Scattering of Negative Mesons by Hydrogen at 130 and 152 Mev*

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The elastic scattering and total cross sections for negative π mesons on hydrogen have been measured at 130 and 152 Mev. At both energies, the number of electrons arising from the charge exchange scattering have been determined with a Čerenkov counter. At 152 Mev, recoil protons were counted, and were distinguished from π mesons by energy loss in a scintillator. The real part of the forward scattering amplitude has been determined to be 0.243 ± 0.015 and 0.218 ± 0.016, in units of the meson Compton wavelength, at 130 and 152 Mev, respectively. These values agree within limits of statistical error with the predictions from dispersion relations.

I. INTRODUCTION

In the last few years there has been renewed interest in the results of pion scattering on hydrogen because of the possibility of comparison with dispersion relations. The dispersion relations as considered by Goldberger, Miyazawa, and Oehme[1] depend on a small set of assumptions of fundamental importance; a reliable test is therefore of great interest. After early tests of the relations showed reasonable agreement,[2] Puppi and Stanghellini were the first to point out a possible discrepancy.[3] Although the π^+ data were in good agreement, these authors exhibited a disagreement between values for the real part of the forward scattering amplitude for the process $\pi^-+p\rightarrow\pi^-+p$. The value of the real part of this amplitude calculated with the dispersion relations disagreed with the value obtained experimentally. This disagreement appeared for scattering of negative mesons above 100 Mev and was statistically most significant for the Carnegie Institute of Technology experiments at 150 and 170 Mev.[4]

Since then the discrepancy has been examined by several authors[5] and it was shown that the discrepancy was reduced if more recent values for the total cross section were used in the calculations with the dispersion relations; also, further uncertainties in the data were examined. To complement this theoretical investigation more accurate experimental values of the total cross section and differential cross sections in this energy range were necessary. This experiment was undertaken to obtain new measurements of these cross sections at 130 and 150 Mev.

One of the principal experimental uncertainties in this energy range had been the number of electrons which arise from the charge exchange process $\pi^-+p\rightarrow\pi^0+n$ through the decay of the π^0. The π^0 may decay into two gamma rays, or alternatively into a gamma ray and an electron pair, or directly into two pairs. The gamma rays may generate electrons in the hydrogen target or its surroundings by Compton scattering or pair production. To obtain an accurate measure of the number of negative mesons coming from the target, the electron contribution must therefore be subtracted from the total number of charged particles counted.

In this experiment, as in the Liverpool experiment at 98 Mev,[6] Čerenkov counters were used to separate electrons from π^- mesons. The separation with this technique becomes increasingly more difficult at higher energies, since the velocity of the pions exceeds the Čerenkov threshold of most convenient media. At high energies an independent measurement of the $\pi^-+p\rightarrow\pi^-+p$ cross section becomes possible by detecting the recoil protons. For backward-scattered high-energy mesons, the recoil protons will have sufficient energy to emerge from the target. In the experiment at 152 Mev it was possible to separate these protons from mesons by observing their larger light output in a scintillator.

The experiments at 130 and 152 Mev will be discussed together in the following sections. Section II describes the experimental arrangement for total cross section and differential cross section measurements. Section III outlines the analysis applied to the data and gives estimates of the uncertainties. In Sec. IV the results of the experiment are summarized and the calculation of the scattering amplitudes is presented. Finally, in Sec. V, these results are compared with other experiments near this energy and the real parts of the forward scattering amplitudes are compared with calculations from the dispersion relations.

II. EXPERIMENTAL ARRANGEMENT

A. Meson Beams

The negative meson beams employed in this experiment were obtained by bombarding a beryllium target

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[1] Goldberger, Miyazawa, and Oehme, Phys. Rev. 99, 986 (1955).
[2] Anderson, Davidon, and Kruse, Phys. Rev. 100, 339 (1955).
[3] G. Puppi and A. Stanghellini, Nuovo cimento 5, 1256 (1957).
[4] Ashkin, Blaser, Feiner, and Stern, Phys. Rev. 101, 1149 (1956).
[5] See, e.g., H. J. Schnitzer and G. Salzman, Phys. Rev. 113, 1153 (1959) and Phys. Rev. 112, 1802 (1959). These authors also refer to the previous literature. H. P. Noyes (private communication).
[6] Edwards, Frank, and Holt (private communication).

with the circulating proton beam of the Chicago cyclotron. The mesons produced emerged from the cyclotron and travelled to the experimental area inside a vacuum pipe which was directly connected to the cyclotron vacuum. They were focused on the way with a double quadrupole strong focus magnet. In the experimental area they were deflected by a bending magnet which gave further horizontal and vertical focusing. Beams of 130- and 152-Mev negative mesons were obtained in this way. The energy for each beam was determined by range curves in copper and aluminum. For 130 Mev an additional range curve in graphite was obtained. The negative μ-meson and electron fractions in the beam were estimated from the range curves.

