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Occurrence of Vavilov-Čerenkov Radiation in a High-Temperature Plasma

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Akhiezer and Sitenko investigated the energy loss of a charged particle moving through a plasma at a velocity considerably lower than the mean thermal velocity of the electrons in the plasma and determined the component of the stopping power due to the transverse electric field produced by the plasma and acting upon the particle. The presence of such a component may in some instances be associated with the occurrence of the Vavilov-Čerenkov radiation. It is shown, however, that in this particular case the field surrounding the particle decreases very rapidly with the distance from the particle and no radiation takes place.

I. INTRODUCTION

THIS investigation is concerned with the possible occurrence of Vavilov-Čerenkov radiation that could be emitted by a particle moving through a high-temperature plasma at a velocity lower than the mean thermal velocity of plasma electrons. It is well known that at a sufficiently low temperature at which the dielectric constant of the plasma may be represented as $\epsilon=1-\omega_0^2/\omega^2$, (where ω_0 is the Langmuir frequency), there is no Vavilov-Čerenkov radiation since the phase velocity of electromagnetic waves always exceeds the velocity of light. We shall now ascertain whether or not this radiation appears at high temperatures. The problem is of definite interest since it has been found that the field exerted by the plasma on a slow-moving particle has a transverse component and the occurrence of such a transverse component may in some instances be associated with the presence of Vavilov-Čerenkov radiation. Subsequently it shall be shown by applying a suitable criterion that the temperature effect does not cause the emission of Vavilov-Čerenkov radiations by slow-moving particles. We are particularly interested in slow particles since the presence of the transverse component has been noted for slow particles only. For fast particles such a transverse component has been found nonexistent and is apparently negligible.[1]

The occurrence of Vavilov-Čerenkov radiation for slow particles may appear to be paradoxical since in conventional dispersive media this radiation is usually emitted by fast particles. It should be noted in that connection that for particles moving through plasma this radiation may in some instances occur at very low velocities, considerably below the mean thermal velocity of plasma electrons. This, in fact, has been found by Kolomenski[2] in a plasma in the presence of a sufficiently strong magnetic field.

II. OCCURRENCE OF VAVILOV-ČERENKOV RADIATION

The stopping power of a particle having charge q and moving in a material medium along the axis z can be represented as

$$-dW/dz=-qE_z^l-qE_z^t, \tag{1}$$

where E_z^l and E_z^t are components along the z axis of the longitudinal and of the transverse electric field produced by the medium and acting on the particle.

In the most general case the term qE_z^t in the expression (1) has two components. One of these, designated as $(qE_z^t)_1$, represents the energy due to the transverse electric field that is absorbed in the immediate neighborhood of the particle track. The other component,

* Operated by Union Carbide Corporation for the U. S. Atomic Energy Commission.

[1] A. I. Akhiezer and A. G. Sitenko, Zhur. Eksptl. i Teoret. Fiz. **23**, 161 (1952). This paper appeared in English translation as British Atomic Energy Project Report AERE Lib/Trans. 759 (unpublished).

[2] A. A. Kolomenski, Doklady Akad. Nauk S.S.S.R. **106**, 982 (1956).

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designated as $(qE_z{}^t)_{\text{V-C}}$, represents the energy that is radiated by the particle in the form of Vavilov-Čerenkov radiation. Occasionally in the literature the term $qE_z{}^t$ is identified with the Vavilov-Čerenkov radiation.[3] Such an interpretation, however, is only valid in a restrictive sense in those cases in which the nonradiative loss due to the transverse component is small when compared to the radiative loss. On the other hand, in those cases in which the radiative loss is negligible, the term $qE_z{}^t$ represents the energy loss absorbed by the medium.

A charged particle moving in a vacuum carries with it fields of "virtual" longitudinal and transverse photons that decrease exponentially at large distances. When this particle is moving in a material medium even at a relatively low velocity, these virtual longitudinal and transverse photons contribute separately to the total stopping power[4] although the contribution due to the "transverse" photons may be very small. Thus the existence of the transverse component in the expression for the stopping power does not mean necessarily that the Vavilov-Čerenkov radiation takes place. On the other hand, the Vavilov-Čerenkov radiation is always accompanied by an energy loss due to the transverse component of the electric field.

It has been shown by Akhiezer and Sitenko[1] that for a particle moving through a plasma with a velocity $v \gg v_0$ [where $v_0=(\kappa T/m)^{\frac{1}{2}}$ is of the order of magnitude of the mean thermal velocity of the plasma having temperature T and κ is the Boltzmann constant], the energy loss is due entirely or almost entirely to the longitudinal component of the electric field. However, for $v \ll v_0$ a small portion of the stopping power is due to the transverse component. It is of interest in that connection to ascertain the physical significance of this component and to determine whether or not the temperature effect in a plasma would cause the occurrence of Vavilov-Čerenkov radiation.

