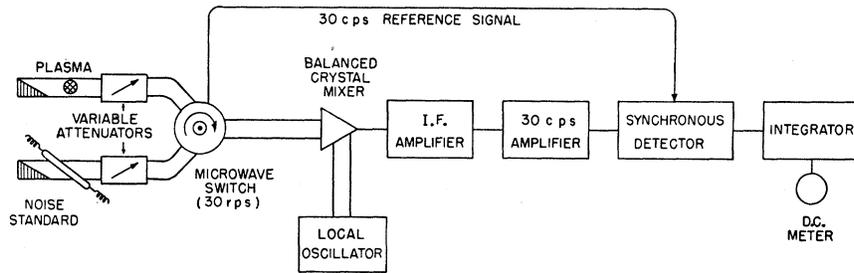


FIG. 2. Microwave radiometer.

absence of reflections. This procedure is useful for comparisons between theory and experiment. If we rewrite (2) in the form  $P/(1-\Gamma) = kTA_e d\omega/2\pi$ , we obtain an effective absorption coefficient  $A_e = A/(1-\Gamma)$ , which is defined as the ratio of the power absorbed to the power in the test wave that actually penetrates the plasma. The limit  $A_e = 1$  then corresponds to a perfectly opaque plasma that is mismatched to its surroundings. Measurements showed that when the plasma is dense,  $A_e$  indeed approaches unity, and values between 0.87 and 0.95 were obtained.

#### MEASUREMENTS

The plasma that was studied was a section of the positive column of a hot-cathode dc glow discharge in helium or hydrogen. The cylindrical discharge tube had an internal diameter of 2.6 cm. The anode-cathode spacing was 50 cm. Probes projecting into the positive column measured the axial voltage drop. Pressures of approximately  $10^{-8}$  mm Hg were attained in the discharge tube after activation of the cathode.

With the discharge tube operated over a range of gas pressures from 0.05 to 25 mm Hg, currents from  $10^{-3}$  to 4 amp gave electron densities from  $10^8$  to  $10^{13}$   $\text{cm}^{-3}$ . The electron density was calculated from the measured tube current and from the axial dc field of the positive column. The dc mobility was determined from the known collision frequency  $\nu$ . In the electron-energy range of our experiments (greater than 2 eV), the electron-atom collision frequency in helium or hydrogen is nearly independent of the electron velocity. We assumed that  $\nu = 2.55 \times 10^9 p_0 \text{ sec}^{-1}$  ( $p_0$  is the gas pressure normalized to  $0^\circ\text{C}$ ) for helium,<sup>21</sup> and  $4.85 \times 10^9 p_0$  for hydrogen.<sup>22</sup> The electron density was assumed to vary in the radial direction  $r$  as  $n = n_0 J_0(2.405r/R)$ , where  $n_0$  is the axial density,  $R$  is the tube radius, and  $J_0(x)$  is the zero-order Bessel function. Independent microwave measurements of electron density were made at a frequency of 4300 Mc/sec by enclosing a section of the positive column in a cylindrical cavity; the density was computed from the observed shift of the resonant frequency of the  $TE_{011}$  mode.<sup>23</sup> The two sets of data for the electron density agreed within 50%.

<sup>21</sup> A. D. MacDonald and S. C. Brown, Phys. Rev. **75**, 411 (1949).

<sup>22</sup> D. J. Rose and S. C. Brown, Phys. Rev. **98**, 310 (1955).

<sup>23</sup> S. J. Buchsbaum and S. C. Brown, Phys. Rev. **106**, 196 (1957).

The discharge tube was oriented with its axis at right angles to the propagation direction of the wave in a rectangular waveguide. The waveguide was provided with two sets of portholes drilled in its opposite faces so that the tube axis could be placed either parallel or perpendicular to the electric field of the dominant  $TE_{01}$  waveguide mode. One end of the waveguide was terminated in a matched load, the other was connected to the waveguide input of the receiver (see Fig. 2). Every effort was made to eliminate standing waves across the plasma, in order to prevent an artificial increase in its optical depth.