B. Target

The measurements of all the cross sections were performed with a common liquid hydrogen target. The target consisted of a Mylar cylinder[7] 9.44 cm in diameter surrounded by a $\frac{1}{2}$-mil aluminum thermal radiation shield, the whole being enclosed in a high vacuum container. The target cylinder could be filled with liquid hydrogen, or emptied by displacing the liquid into a reservoir located above the target. Small resistors near the top and bottom of the target cylinder gave reliable indication of the liquid level in the target.

Two vacuum enclosures for the target were used in this experiment. For the total cross section measurement it is desirable to be able to move counters close to the target; a small box 9 in. square with Mylar entrance window and large aluminum exit window was employed for these measurements. For measurements of the differential scattering cross section it is desirable to have a minimum of material in the beam, and access at a wide range of scattering angles. A can of 20-in. diameter was constructed for these measurements. The beam entered through a 3-mil Mylar window. The scattered particles could be observed after leaving through a 12-mil aluminum window. The aluminum window extended all the way around the vacuum can with the exception of a small frame for the Mylar entrance window, and allowed observation of scattered particles at any angle back to 155°.

The whole target assembly was mounted on a post at the center of a scattering table; by removing the Mylar entrance window and aluminum thermal radiation shield the target could be optically aligned in the meson beam path.

C. Total Cross-Section Counter Arrangement

A standard arrangement for transmission measurements was employed for the total cross sections. The incident beam was defined by two scintillation counters, each 2 in.×2 in. The transmission counters were mounted behind the total cross-section vacuum box. The transmission defining counter was $4\frac{1}{2}$ in.×$4\frac{1}{2}$ in. and its distance from the center of the target was varied to check the corrections for geometry.

[7] V. O. Nicolai, Rev. Sci. Instr. **26**, 1203 (1955).

D. Differential Cross-Section Counter Arrangement

1. Monitor

The counters for the differential cross section were arranged to minimize the counting rate with the target empty. Only one 2 in.×2 in. counter was placed in the incident beam, approximately 32 in. from the center of the target. The beam was further defined by a set of anticoincidence counters immediately in front of the Mylar entrance window. The anticoincidence counters surrounded a 2 in.×2 in. aperture and greatly reduced all the rates taken with target empty. The elimination of a second defining counter with its associated scattering made it possible to measure at small forward angles while maintaining good hydrogen-in to hydrogen-out counting ratios.

The incident beam at both energies was roughly 10^4 mesons per second. This flux was monitored by counting the number of particles traversing the incident beam counter which did not strike the anticoincidence counters. Because of dead times of roughly 10^{-7} second in the counting equipment, the high instantaneous flux during the beam bursts lead to counting losses of roughly 4%. During the 130-Mev experiment this loss rate was checked continuously by counting a small fraction of the incident beam scattered by the 2 in.×2 in. beam defining counter. In the 152-Mev experiment the loss rate was checked at regular intervals. At all times the loss rate was a linear function of the beam intensity.

2. Scattered Mesons

The particles scattered by the hydrogen were detected by two scintillation counter telescopes mounted just outside the aluminum window of the scattering chamber. The telescopes consisted of two scintillation counters and a Čerenkov counter between them. The first scintillation counter was 4 in.×4 in., the Čerenkov cell was $4\frac{1}{2}$ in.×$4\frac{1}{2}$ in. and 2 in. deep, and the last scintillation counter, which defined the solid angle, was 3 in.×3 in. The counters in front of the defining counter were sufficiently large so that in- and out-scattering cancelled. A typical arrangement is shown in Fig. 1.

The Čerenkov counter was used to determine the number of electrons coming through the scintillation counters. In the 130-Mev experiment the Čerenkov medium was water, which gave only small pulses for π mesons and larger pulses for the electrons. In the 152-Mev experiment fluorochemical *FC* 75[8] was used.

[8] Manufactured by Minnesota Mining and Manufacturing Company with refractive index of nearly 1.28.