We shall use the conventional representation of a plasma by means of the expressions commonly designated as ϵ_l and ϵ_t.[5] These expressions represent the dielectric capacitivity of the medium occupied by the plasma and they are, respectively, applicable to the longitudinal and transverse component of the field. Using cylindrical coordinates r, φ, z, with the z axis aligned along the direction of motion, we obtain for the vector potential of the field produced by a moving charged particle the following expression:

$$\mathbf{A}=\int e^{i(\omega/v)(z-vt)}\mathbf{a}(\omega,\mathbf{r})d\omega, \tag{2}$$

where the components of the vector $\mathbf{a}(\omega,\mathbf{r})$ that are perpendicular and parallel to the velocity $\mathbf{v}$ can be represented, respectively, as follows:

$$a_r(\omega,r)=\left\{\frac{iqe}{\epsilon_t\pi v^2}\left\{K_1\left(\frac{\omega}{v}r\right) - (1-\epsilon_t\beta^2)^{\frac{1}{2}}K_1\left[\frac{\omega}{v}(1-\epsilon_t\beta^2)^{\frac{1}{2}}r\right]\right\}\right.,$$
$$\text{and complex conjugate for } \omega<0, \tag{3}$$
$$a_z(\omega,r)=\left\{\frac{qe}{\epsilon_t\pi v^2}\left\{K_0\left(\frac{\omega}{v}r\right) - (1-\epsilon_t\beta^2)^{\frac{1}{2}}K_0\left[\frac{\omega}{v}(1-\epsilon_t\beta^2)^{\frac{1}{2}}r\right]\right\}\right.,$$
$$\text{and complex conjugate for } \omega<0.$$

In the above expressions $\beta=v/e$ and K_0 and K_1 are modified Bessel functions of the second kind of the order 0 and 1, respectively. For a Maxwellian plasma we have

$$\epsilon_t=1-\frac{\omega_0{}^2}{(2\pi)^{\frac{1}{2}}\omega k v_0}\int_\Gamma \frac{\exp(-y^2)}{\eta-y}dy, \tag{4}$$

where $\omega_0{}^2=4\pi Ne^2/m$, $\eta=\omega/\sqrt{2}kv_0$ has a negative imaginary part, and Γ is an infinite contour which dips below the pole slightly.

Akhiezer and Sitenko determined the energy loss due to the transverse component of the field for those frequencies that satisfy the relationship $\sqrt{3}\omega/\sqrt{2}kv_0 \ll 1$. This relationship corresponds, however, to $\eta \ll 1$. It is of interest to note that for a given ω and k the term η is inversely proportional to v_0, i.e., the term η decreases with the increase in temperature. Therefore, the assumption that $|\eta| \ll 1$ is associated with a plasma at a high temperature.

Taking into account $\eta \ll 1$ and using a simple residue argument, Eq. (4) can be represented as[6]

$$\epsilon_t=1-\frac{\omega_0{}^2}{k^2v_0{}^2}+i\left(\frac{\pi}{2}\right)^{\frac{1}{2}}\frac{\omega_0{}^2}{kv_0\omega}. \tag{5}$$

The occurrence of the Vavilov-Čerenkov radiation depends upon the behavior of functions K_0 and K_1 for $r \to \infty$ in the expression (3).

Thus, the behavior of $(1-\epsilon_t\beta^2)^{\frac{1}{2}}$ determines the nature of the field surrounding the particle. The Vavilov-Čerenkov effect takes place only if this field represents an outgoing radiation, i.e., if for $r \to \infty$

$$\text{Re}[(1-\epsilon_t\beta^2)^{\frac{1}{2}}]\sim 0, \quad \text{Im}[(1-\epsilon_t\beta^2)^{\frac{1}{2}}]<0. \tag{6}$$

[3] See in that connection, B. M. Bolotovski, Uspekhi Fiz. Nauk **62**, 201 (1957), p. 214.
[4] See, for instance, E. J. Williams, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **13**, No. 4 (1935).
[5] M. E. Gertzenshtein, Zhur. Eksptl. i Teoret. Fiz. **22**, 303 (1952). Also see J. Neufeld and R. H. Ritchie, Phys. Rev. **98**, 1632 (1955).
[6] See, for instance, A. I. Akheizer and A. B. Fainberg, Zhur. Eksptl. i Teoret. Fiz. **21**, 1262 (1951).

As stated above, there is no Vavilov-Čerenkov radiation in a plasma at a low temperature. We shall now ascertain whether or not this radiation appears at high temperatures. Substituting (5) into the expression $(1-\epsilon_t\beta^2)$, we obtain $\mathrm{Re}[(1-\epsilon_t\beta^2)^{\frac{1}{2}}]>0$ and $\mathrm{Im}[(1-\epsilon_t\beta^2)^{\frac{1}{2}}]<0$.