The receiver used in the noise measurements was a form of Dicke radiometer<sup>24</sup> (Fig. 2) operating at a frequency of 3000 Mc/sec with a 2 Mc/sec band width. The noise emitted from the plasma was made to alternate at 30 cps with the noise from a standard source. By adjusting the precision-calibrated attenuators (accuracy  $\pm 0.2$  db) to null the radiometer output, the measurements were rendered independent of the receiver characteristics. The radiometer had an over-all noise figure of 9 db and was capable of measuring power levels down to  $5 \times 10^{-17}$  watt. The noise standard was a fluorescent lamp calibrated against a hot load; its radiation temperature was determined as  $10\,700 \pm 300^\circ\text{K}$ .<sup>25</sup>

The absolute noise power  $P_r$  that reaches the detector cannot be entirely accounted for by the power  $P$  emitted by the plasma when it is reduced by the attenuation  $\beta$  of the precision attenuator. The power  $P_r$  differs from  $\beta P$  because of the power radiated by the attenuator and because of contributions that arise from reflections of this power from the plasma. If  $T_0$  is the radiation temperature of the attenuator (approximately  $300^\circ\text{K}$ ),  $T$  is the radiation temperature of the plasma (in excess of  $10^4$   $^\circ\text{K}$ ), and  $\Gamma$  is the power-reflection coefficient from the plasma, it can be shown that  $P_r = \beta P [1 - (T_0/T)] + kT_0(1 - \beta^2\Gamma)$ . A similar equation, which is applicable to the noise radiated by the standard source, can be equated to  $P_r$  when a null is obtained on the detector output. Calculations from measured values of  $\Gamma$  indicate that correction terms to the relation

<sup>24</sup> J. P. Wild, *Advances in Electronics and Electron Physics*, edited by L. Marton (Academic Press, Inc., New York, 1955), Vol. 7, pp. 299-362.

<sup>25</sup> W. C. Schwab, S.M. thesis, Department of Electrical Engineering, Massachusetts Institute of Technology, August 22, 1955 (unpublished).

$P_r = \beta P$  become significant only when  $P/kT \ll 1$ . When  $P/kT = 0.01$  the correction does not exceed 5%.

In conjunction with the radiation measurements, a simultaneous determination of the reflection and transmission coefficients of the plasma was made. This was done by sending a test wave down the waveguide from the position of the detector and by measuring the voltage-standing-wave ratio and the fraction of the incident power transmitted through the discharge.

### RESULTS

Measurements of the noise power radiated from helium discharges are plotted in Figs. 3 and 4. Unless