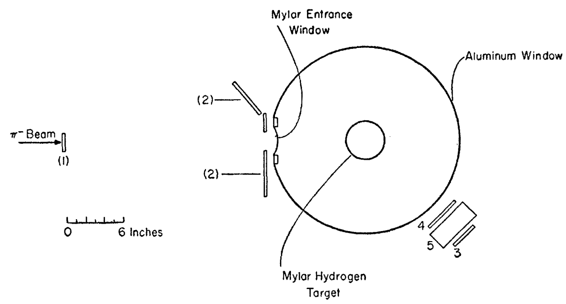

FIG. 1. Typical arrangement of apparatus for differential cross sections. Counter 1: beam-defining counter 2 in.×2 in. Counter 2: beam-defining anticoincidence counters: 2 in.×2 in. aperture. Counter 3: scattered particle defining counter 3 in.×3 in. Counter 4: scattered particle counter 4 in.×4 in. Counter 5: Čerenkov counter $4\frac{1}{2}$ in.×$4\frac{1}{2}$ in.×2 in.

The Čerenkov counters were calibrated with electrons produced in a lead converter by gamma rays from the charge exchange process $\pi^-+p\rightarrow\pi^0+n$, and with cosmic rays. The pulses from the Čerenkov counter were passed through a gate opened whenever a charged particle traversed the beam defining counter and both scintillation counters in the telescope. Then they were pulse-height analyzed. In the 130-Mev experiment the analyzer developed by E. L. Garwin and A. Penfold of this laboratory was used. We are grateful to them for the loan of the equipment and instruction in its use.

3. *Scattered Protons*

In the 152-Mev experiment it was possible to count protons recoiling from backward scattered mesons. The protons were separated from the mesons by the difference in energy loss in a scintillator. The proton telescope consisted of a defining counter 2 in.×3 in. followed by a second scintillator 4 in.×4 in. The coincidence between these and the beam defining counters opened a gate, which allowed slow pulses from the 2 in.×3 in. defining counter to be analyzed by the pulse-height analyzer. A clean separation of meson and proton pulse heights was achieved, as shown in Fig. 2.

E. Beam Distributions

The beam distribution at the target position was checked at various times during the run by small sampling counters. This permitted an accurate estimate of the effective number of scattering centers, taking into account the variation of the target thickness across the beam. The beam distribution was also measured behind the target during the transmission measurement to allow an estimate of the scattering losses outside the transmission counters. An automatic beam scanning device behind the target was used at various intervals during the experiments to check on the stability of the beam location.

III. DATA ANALYSIS

A. Total Cross-Section Measurement

The total cross section was determined in the usual manner from the transmission measurement. From an estimate of the number of μ mesons and electrons in the beam and from the observed rates with the target empty the effective number of π mesons striking the target per unit monitor was determined. The additional attenuation observed with hydrogen in the target then yielded an effective total cross section for the π mesons.

1. *Corrections*

There are a number of corrections required to obtain the true nuclear cross section. The Coulomb scattering losses must be estimated. A preliminary approximation made use of the phase shifts of Anderson and Metropolis[9] to obtain the Coulomb amplitudes, according to the prescription of Solmitz.[10] The resulting scattering distribution was folded into the observed distribution of mesons behind the empty target to obtain the number lost due to Coulomb scattering. The number of mesons and recoil protons which are scattered into the transmission counter must be calculated; this effect is not insignificant due to the close geometry used.

Further small corrections were necessary to take into account the recoil neutrons and electrons produced by the charge exchange reaction. All of the above corrections depend on the spacing between the target and the transmission counter; this spacing was varied to obtain a check on the consistency of the above corrections. The cross sections were in agreement within statistical accuracy at each of the three spacings used. The final correction was a subtraction of the cross section for the reaction $\pi^-+p\rightarrow\gamma+n$ to get a total cross section due to purely nuclear interactions.

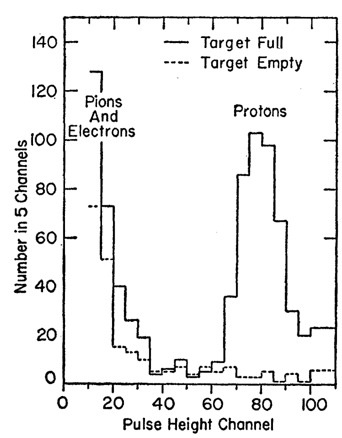

FIG. 2. Pulse-height analyzer spectrum for charged particles at 30°.

[9] H. L. Anderson and N. Metropolis, *Proceedings of the Sixth Annual Rochester Conference on High-Energy Nuclear Physics, 1956* (Interscience Publishers, Inc., New York, 1956), Sec. I, p. 20.

[10] F. T. Solmitz, Phys. Rev. 94, 1799 (1954).

2. *Uncertainties*

We have classified the uncertainties and errors of the total and differential cross sections into two groups; those which are common to all measurements, and hence affect both total and differential measurements, and those which are peculiar to each measurement and may be considered statistically independent for the purpose of this analysis.

