Numerical Evaluation of Deuteron Stripping Cross Sections and Polarizations*

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The distorted-wave Born approximation is applied to deuteron stripping reactions. Optical potentials with rounded edges are used to distort the wave functions used in our calculations. It is found possible to get a fair fit to the (d,p) cross section in four cases and to the (d,p) polarization in one case with this treatment without introducing a cutoff.

INTRODUCTION

THE Butler theory[1] has had considerable success in the interpretation of the angular distributions of deuteron stripping reactions. Given a (d,p) or (d,n) reaction in which the incident deuteron has an energy well above the Coulomb barrier, the cutoff radius R in the Butler theory can be adjusted so that the Butler theory correctly describes the position and shape of the first peak in the experimental angular distribution. This theory is not nearly so successful in describing some other aspects of stripping reactions. It generallly underestimates the differential cross section in the backward angles relative to that for the forward angles, and contrary to experiment Butler theory predicts that the outcoming neutrons and protons are unpolarized.

The Butler theory is based on the plane wave, cutoff Born approximation.[2] By "cutoff" is meant that the Born overlap integral is limited to the region of configuration space where the captured particle is outside a sphere of radius R which is concentric with the target nucleus. It has been shown[3] that there is some hope of remedying the inadequacies of the Butler theory by using distorted waves instead of plane waves in the Born approximation integral. The hope of learning something of the extent to which the distorted-wave Born approximation (DWBA) treatment can provide an adequate description of the deuteron stripping reaction has been the motivation for this work.

In this paper we will present the results of a DWBA calculation of the deuteron stripping cross section and polarization. The wave functions used in the Born integral are distorted by an optical potential with rounded edges. As a check on these wave functions we

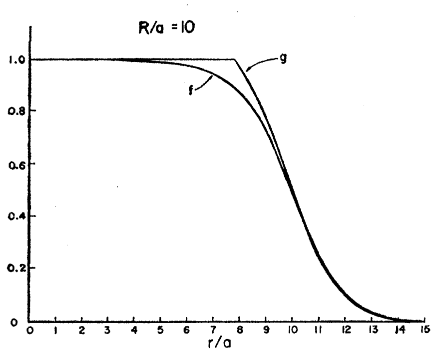

FIG. 1. Comparison of the usual optical potential form factor $f=\{1+\exp[i(r-R)/a]\}^{-1}$ with the form factor used in our calculations, $g=1$ for $r<R-2.239a$ and $g=1.135\{1+\exp[i(r-R<0.239)/a]\}^{-1}$ for $r>R-2.239a$.

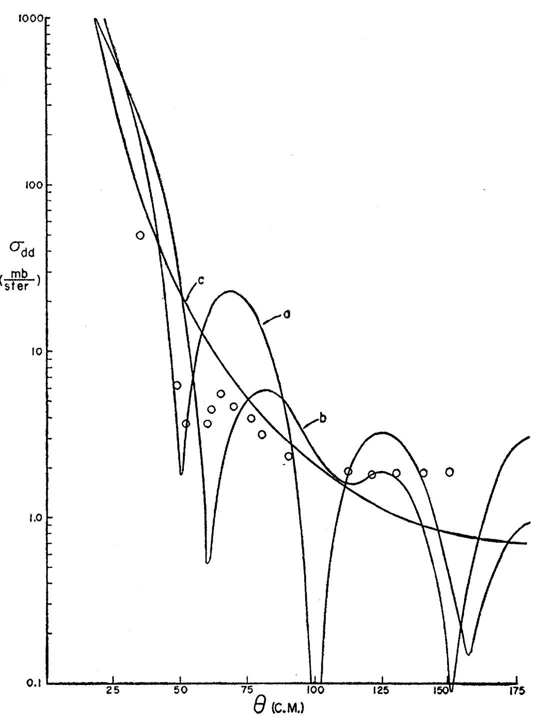

FIG. 2. Cross section for the elastic scattering of 8.10-Mev (lab) deuterons on B^{10}. Curves a and b are the cross sections predicted by optical potentials a and b (see Table I), respectively, while curve c is the Rutherford cross section. The circles represent the experimental elastic cross section for 7.7-Mev (lab) deuterons on Be^9.[7]

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[1] S. T. Butler, Proc. Roy. Soc. (London) **A208**, 559 (1951); R. Huby, Proc. Roy. Soc. (London) **A215**, 385 (1952); F. L. Friedman and W. Tobocman, Phys. Rev. **92**, 93 (1953); Fujimoto, Hayakawa, and Nishyima, Progr. Theoret. Phys. (Kyoto) **10**, 113 (1953).

[2] P. B. Daitch and J. B. French, Phys. Rev. **87**, 900 (1952); E. Gerjuoy, Phys. Rev. **91**, 645 (1953); W. Tobocman, Phys. Rev. **94**, 1655 (1954).

[3] W. Tobocman and M. H. Kalos, Phys. Rev. **97**, 132 (1955).

calculate the elastic differential cross sections predicted by them.

The calculation was programmed for an IBM-650 automatic computer. The time required to calculate a single case varied from 10 to 30 hours so that it was not practical for us to vary the parameters entering into the calculation to any great extent. Being unable to vary the parameters prevents us from determining the extent to which the DWBA can be made to provide a completely satisfactory description of deuteron stripping. The attainment of this goal is also hampered by the present lack of appropriate data. Nevertheless our calculations do indicate that the DWBA provides a better description of deuteron stripping than does the cutoff, plane wave Born approximation (Butler theory). This result is achieved without using a cutoff.