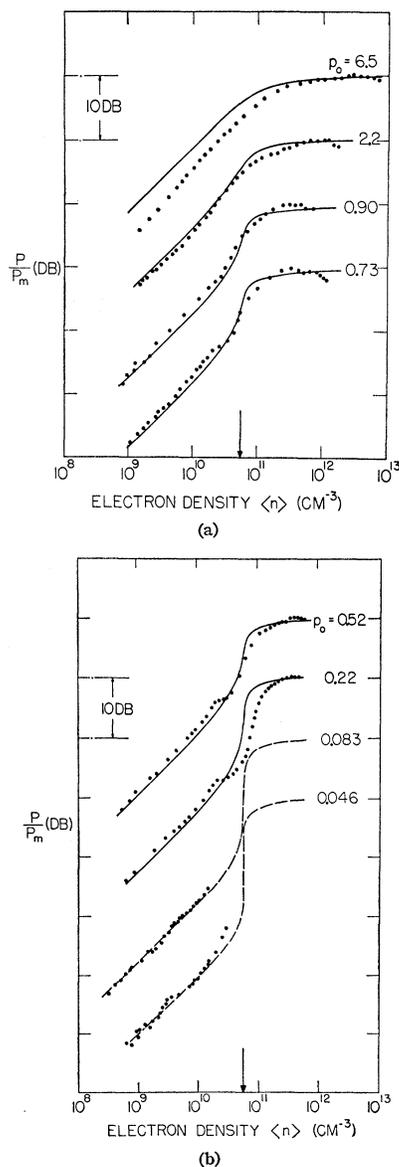


FIG. 3. (a) and (b) Normalized noise power as a function of electron density for various gas pressures,  $p_0$  (mm Hg). ●● measured; — geometrical-optics calculations. (For clarity, curves are displaced vertically 10 db relative to each other.)

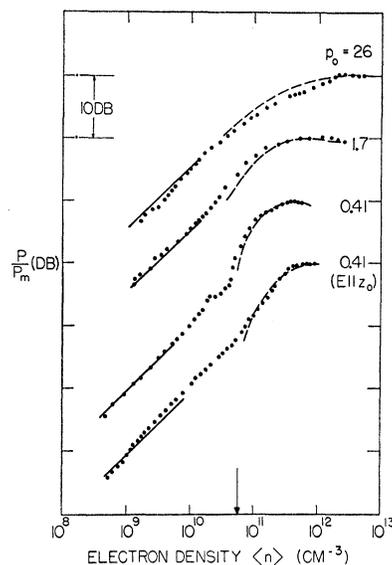


FIG. 4. Normalized noise power as a function of electron density. ●● measured; — exact theory for transparent limit; --- measured values of absorption coefficient  $A_e$ . (For clarity, curves are displaced vertically 10 db relative to each other.)

the contrary is stated, the measurements were made with the plasma cylinder oriented perpendicular to the electric field in the waveguide. Along the abscissa, the average electron density  $\langle n \rangle$ , defined as  $\langle n \rangle = (n_0/\pi R^2) \times \int_0^R J_0(2.405r/R) 2\pi r dr$ , is plotted. Along the ordinate, the measured noise power  $P$ , corrected for reflections and normalized to the noise emitted by an opaque plasma {i.e.,  $P/P_m = P/[kT(1-\Gamma)d\omega/2\pi]^{-1}$ }, is plotted. The scale is in decibels below the maximum power radiated at any given pressure. For convenience of presentation, the curves are displaced vertically 10 db with respect to each other. We note that the dependence of the noise power on electron density falls into the three regions discussed in the previous section. The linear increase of  $P/P_m$  with electron density for low values of  $\langle n \rangle$  falls into the region in which the plasma is essentially transparent; the nonlinear increase falls into the semiopaque region that occurs in the vicinity of  $(\omega_p/\omega)^2 = 1 - (\lambda/\lambda_e)^2$  (shown by a vertical arrow in Figs. 3 and 4); and the essentially density-independent region falls into the opaque limit  $A_e \rightarrow 1$ . As the pressure and collision frequency decrease, the transition from the transparent to the opaque limit becomes more pronounced. The radiation temperatures  $T$  that were measured in the limit as  $A_e \rightarrow 1$  are listed in Table I for various gas pressures  $p_0$ , and for the ratios  $\mathcal{E}/p_0$ , where  $\mathcal{E}$  is the axial electric field in the positive column. These measurements of  $T$  agree with Knol's<sup>7</sup> for values of  $p_0 R$  (mm Hg-cm) greater than 0.5, but they disagree for lower values.

The solid lines in Fig. 3(a) and (b) were calculated from the geometrical-optics approximation of Sec. 2 [see Fig. 1(a)]. In the transparent limit this calculation reduces to the exact solution outlined in Sec. 1. At the

TABLE I. Measured radiation temperatures as a function of gas pressure.