The common uncertainties in the total cross-section measurement arise from possible error in estimating the effective number of scatterers, N, in the target, and from the uncertainty in the number of mesons striking the target per monitor. The largest contribution in the latter uncertainty is from the uncertainty in the number of μ mesons and electrons in the beam.

The uncertainties peculiar to the total cross-section measurement are due to the possibility of error in the various corrections applied, and from the statistical errors in counting. In Table V the total cross sections are presented, with the common and independent errors associated with them.

B. Differential Cross Sections: Measurement of Scattered Pions

The differential cross sections were obtained from the measurements with the pion telescopes by subtracting the number of electrons observed from the total number of charged particles passing through the scintillator telescope. The scintillation counter efficiencies were checked periodically throughout the run and were established to be better than 99.5%.

1. *Correction for Accidentals*

Determination of the rate of accidental coincidences was complicated by the anticoincidence defining system used. The usual method of delaying the counter telescope relative to the monitoring counters was impractical, since the anticoincidence system would then not shield the telescope from the direct beam, and the counting rate would have no simple relationship to the usual accidental rate. Hence, for this purpose, the rate dependence of the cross sections was checked at several scattering angles. The net hydrogen rate was found to be independent of beam rate; so no corrections for accidentals were necessary.

TABLE I. Typical experimental numbers for π^--scattering measurement at 152 Mev.

Lab angle	Charged particles[a] Target full	Charged particles[a] Target empty	Electrons[b] Target full	Electrons[b] Target empty	Net π mesons
31.7°	928	420	84	16	429
60°	199	29	29	4	145
90°	140	18	21	2	104
110°	164	20.5	20	1.5	122
135°	203	31	34	4	142

[a] Total charged particles for unit monitor.
[b] Total electron number estimated from pulse-height analysis, for unit monitor.

TABLE II. Common uncertainties (ϵ) and independent uncertainties (Δ) in π^--scattering measurements.

Uncertainty cause	ϵ 130 Mev	ϵ 152 Mev
$N_{\rm inc}$	3%	3%
N	1.5%	1%
e^- subtraction	15% of e^-	10% of e^-
Light pipes	0	1%
π^- absorption	1%	1%

Uncertainty cause	Δ 130 Mev 30°	75°	120°	152 Mev 30°	90°	120°
N_π	5%	5.9%	8%	3%	8%	7%
$\Delta\Omega$ (radius)	1.5%	1.5%	1.5%	1%	1%	1%
Angle mis-set	2%	1%	0.5%	1%	0	0.5%
g		Negligible			Negligible	

2. *Corrections for Čerenkov Effect in Light Pipes*

Extra counts may be observed if a charged particle delivers enough Čerenkov radiation in the Lucite light pipe of the defining counter to trigger the coincidence circuits. This effect increases the active area of the defining counter beyond the dimensions of the scintillator. The magnitude was checked by substituting a matched light pipe without a scintillator for the usual defining scintillator and light pipe. The effect was negligible at 130 Mev; however, with slightly changed electronics, there was a small contribution at 152 Mev which was subtracted from that data.

3. *Electron Subtraction*

From the Čerenkov counter calibration mentioned in Sec. II-D, a cutoff channel was established for the pulse-height spectra at each scattering angle. Above this channel, the number of observed counts were taken to be electrons. A correction was made for the fraction lost below this channel. The required pulse was made large enough so that only a negligible number of pions were included. The net number of electrons was subtracted from the net number of charged particles to get the number of pions counted by the telescope.

4. *Nuclear Absorption of Pions*

The net scattered pion rate was corrected for the nuclear absorption taking place when the pions left the target and traversed the first scintillation counter and the Čerenkov counter. These absorptions were determined separately by measuring the attenuation in the direct beam, and were in good agreement with expected nuclear cross sections. The number of pions counted by the telescope multiplied by the correction for absorption gives the number of pions N_π which are scattered toward the defining counter.

TABLE III. Center-of-mass cross sections at 130 Mev, in mb/sterad.

C.m. angle (degrees)	$(d\sigma/d\Omega)^a$	Δ^b	ϵ^c	$(d\sigma/d\Omega)^d$
39.2	1.94	0.115	0.085	1.84
55.4	1.22	0.067	0.054	1.28
72.5	0.850	0.045	0.037	0.84
88.7	0.592	0.036	0.026	0.59
103.9	0.604	0.085	0.027	0.53
118.2	0.607	0.045	0.027	0.61
131.6	0.698	0.059	0.031	0.77
144.4	1.04	0.061	0.046	0.96
154.0	1.02	0.083	0.045	1.11
157.7	1.20	0.089	0.053	1.15

[a] Differential cross sections, experimental.
[b] Δ—independent uncertainties.
[c] ε—common uncertainties.
[d] Differential cross section from least-squares fit of Sec. IV, B.