CALCULATION

According to the distorted-wave Born approximation treatment,[4] the deuteron stripping cross section for the (d,p) reaction is given by

$$\sigma_{d,p}(\theta)=\frac{(2J_F+1)M_{PF}M_{ID}\gamma_{lj}K_P}{2(2J_I+1)(2l+1)(2\pi\hbar^2)^2K_D}\sum_{m=-l}^{l}|B^m|^2, \quad (1)$$

while the polarization of the emerging protons is just

$$P(\theta)=\frac{4}{3}\frac{(j-l)}{(2j+1)}\frac{\sum_m [l(l+1)-m(m+1)]^{\frac{1}{2}}\,\mathrm{Im}B^mB^{m+1*}}{\sum_m B^mB^{m+1*}}, \quad (2)$$

where

$$B^m=\frac{(2M_{IN}R)^{\frac{1}{2}}}{\hbar\zeta_{lj}(R)}\int d\mathbf{r}_{PF}\int d\mathbf{r}_{NI}\Psi_{FP}(\mathbf{r}_{PF})^*\frac{\zeta_{lj}(r_{NI})}{r_{NI}}Y^m{}_l(\Omega_{IN})^*\times V_{NP}\chi_D(\mathbf{r}_{NP})\Psi_{DI}(\mathbf{r}_{DI}), \quad (3)$$

γ_{lj} = the reduced width, $M_{AB}=M_AM_B\div(M_A+M_B)$, M_I = mass of the target nucleus, M_F = mass of the residual nucleus, M_D = mass of the deuteron, M_P = mass of the proton, M_N = mass of the neutron, $\mathbf{r}_{AB}$ = separation of particle A from particle B, $\hbar\mathbf{K}_P$ = relative momentum of the proton and the residual nucleus, $\hbar\mathbf{K}_D$ = relative momentum of the deuteron and the target nucleus, V_{NP} = neutron-proton interaction, χ_D

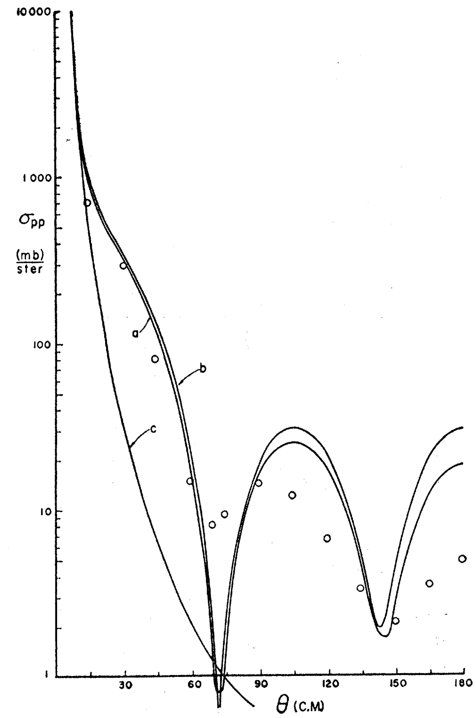
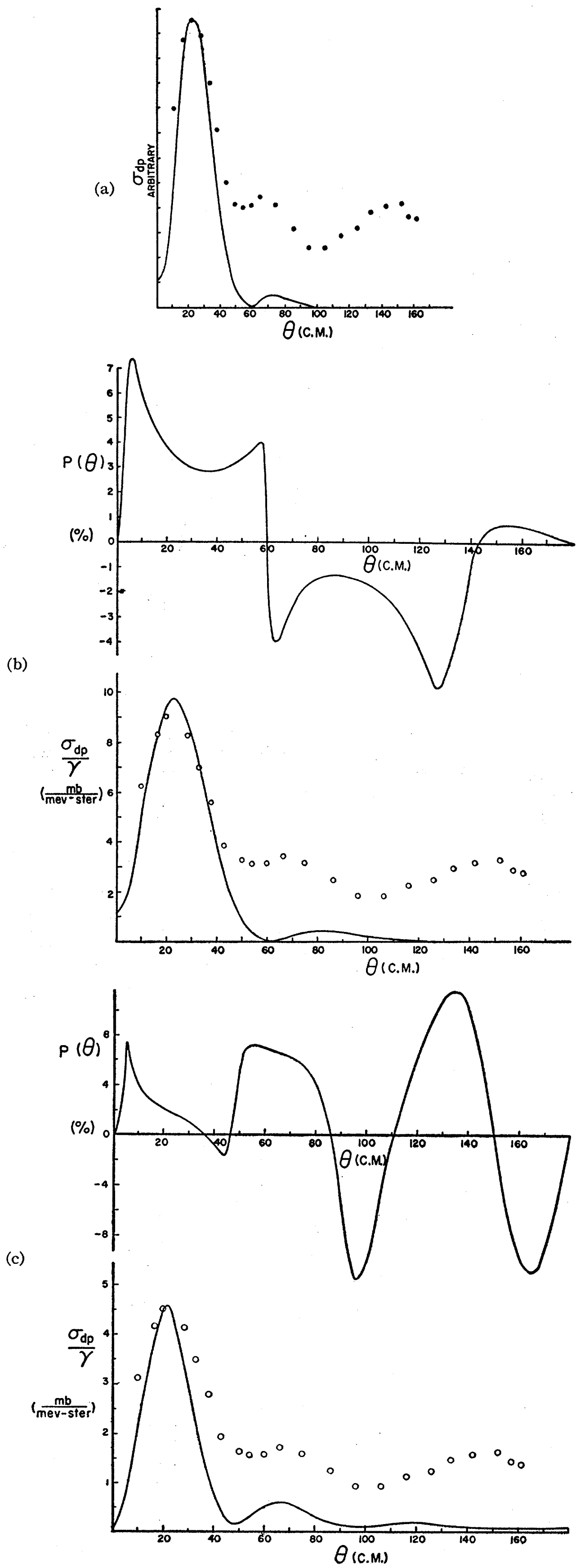


FIG. 3. Cross section for the elastic scattering of 17.44-Mev (lab) protons on B^{11}. Curves a and b are the cross sections predicted by optical potentials a and b (see Table I), respectively, while curve c is the Rutherford cross section. The circles represent the experimental elastic cross section for 17.0-Mev (lab) protons on B^{10}.[8]

TABLE I. Important parameters characterizing the four cases calculated.