$p_0$ (mm Hg)	$\mathcal{E}/p_0$ [volts (cm mm Hg) <sup>-1</sup> ]	$T$ (10 <sup>4</sup> °K)
26	0.67	2.1
19	0.72	2.3
14	0.93	2.5
9.4	0.95	2.8
6.5	0.98	2.7
4.3	1.6	2.9
2.2	1.9	3.3
1.7	2.3	2.8
0.90	4.6	3.6
0.73	5.1	3.4
0.52	6.8	4.0
0.41	8.9	4.4
0.25	15	3.9
0.22	16	4.0
0.083	62	(4.8)
0.046	95	(9.2)

lowest gas pressures ( $p_0=0.046$  and  $0.083$  mm Hg) the opaque limit could not be reached with the available tube currents, and so this comparison could not be made. However, the radiation temperature was calculated in the transparent limit from the known noise power,  $\langle n \rangle$  and  $\nu$ . The temperatures thus obtained are shown in parentheses in Table I. The hatched curves in Fig. 3(b) are geometrical-optics calculations fitted to the experimental points.

In the semi-opaque region the agreement between experiment and geometrical-optics calculations is better than would be expected from the comparison between approximate and exact solutions discussed in Sec. 2 [see Fig. 1(e)]. This must be the result of the gradual radial variation of the electron density in the experiment, as compared with the sharp discontinuity assumed in the computations for the plasma slab. The slow variation of electron density leads to a great reduction of the reflection coefficient (confirmed by experiment) and hence to better agreement between geometrical-optics calculations and noise measurements.

Further comparison with noise observations was made by an actual measurement of the power attenuation coefficient  $A$  of Eq. (2). This attenuation coefficient is derived from measurements of the reflection coefficient  $\Gamma$  and from the transmission coefficient  $\tau$ . In terms of these quantities, the ratio  $P/P_m=A_e$  is given

by  $(1-\Gamma-\tau)/(1-\Gamma)$ . The comparison is shown in Fig. 4 by the dashed line. The two curves in Fig. 4 for  $p_0=0.41$  mm Hg indicate the differences that were observed with the plasma cylinder oriented parallel (the curve denoted by  $E_{\parallel z_0}$ ) and with it oriented perpendicular to the electric vector of the dominant waveguide mode. In the second orientation ac space-charge polarization of a homogeneous plasma cylinder manifests itself as a resonance-scattering phenomenon<sup>15,26</sup> in the vicinity of  $(\omega_p/\omega)^2=1-(\lambda/\lambda_c)^2$ , which is absent when the cylinder axis is parallel to the electric vector. The sharper increase of noise power toward the opaque limit observed when the cylinder is perpendicular to the electric field and similar observations of the variation of  $\Gamma$  with  $\langle n \rangle$ , support these results. However, in our inhomogeneous plasma cylinder the resolution is insufficient to identify these effects accurately.

Our determination of  $A_e$  is limited in range, chiefly by the precision of the attenuators; when  $A_e < 0.1$ , the error in the measurement can exceed  $\pm 90\%$ . Measurements of noise power and of  $A_e$  similar to those described were made in hydrogen, which has a collision frequency approximately twice that for helium. The agreement between experiment and theory was substantially the same as for helium.

Finally, we wish to point to the characteristic falling off of the noise power at very high electron densities. Similar observations have been reported elsewhere.<sup>6,7</sup> It has been suggested that this effect may be the result of electron-electron interactions that lead to a decrease in the electron temperature.<sup>27</sup> However, we have observed that the decrease in noise power is accompanied by a similar decrease in the absorption coefficient (see Fig. 4). Unless the collision frequency, and hence the conductivity and  $A_e$ , become temperature-sensitive in this range of electron densities, we must conclude that the radiation temperature is not diminished in our discharges.

#### ACKNOWLEDGMENT

Computations for Fig. 1 were made at the M.I.T. Computation Center, Cambridge, Massachusetts.

<sup>26</sup> T. R. Keiser and R. L. Closs, *Phil. Mag.* **43**, 1 (1952).

<sup>27</sup> J. H. Cahn, *Phys. Rev.* **75**, 293 (1949).