5. *Solid Angle Determination; ΔΩ*

The radius or distance of the defining counter from the center of the target support post was measured for each individual angular setting.

6. *Geometrical Corrections*

It was necessary to apply further corrections to the observed angular distribution to correct for the finite size of the target and counter. The analysis was the same as has been described previously[11]; corrections were included from the first and second derivatives of the cross section with respect to scattering angle. We use

$$g=[1+\alpha+\beta(\sigma'/\sigma)+\tfrac{1}{2}\gamma(\sigma''/\sigma)] \qquad (1)$$

to represent the geometrical correction factor.

TABLE IV. Center-of-mass cross sections at 152 Mev, in mb/sterad.

C.m. angle (degrees)	$(d\sigma/d\Omega)^a$	Δ^b	ϵ^c	$(d\sigma/d\Omega)^d$
32	3.07	0.139	0.127	2.96
37.5	2.59	0.126	0.103	2.66
40	2.61	0.049	0.104	2.52
47	2.24	0.059	0.089	2.18
56	1.65	0.061	0.066	1.78
65	1.39	0.066	0.056	1.42
73.5	1.06	0.035	0.042	1.15
89.9	0.84	0.057	0.036	0.84
105	0.83	0.071	0.040	0.85
108[e]	0.88	0.064	0.032	0.88
111[e]	0.92	0.047	0.033	0.92
116[e]	1.16	0.063	0.042	1.01
118[e]	1.12	0.024	0.040	1.05
124	1.15	0.040	0.051	1.20
130[e]	1.34	0.033	0.048	1.36
132	1.34	0.071	0.060	1.42
145	1.67	0.092	0.075	1.79
151	1.80	0.089	0.078	1.95
158	2.09	0.149	0.091	2.12

[a] Differential cross sections, experimental.
[b] Δ—independent uncertainties.
[c] ε—common uncertainties.
[d] Differential cross section from least-squares fit of Sec. IV, B.
[e] Recoil proton measurements.

[11] Anderson, Davidon, Glicksman, and Kruse, Phys. Rev. **100**, 279 (1955).

7. *Effective Number of Incident Pions; N_{inc}*

Absolute cross sections were obtained by converting the monitor counts into mesons effectively incident on the target, by taking into account the monitor normalization and beam dependence, correcting for μ and e contamination of the beam, and for the attenuation of the pions in the hydrogen before reaching the center of the target.

8. *Calculation of Laboratory Cross Section*

After determining N, the number of effective scattering centers in the target, the laboratory cross section is given by:

$$\frac{d\sigma}{d\Omega}=\frac{N_\pi}{\Delta\Omega N_{inc}Ng}. \qquad (2)$$

Some typical values for the quantities described above are given in Table I.

9. *Uncertainties in Differential Cross Section*

The common and independent uncertainties are again grouped separately. The common uncertainties include: the number of incident mesons N_{inc}, the number of scatterers N, the possible error in the electron subtraction criterion, error in determining the Čerenkov effect in the light pipes, and in correcting for absorption of the pions in the counter telescope.

TABLE V. Total cross sections, in mb.

Energy (Mev)	σ_{tot}^a	ϵ^b	Δ^c	σ_{tot}^d
130	42.7	1.24	0.77	42.6
152	60.0	2.0	1.2	59.6

[a] Experimental value.
[b] ε—common uncertainties.
[c] Δ—independent uncertainties.
[d] Least-squares fit of Sec. IV, B.

The uncertainties which are independent at each angle include the statistical error in N_π, the finite-size target corrections g, possible errors in ΔΩ due to errors in radius measurement, and possible error in angle setting causing an apparent shift of cross section. Some typical values for these uncertainties are presented in Table II, illustrating the relative magnitudes of these errors.

C. Differential Cross Sections; Recoil Proton Method (152 Mev)

The number of protons intercepted by the proton-defining counter was determined by selecting a cutoff channel for the pulse-height analyzer, and counting all events above that channel as protons. The net hydrogen minus empty rate as determined by this method was quite independent of the choice of cutoff channel, as can be seen from the spectrum shown in Fig. 2.

1. *Corrections*

In the proton analysis corrections corresponding to those described above for the π mesons were made. Two corrections, the electron subtraction and the Čerenkov effect in the light pipes, do not apply for protons. One additional uncertainty has been introduced, the possibility of losing some of the protons below the cutoff channel. The latter, however, appears very small, as indicated by Fig. 2.

IV. RESULTS

A. Center-of-Mass Cross Sections

The results of the foregoing analyses are summarized in Table III for the 130-Mev data, and Table VI for the 152-Mev data. The common and independent uncertainties are given, as well as the cross sections obtained from the best fits to the data, as described below.