	A $B^{10}(d,p)B^{11}$	B $C^{44}(d,p)Ca^{45}$	C $Pb^{207}(d,p)Pb^{208}$	D $Ti^{48}(d,p)Ti^{49*}$
E_D(Mev)	8.1	7.01	15.1	2.6
Q(Mev)	9.24	3.30	5.41	4.46
l	1	1	1	0 1 2
j	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{1}{2}$ $\frac{3}{2}$ $\frac{3}{2}$
$R(10^{-13}$ cm)	5.41	6.15	8.74	6.18
	a b			
V_{ID}(Mev)	−60 −50	−50	−50	−44
W_{ID}(Mev)	−17 −14	−14	−15	−13
$a_{ID}(10^{-13}$ cm)	0.70 0.68	0.68	0.70	0.70
$R_{ID}(19^{-13}$ cm)	3.66 3.23	5.30	8.60	5.34
V_{FP}(Mev)	−50 −50	−60	−60	−60
W_{FP}(Mev)	−11 − 8	−10	−10	− 7
$a_{FP}(10^{-13}$ cm)	0.40 0.40	0.40	0.40	0.45
$R_{FP}(10^{-13}$ cm)	2.9 2.9	4.26	7.12	4.36
$\gamma_{lj}{}^s(R)$ (Mev)	0.0334	0.114	0.123	0.293
$\gamma_{lj}{}^s(1.47A^{\frac{1}{3}})$ (Mev)	1.79	0.395	0.123	0.891

[4] S. T. Butler, *Nuclear Stripping Reactions* (Horowitz Publications Inc., Sidney, Australia, 1957); W. Tobocman, Technical Report No. 29, Nuclear Physics Laboratory, Case Institute of Technology (unpublished).



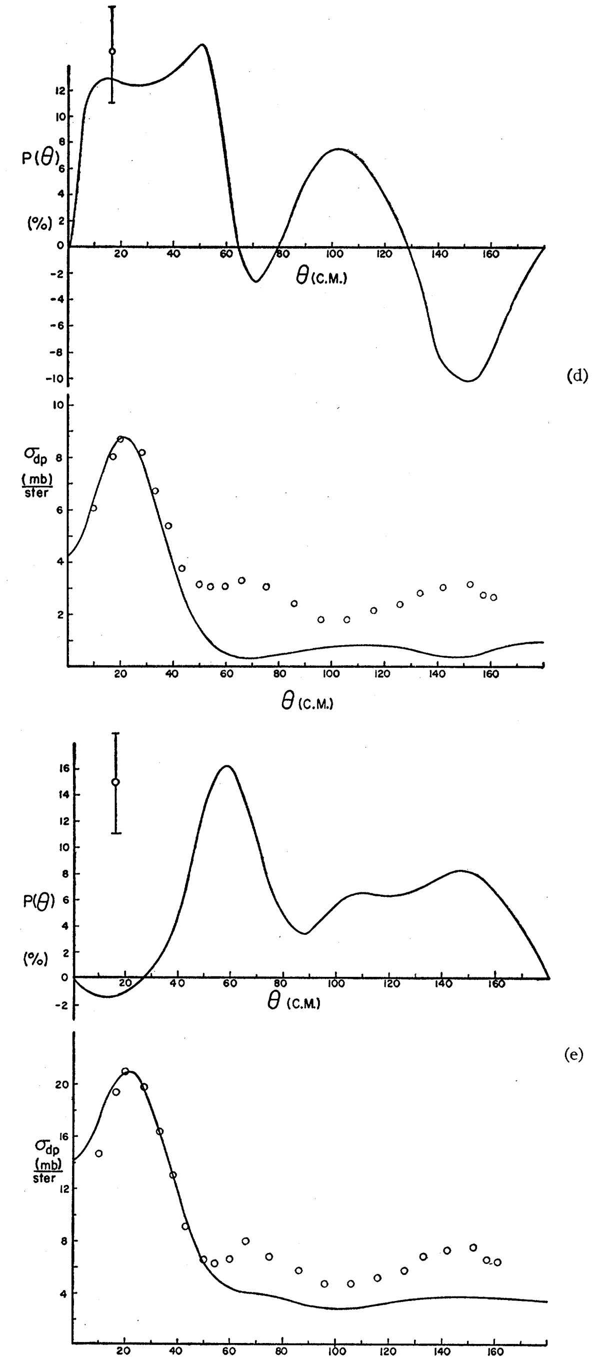


FIG. 4. $B^{10}(d,p)B^{11}$ cross section and polarization for 8.1-Mev (lab) deuterons. $Q=9.24$ Mev. The experimental points are taken from references 5 and 6. The experimental points for the cross section are arbitrarily normalized to fit the theoretical curves since the magnitude of the cross section was not measured. The curves were calculated with $l=1$, $j=3/2$, $R=5.2\times10^{-13}$ cm. (a) The Butler theory. (b) Butler theory modified to include Coulomb effects. (c) The cutoff distorted-wave Born approximation using the optical potentials of set *a*. (d) The distorted-wave Born approximation using the optical potentials of set *a*. (e) The distorted-wave Born approximation using the optical potentials of set *b*.

=deuteron internal wave function, $\zeta_{lj}(r)/r$=radial wave function of the captured neutron, $Y_l^m(\Omega)$=spherical harmonic, $\hbar l$=orbital angular momentum of the captured neutron, $\hbar j$=total angular momentum of the captured neutron, J_F=spin of the residual nucleus, J_I=spin of the target nucleus, Ψ_{FP}=wave function for the relative motion of the outgoing proton and the residual nucleus, Ψ_{ID}=wave function for the relative motion of the incident deuteron and the target nucleus, and R=the cutoff radius.