Therefore, the moving particle produces an oscillatory field, the amplitude of which decreases exponentially at large distances. Thus, there is no radiation. More specifically, the relative magnitude of $\mathrm{Re}[(1-\epsilon_t\beta^2)^{\frac{1}{2}}]$ as compared to $\mathrm{Im}[(1-\epsilon_t\beta^2)^{\frac{1}{2}}]$ shows the effectiveness of the exponential damping per wavelength of the oscillatory motion. Let ψ be the argument of $(1-\epsilon_t\beta^2)$. Then using the expression (5), we obtain

$$\operatorname{arc\,cot}\psi=-\frac{1-\beta^2}{\beta^2}\frac{1}{\eta}\frac{\omega^2}{\omega_0^2}-\frac{2}{\sqrt{\pi}}\eta. \qquad (7)$$

Since $\eta\ll1$, the second term in the above expression can be neglected. For $0\leq\beta\ll1$, ψ is negative and $0<|\psi|\ll\pi/2$. Consequently,

$$\frac{\mathrm{Re}[(1-\epsilon_t\beta^2)^{\frac{1}{2}}]}{\mathrm{Im}[(1-\epsilon_t\beta^2)^{\frac{1}{2}}]}<0 \quad\text{and}\quad \left|\frac{\mathrm{Re}[(1-\epsilon_t\beta^2)^{\frac{1}{2}}]}{\mathrm{Im}[(1-\epsilon_t\beta^2)^{\frac{1}{2}}]}\right|\gg1. \qquad (8)$$

The expression (8) is not in agreement with the conditions as shown in (6) and, therefore, there is no Vavilov-Čerenkov radiation.

III. BEHAVIOR OF THE TRANSVERSE FIELD IN THE NEIGHBORHOOD OF THE PARTICLE TRACK

Equation (3) can be put in a simplified form. Since $\beta\ll1$ we expand $(1-\epsilon_t\beta^2)^{\frac{1}{2}}$ in power series and each of the terms $K_0[(\omega/v)(1-\beta^2)^{\frac{1}{2}}r]$ and $K_1[(\omega/v)(1-\beta^2)^{\frac{1}{2}}r]$ in Taylor series. We obtain then:

$$a_r(\omega,r)=\frac{iq}{2\pi c}\left(1+\frac{\epsilon_0\beta^2}{4}\right)\left\{K_1\left(\frac{\omega r}{\beta c}\right)+\left(1-\frac{\epsilon_t\beta^2}{2}-\frac{\epsilon_t^2\beta^4}{8}\right)\frac{\omega r}{\beta c}K'\left(\frac{\omega r}{\beta c}\right)\right\},\ \text{and complex conjugate for } \omega<0,$$
$$a_z(\omega,r)=\frac{q}{2\pi c}\left(1+\frac{\epsilon_t\beta^2}{4}\right)\left\{K_0\left(\frac{\omega r}{\beta c}\right)+\left(1-\frac{\epsilon_t\beta^2}{2}-\frac{\epsilon_t^2\beta^4}{8}\right)\frac{\omega r}{\beta c}K_0'\left(\frac{\omega r}{\beta c}\right)\right\},\ \text{and complex conjugate for } \omega<0. \qquad (9)$$

In the above expression $(d/dx)K_0(x)=K_0'(x)$ and $(d/dx)K_1(x)=K_1'(x)$. We shall consider two limiting cases as follows: (a) $1\gg\beta\gg(\omega r/c)$ and (b) $\beta\ll(\omega r/c)\ll1$. For the case (a), we use the simplified expressions $K_0(x)\sim\frac{1}{2}\ln(4/3.17x^2)$ and $K_1(x)\sim1/x$. Then the expressions (9) can be represented as follows:

$$a_r(\omega,r)=\frac{iq\epsilon_t\beta^3}{4\pi\omega}\left(1+\frac{\epsilon_t\beta^2}{4}\right)\frac{1}{r}$$
and complex conjugate for $\omega<0$,
$$a_z(\omega,r)=\frac{q}{2\pi\epsilon_t\beta^2c}\left(1+\ln\frac{4\beta c}{3.17\omega r}\right) \qquad (10)$$
and complex conjugate for $\omega<0$.

For the case (b) we put $K_0(x)\sim K_1(x)\sim(\pi/2x)^{\frac{1}{2}}e^{-x}$ and then the expressions (9) can be represented as

$$a_r(\omega,r)=\frac{iq}{2\sqrt{2}\pi c}\frac{\epsilon_t\beta^2}{4}\left(1+\frac{\epsilon_t\beta^2}{4}\right)\left(\frac{\omega r}{\beta c}\right)^{\frac{1}{2}}e^{-\omega r/\beta c}$$
and complex conjugate for $\omega<0$, (11)
$$a_z(\omega,r)=-ia_r(\omega,r).$$

The exponential term that appears in the above expression for $a_r(\omega,r)$ and $a_z(\omega,r)$ decreases by $1/e$ for $r_1=\beta c/\omega$. We can therefore assume that the energy of the transverse field is contained to a large extent within the distance $r<r_1$.

We wish to state in conclusion that the transverse dissipation is a real effect, but it leads to the heating of the plasma in the immediate vicinity of the trajectory rather than to the excitation of waves having small attenuation.