TABLE VI. Results of least-squares computation at 130 Mev.

	Scattering amplitudes[a]				
	A	*B*	*C*	*D*	*E*
	0.112	0.128	0.977	0.865	0.737
	Error matrix[b] for scattering amplitudes[a] $=10^4(G^{-1})_{ij}$				
	A	*B*	*C*	*D*	*E*
A	9.914	−2.210	9.642	−0.432	−3.238
B		3.334	−3.811	−0.155	0.956
C			16.844	1.191	−5.110
D				3.392	−1.071
E					6.792
	$\chi^2=3.62$[c]		Goodness of fit Degrees of freedom = 6		

[a] The units of *A*, *B*, *C*, *D*, and *E* are $(10^{-27}\ \text{cm}^2)^{\frac{1}{2}}$, so that $d\sigma/d\Omega$ of formula (3) is in units of millibarns per steradian.
[b] See reference 11.
[c] See formula (6).

B. Determination of the Scattering Parameters

In order to determine the forward scattering amplitudes and other scattering parameters with the least dependence on other experiments, the following scheme was used to obtain values for the scattering amplitudes.

If we include only *s*- and *p*-wave scattering, the cross sections may be written as:

$$d\sigma/d\Omega=|f(\theta)+A+iB+(C+iD)\cos\theta|^2+E^2\sin^2\theta, \quad (3)$$

where $f(\theta)$ is the Coulomb scattering amplitude and was evaluated following Solmitz,[10] using $f(\theta)=(a+b\cos\theta)/(1-\cos\theta)$.

The total nuclear cross section may be expressed with the aid of the optical theorem by

$$\sigma_{\text{tot}}=(4\pi/k)(B+D). \quad (4)$$

TABLE VII. Results of least-squares computation at 152 Mev.

	Scattering amplitudes[a]				
	A	*B*	*C*	*D*	*E*
	0.0230	0.146	0.953	0.377	0.903
	Error matrix[b] for scattering amplitudes[a] $=10^4(G^{-1})_{ij}$				
	A	*B*	*C*	*D*	*E*
A	12.360	−7.010	10.466	−4.574	0.332
B		5.176	−7.514	2.892	0.320
C			15.560	−7.020	−1.691
D				8.081	−1.133
E					2.688
	$\chi^2=18.1$[c]		Goodness of fit Degrees of freedom = 15		

[a] The units of *A*, *B*, *C*, *D*, and *E* are $(10^{-27}\ \text{cm}^2)^{\frac{1}{2}}$, so that $d\sigma/d\Omega$ of formula (3) is in units of millibarns per steradian.
[b] See reference 11.
[c] See formula (6).

In the usual least-squares calculation, the quantity

$$\chi^2=\sum_i \frac{(O_i-C_i)^2}{\Delta_i^2} \quad (5)$$

is minimized,[11] where O_i is the observed experimental cross section and C_i is the calculated cross section. The quantity Δ_i represents the independent uncertainty at each observed point.

This expression may be generalized by writing

$$\chi^2=\sum_{i,j} H_{ij}(O_i-C_i)(O_j-C_j), \quad (6)$$

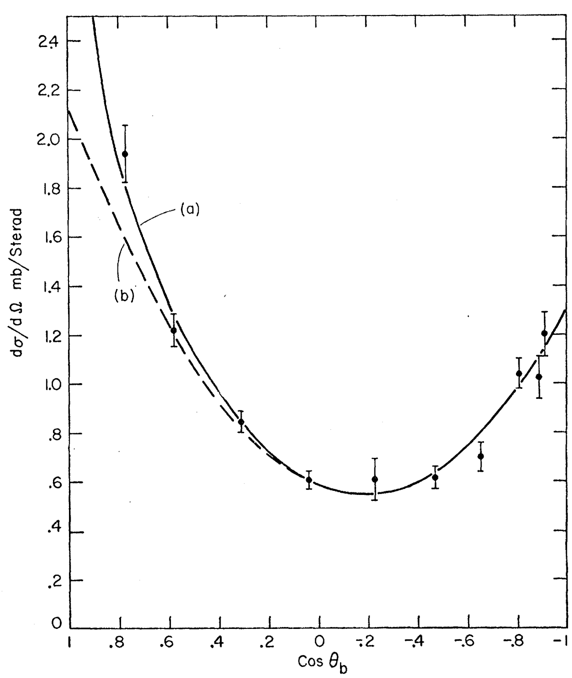

FIG. 3. Center-of-mass differential cross section at 130 Mev. Only independent errors Δ are shown. Curve *a*, least-squares fit to differential and total cross section. Curve *b*, least-squares fit with Coulomb contributions subtracted.