Although in this paper we present results only for (d,p) reactions, the program can be used for (d,n) reactions as well.

The cutoff theory results when the integration over r_{NI} in Eq. (3) is limited to the region where $r_{NI}>R$. To get the Butler theory one replaces $\Psi^*_{PF}\Psi_{DI}$ by $\exp i(\mathbf{K}_D\cdot\mathbf{r}_{DI}-\mathbf{K}_P\cdot\mathbf{r}_{PF})$ in the cutoff theory.

If the captured neutron can be described by a pure single particle state, then the reduced width is given by

$$\gamma_{lj}=\frac{\hbar^2\zeta_{lj}(R)^2}{2M_{IN}R}=\gamma_{lj}{}^s(R). \tag{4}$$

If not, then this expression must be reduced by a factor representing the probability for finding the captured neutron in the single particle state characterized by the quantum numbers l and j. For those cases calculated with a cutoff, we give σ_{dp}/γ_{lj}, the stripping cross section divided by the reduced width. For those cases calculated without a cutoff we use Eq. (4) for the reduced width. Thus for the non cutoff cases the calculated cross sections are upper limits.

Another assumption used in the evaluation of the above expressions is the replacement of V_{NP} by a zero-range potential. It can be shown that the Butler theory results are quite insensitive to the range of V_{NP}, and this leads us to hope that the same is true for the distorted wave theory. Nevertheless, the use of the zero-range potential must be regarded as a shortcoming of our calculation.

The wave functions for the relative motion of the incident deuteron and the target nucleus and for the relative motion of the emerging proton and residual nucleus are solutions of the following differential equations:

$$\left\{-\frac{\hbar^2\nabla^2}{2M_{ID}}+(V_{ID}+iW_{ID})g_{DI}+U_{DI}-\frac{\hbar^2K_D{}^2}{2M_{ID}}\right\}\Psi_{ID}=0, \tag{5}$$

$$\left\{-\frac{\hbar^2\nabla^2}{2M_{PF}}+(V_{PF}+iW_{PF})g_{PF}+U_{PF}-\frac{\hbar^2K_P{}^2}{2M_{PF}}\right\}\Psi_{PF}=0, \tag{6}$$

where U_{XY} is the Coulomb potential of a point charge e located a distance r_{XY} from a sphere of charge of radius R_{XY} and charge Z_Ye, and $U+(V+iW)g$ is the optical potential. The form factor g is a flat-bottomed approximation to the usual form factor f. Given

$$f=\{1+e^{(r-R)/a}\}^{-1}, \tag{7}$$

we take

$$g=1,\quad r<R-2.239a$$
$$g=1.135\{1+e^{(r-R+0.239)/a}\}^{-1},\quad R-2.239a<r. \tag{8}$$

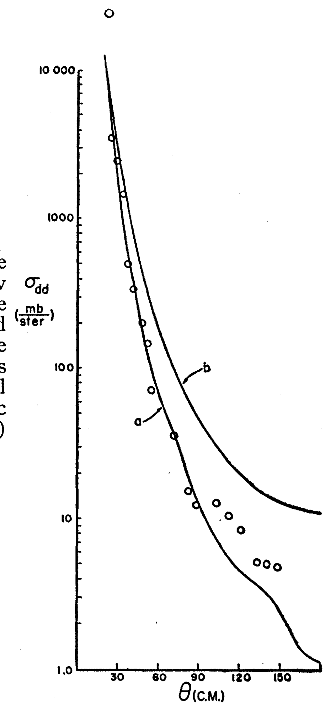


FIG. 5. Cross section for the elastic scattering of 7.01-Mev (lab) deuterons on Ca^{44}. Curve a is the cross section predicted by the optical potential while curve b is the Rutherford cross section. The experimental points are for the elastic scattering of 8-Mev (lab) deuterons on A^{40}.[1]

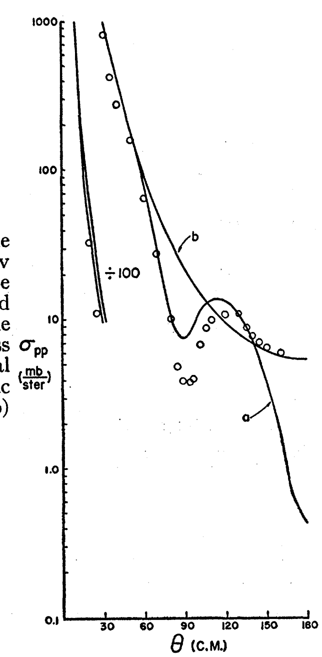


FIG. 6. Cross section for the elastic scattering of 10.22-Mev (lab) protons on Ca^{45}. Curve a is the cross section predicted by the optical potential while curve b is Rutherford cross section. The experimental points are for the elastic scattering of 9.5-Mev (lab) protons on A^{40}.[10]

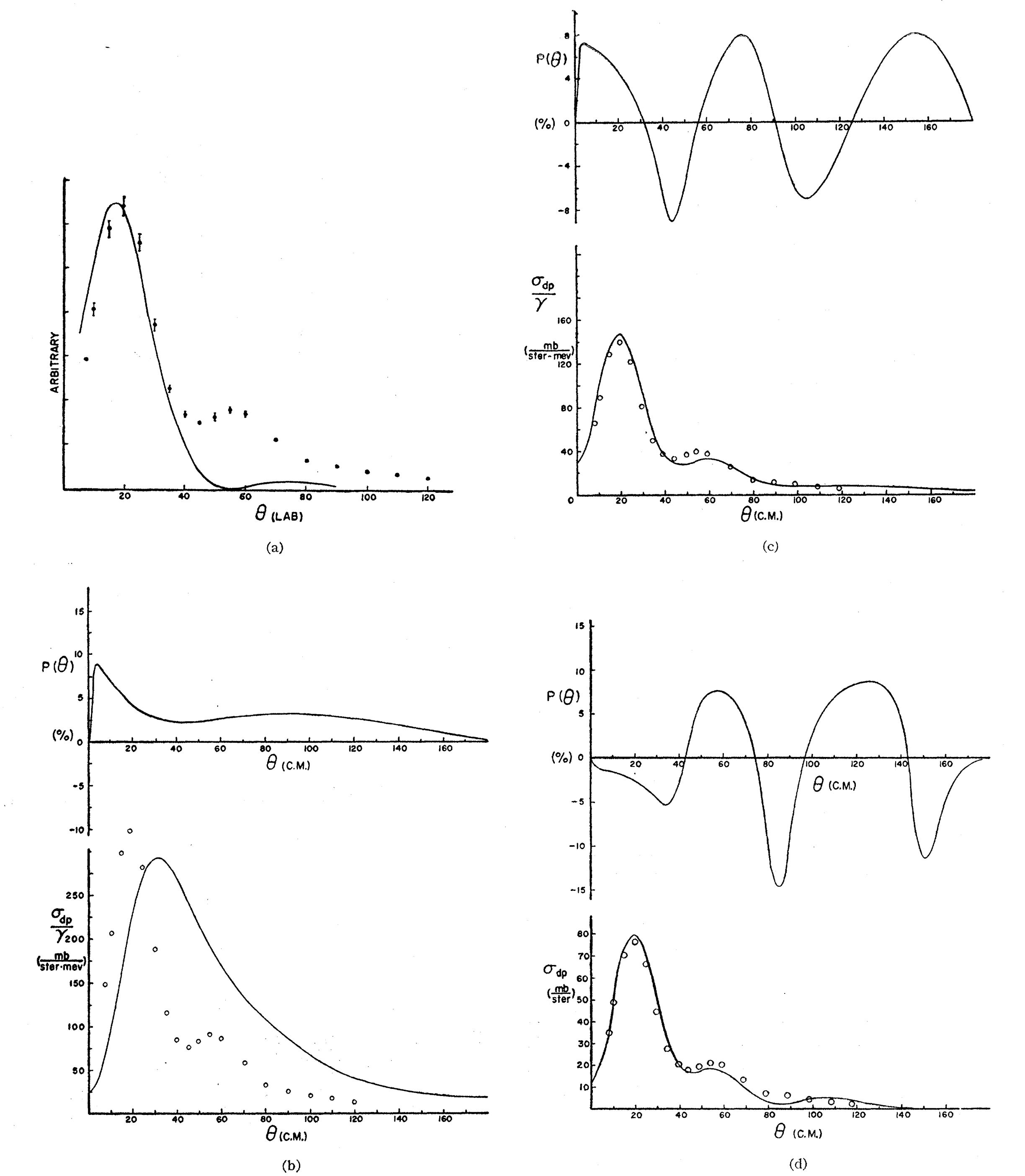


FIG. 7. $Ca^{44}(d,p)Ca^{45}$ cross section and polarization for 7.01-Mev (lab) deuterons. $Q=3.30$ Mev. The experimental points are taken from reference 9. The experimental points are arbitrarily normalized to fit the theoretical curves since the magnitude of the cross section was not measured. The curves were calculated with $l=1, j=3/2, R=6.0\times10^{-13}$ cm. (a) The Butler theory. (b) Butler theory modified to include Coulomb effects. (c) The cutoff distorted-wave Born approximation. (d) The distorted wave Born approximation.

In Fig. 1 is a comparison of the two form factors for the case where $R/a=10$.

For the region outside the target nucleus, the radial wave function of the captured neutron is taken to be the asymptotically vanishing solution of

$$\left\{\frac{d^2}{dr^2}-K_N{}^2-\frac{l(l+1)}{r^2}\right\}\zeta_l=0, \tag{9}$$

where

$$\frac{\hbar K_N^2}{2M_{IN}} = Q + 2.226 \text{ Mev},$$

Q being the Q value for the reaction. Inside the target nucleus we take ζ_l to be a harmonic oscillator wave function

$$\zeta_l(r) = Ar^{l+1}\exp(-\beta r^2/2)F(\tfrac{1}{2}(l+1-n)\,|\,l+\tfrac{3}{2}\,|\,\beta r^2), \quad (10)$$

where A and β are chosen so that the value and derivative of ζ_l are continuous at the nuclear surface. n is chosen on the basis of the shell model assignment.

RESULTS

In Table I are listed the four cases we have calculated and the important parameters involved. For each case we calculate

(1) the cross section for elastic scattering of deuterons by the target nucleus,

(2) the cross section for elastic scattering of protons by the residual nucleus,

(3) the stripping cross section predicted by Butler theory,

(4) the stripping cross section and the polarization predicted by Butler theory modified to include Coulomb interactions,

(5) the stripping cross section and the polarization predicted by the cutoff distorted-wave Born approximation, and

(6) the stripping cross section and the polarization predicted by the distorted-wave Born approximation.

Thus (1), (2), and (6) of each set give the theoretical predictions associated with a particular set of optical potential parameters. The difference between (3) and (4) provides a measure of the effects of the Coulomb interaction. The difference between (4) and (5) provides a measure of the effects of the nuclear interactions $(V+iW)g$. Finally, the difference between (5) and (6) provides a measure of the contribution to the stripping cross section and polarization due to interactions that occur inside the cutoff radius R, the so called inside contribution.

The calculated cross sections are plotted in Figs. 2–13, together with appropriately normalized experimental cross sections.