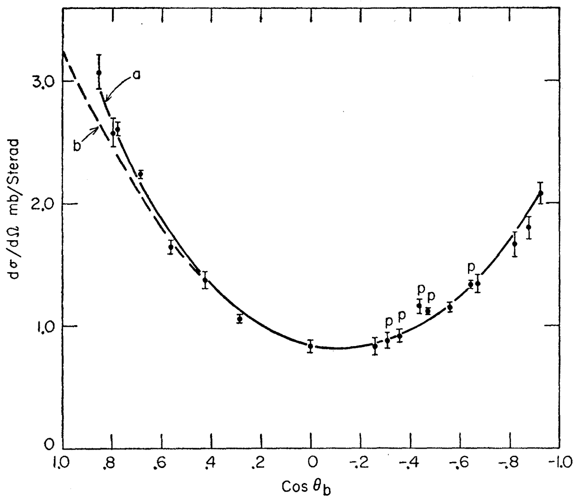

FIG. 4. Center-of-mass differential cross section at 152 Mev. Only independent errors Δ are shown. Curve *a*, least-squares fit to differential and total cross section. Curve *b*, least-squares fit with Coulomb contributions subtracted. Points *p* indicate data obtained by measuring recoil protons.

where, in the above case, the off-diagonal elements of H were assumed to vanish, corresponding to the statement that there are no common errors. With the possibility of common errors, these off-diagonal terms will in general not vanish, and a proper generalization is found by setting[12]

$$H_{ij}=\frac{1}{\Delta_i\Delta_j}\left(\delta_{ij}-\frac{\epsilon_i\epsilon_j/\Delta_i\Delta_j}{1+\sum_k(\epsilon_k^2/\Delta_k^2)}\right), \tag{7}$$

where the ϵ_i are the common errors and the Δ_i are the independent errors;

$$O_i=O_i\pm\Delta_i\pm\epsilon_i. \tag{8}$$

In our analysis we have included the off-diagonal terms and thereby the effect of the common errors.

Minimization of χ^2 was accomplished by varying A, B, C, D, and E.[13] The resulting best values and the corresponding error matrices are given in Tables VI and VII. The best-fit cross sections have been compared with the experimental data in Tables III and IV. The cross sections together with least squares fits have been plotted in Figs. 3 and 4.

V. CONCLUSIONS

The total cross sections of the present experiment can be compared to the results of previous experiments.[4,6,14–18] As can be seen from Fig. 5, our total cross sections fit reasonably well with our measurements. In this connection it should be noted that the Carnegie measurements at 150 and 170 Mev[4] are somewhat below the general trend. The coefficients in the angular distribution $d\sigma/d\Omega=a_-+b_-\cos\theta+c_-\cos^2\theta$ also agree reasonably well with other experiments. Our slightly low value for a_- at 152 Mev is probably associated with the electron subtraction, which is somewhat larger than that estimated previously at 150 Mev.

Our results for the real part of the forward scattering

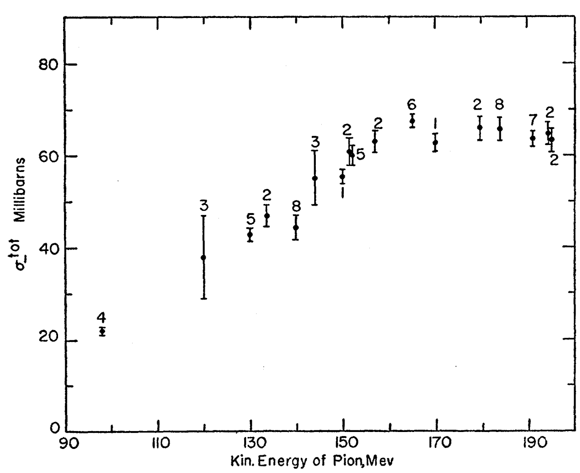

FIG. 5. Total cross section for negative mesons on hydrogen. 1: Ashkin *et al.*, reference 4. 2: Ashkin *et al.*, reference 14. 3: Anderson *et al.*, reference 15. 4: Edwards *et al.*, reference 6. 5: Present data. 6: Anderson and Glicksman, reference 16. 7: Glicksman, reference 17. 8: Ignatenko *et al.*, reference 18.