The B^{10} case [5–8] is an example of the kind of situation where one would expect the Butler theory to be valid. In this case the energy of the incident deuterons is well above the Coulomb barrier. Comparing Figs. 4(a) and 4(b) we see that, indeed, the Coulomb interactions have a negligible effect on the angular dependence of the stripping cross section and give rise to a relatively small polarization. Comparing Figs. 4(b) and 4(c), we see that the nuclear interactions do not affect the cross section angular distribution strongly but do have considerable effect on the polarization. Finally, inspection of Fig. 4(d) reveals that the inside contribution, that is the contribution due to interactions occurring at $r<R$, does have a significant effect on both the cross

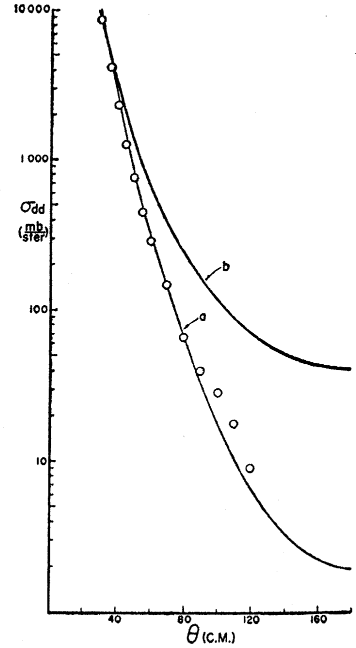


FIG. 8. Cross section for the elastic scattering of 15.1-Mev (lab) deuterons on Pb^{207}. Curve a is the cross section predicted by the optical potential while curve b is the Rutherford cross section. The experimental points are for the elastic scattering of 15.1-Mev deuterons on Pb.[13]

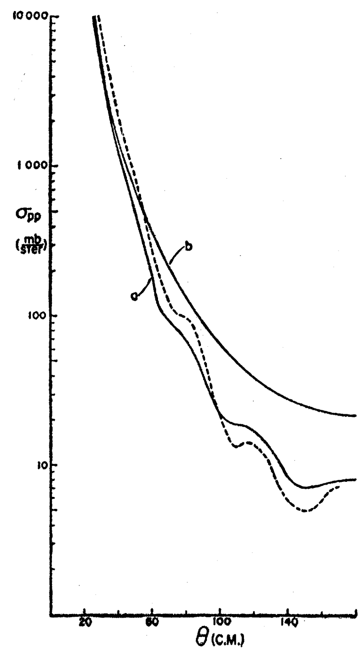


FIG. 9. Cross section for the elastic scattering of 20.4-Mev (lab) protons on Pb^{208}. Curve a is the cross section predicted by the optical potential while curve b is the Rutherford cross section. The broken curve is the experimental cross section for 17-Mev (lab) protons on Pb.[8]

[5] B. Zeidman and J. M. Fowler, Phys. Rev. **112**, 2020 (1958).
[6] J. C. Hensel and W. C. Parkinson, Phys. Rev. **110**, 128 (1958).
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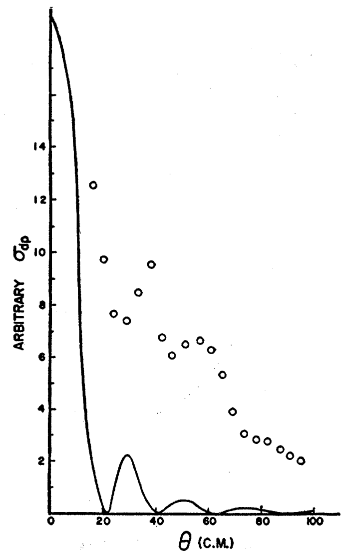
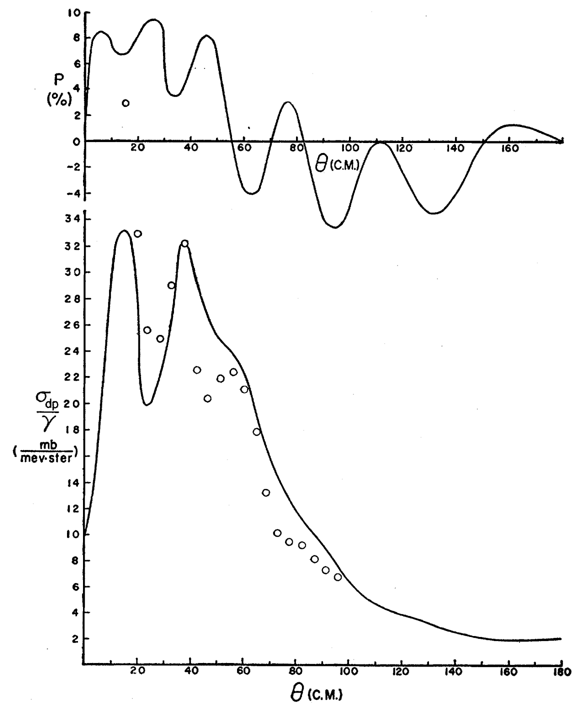
[8] G. Schrank (private communication).

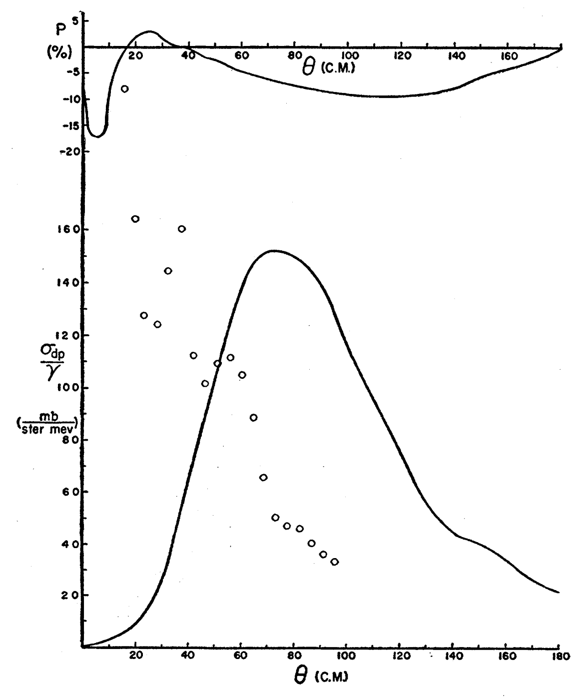


(a)

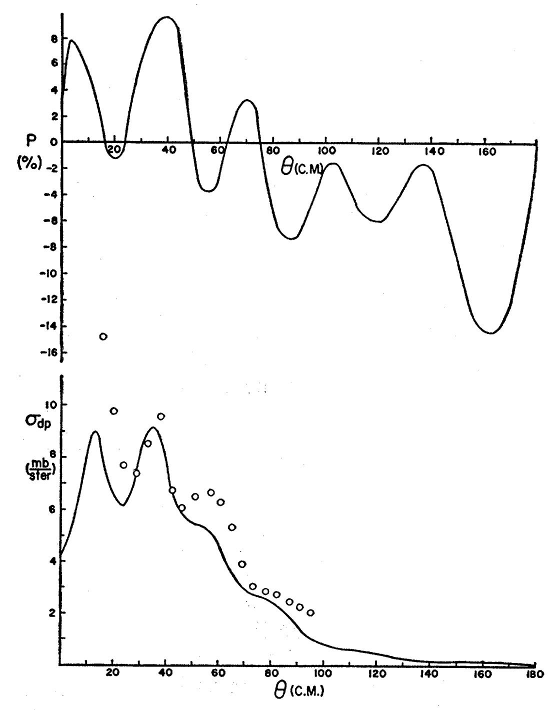


(c)

FIG. 10. $Pb^{207}(d,p)$ Pb^{208} cross section and polarization for 15.1-Mev (lab) deuterons. $Q=5.41$ Mev. The experimental points are taken from reference 12. The experimental points are arbitrarily normalized to fit th theoretical curves since the magnitude of the cross section was not measured. The curves were calculated with $l=1$, $j=3/2$, $R=8.6\times10^{-13}$ cm. (a) The Butler theory. (b) Butler theory modified to include Coulomb effects. (c) The cutoff distorted-wave Born approximation. (d) The distorted-wave Born approximation.



(b)



(d)

section and the polarization. Although the position of the main peak of the cross section is unchanged, the inside contribution increases the width of the peak and the cross section off the peak is considerably changed. Comparing 4(d) and 4(e), we see that the polarization and the details of the cross section are sensitive to the optical potentials used to distort the deuteron and proton wave functions. In view of this fact, it would seem possible to improve the fit of the distorted-wave Born approximation results with experiment by varying the optical potentials. Figure 4(d) and 4(e) already represent a considerable improvement over Fig. 4(a), Fig. 4(d) giving the right value for the polarization at 18° as well as giving an improved fit to the cross-section angular distribution. However, we lack the appropriate elastic scattering data to guide us in the choice of the optical potentials. In this case, as in most of our other cases, the data that appear on the elastic cross-section curves are not the appropriate data. In Fig. 3 the theoretical curve is for the elastic scattering of 17.44-Mev protons on B^{11} while the experimental points are for 17.0-Mev protons on B^{10}. Similarly, in Fig. 2 the theoretical curve is for 8.1–Mev deuterons on B^{10} while the experimental points are for 7.7-Mev deuterons on Be^9. Such discrepancies are likely to be less important for the heavier nuclei involved in our other three cases than for this case.

To sum up the results of the $B^{10}(d,p)B^{11}$ case, we find that the Butler theory correctly describes the first peak of the angular distribution. Nevertheless, the distorted-wave Born approximation gives a better fit and predicts large polarizations. It is interesting that the use of distorted waves have relatively little effect on the outside contribution so that the Butler treatment of this part of the interaction is adequate so far as the cross section is concerned. Still the inside contribution, which is neglected in Butler theory and which is strongly affected by the optical potentials, is found to play an important role in the reaction.

The $Ca^{44}(d,p)Ca^{45}$ case[9–11] is one for which we would expect the Butler theory to fail because the incident energy is only about 1 Mev above the Coulomb barrier. However, in Fig. 7(a) we see that the Butler theory does give a fair representation of the first peak of the cross section. Comparing Figs. 7(b) and 7(c), we see that the success of the Butler theory in this case is due to a cancellation between the effects of the Coulomb interactions and the nuclear interactions. Comparison of Figs. 7(c) and 7(d) reveals that the inside contribution has very little effect on the cross section but does have a strong effect on the polarization. In this case we find that with our choice of optical model parameters the DWBA gives an essentially perfect fit to the cross section.

In the Pb^{207} case[8,12,13] the Coulomb effects are so important that the experimental cross section bears only a very slight resemblance to the Butler curve. The DWBA gives a fair fit to the data. It is to be expected that these results will be quite sensitive to variation of the optical model parameters.

The Ti^{48} case[14] is even more extreme than the Pb^{207} case since the incident deuterons are far below the Coulomb barrier. Thus it is not surprising that the experimental cross section has no resemblance at all to the Butler theory cross section. Nevertheless, the distorted-wave Born approximation does give a fair representation of the experimental cross section.

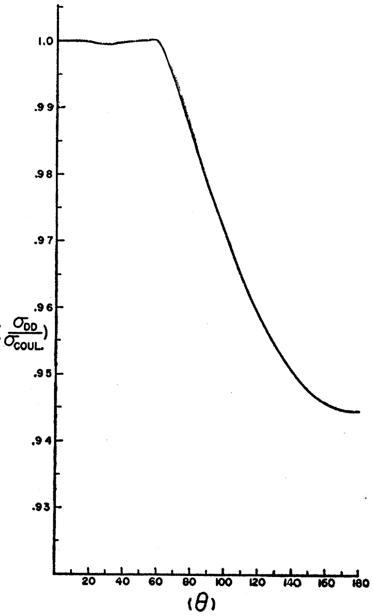


FIG. 11. Ratio of the cross section for the elastic scattering of 2.6-Mev (lab) deuterons on Ti^{48} predicted by the optical potential to the Rutherford cross section.