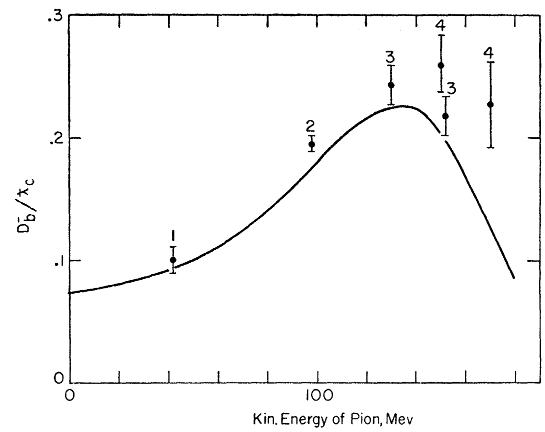

FIG. 6. Recent values for real part of forward scattering amplitude in center-of-mass system in units of Compton wavelength. 1: Barnes *et al.*, reference 19. 2: Edwards *et al.*, reference 6. 3: Present experiment. 4: Ashkin *et al.*, reference 4. Curve is calculated from dispersion relations by Schnitzer and Saltzmann in reference 5.

[12] We are indebted to W. C. Davidon for suggesting this form.

[13] The computer calculations were performed at the Argonne National Laboratory and we would like to thank W. C. Miller and K. Hillstrom for their assistance.

[14] Ashkin, Blaser, Feiner, Gorman and Stern, Phys. Rev. **96**, 1104 (1954).

[15] Anderson, Fermi, Martin, and Nagle, Phys. Rev. **91**, 155 (1953).

[16] H. L. Anderson and M. Glicksman, Phys. Rev. **100**, 268 (1955).

[17] M. Glicksman, Phys. Rev. **95**, 1045 (1954).

[18] Ignatenko, Mukhin, Ozerov, and Pontecorvo, Doklady Akad. Nauk. S.S.S.R. **103**, 45 (1955).

amplitude may be compared with other recent experimental determinations.[4,6,19] A curve calculated from the dispersion relations by Salzman and Schnitzer[5] is chosen for comparison in Fig. 6. It can be seen that the values obtained in this experiment are in statistical agreement with the values calculated from the dispersion relations. The change between the present experiment and the earlier results of the Carnegie group[4] is due primarily to the increase in total cross section.

[19] Barnes, Rose, Giacomelli, Ring, and Miyake (private communication).

The largest single uncertainty in this experiment is the fraction of μ mesons and electrons in the incident beam. An improved technique for determining this fraction would clearly be desirable for future experiments in this energy region.

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Scattering of μ Mesons from Lead Nuclei*

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The angular distribution of 23-Mev μ^- mesons scattered by lead nuclei has been measured by using a counter arrangement and also by using a propane bubble chamber. The results agree, to the accuracy of the experiments, with the distribution predicted by the ordinary Coulomb interaction of the μ meson and the lead nucleus.

INTRODUCTION

FOR many years the interactions of the μ meson with matter have been the subject of experimental investigations.

Measurements[1] on μ-mesonic atoms, on the production of stars by energetic muons, and on the creation of muon pairs by gamma rays have revealed an interaction between the muon and the electromagnetic field that is compatible with the assumption that the muon behaves in the same way as an electron except that it has a larger mass. More recent measurements have established that the magnetic dipole moment has the expected value[2] and that there is no evidence for a parity violating interaction of the electric dipole type.[3] The strength of the nuclear interaction has been established by a study of the lifetimes of negative muons in numerous materials,[1] and the result is that this interaction is of the "weak" type. All of these results lead to the conclusion that the μ meson has interactions that are well understood.

One series of experiments on the μ meson, however, has not uniformly given results in accord with the interactions mentioned above. These are the experiments on the elastic scattering of muons from nuclei. Several experiments[4] have confirmed the simple interpretation of the scattering in terms of the Coulomb interaction, but several other investigations[5] have revealed an excess of large-angle scatterings over the number expected.

All of these experiments have been carried out using cosmic-ray μ mesons. The beams obtainable from this source are very weak and polychromatic. The experiment described in this paper[6] utilized a μ-meson beam from an accelerator, the 450-Mev synchrocyclotron at Carnegie Institute of Technology. The beam was more intense and monoenergetic than cosmic rays can provide, but the energy was much lower, about 23 Mev. At such a low energy the scattered μ mesons detected in the experiment did not appreciably penetrate the nucleus. Although it would seem most probable that an anomaloy, if it does indeed exist, would arise from an interaction of a relatively short range, never-

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† Now at Princeton University, Princeton, New Jersey. This work submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy at Carnegie Institute of Technology.
[1] μ-meson physics has been summarized recently by J. Rainwater, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Palo Alto, 1957), Vol. 7, p. 1.
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[5] Lloyd, Rössle, and Wolfendale, Proc. Phys. Soc. (London) **A70**, 421 (1957); this article summarizes most of the early work.
[6] A preliminary report on this work was published [B. Chidley *et al.* Can. J. Phys. **36**, 801 (1958)].