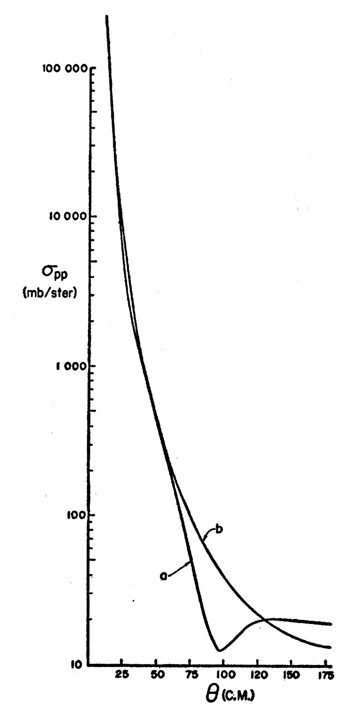


FIG. 12. Cross section for the elastic scattering of 7.2-Mev (lab) protons on Ti^{49*}. Curve a is the cross section predicted by the optical potential while curve b is the Rutherford cross section.

[9] W. R. Cobb and D. B. Guthe, Phys. Rev. **107**, 181 (1957).
[10] Freemantle, Prowse, Hossain, and Rotblat, Phys. Rev. **96**, 1270 (1954).
[11] W. M. Gibson and E. E. Thomas, Proc. Roy. Soc. (London) **A210**, 543 (1952).
[12] N. S. Wall, Phys. Rev. **96**, 670 (1954).
[13] H. E. Gove, Phys. Rev. **99**, 1353 (1955).
[14] W. W. Pratt, Phys. Rev. **97**, 131 (1954).

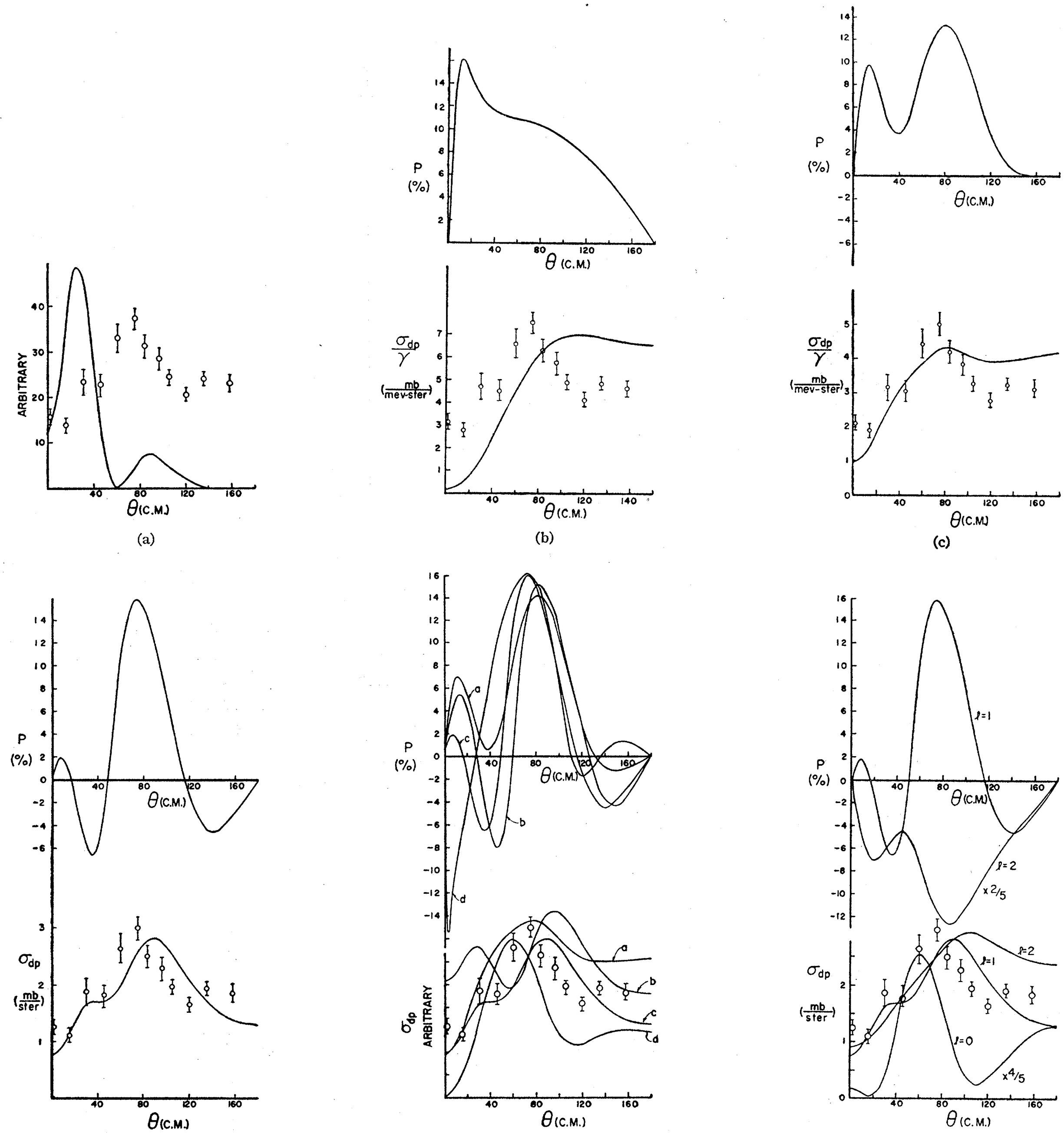


FIG. 13. $Ti^{48}(d,p)Ti^{49*}$ cross section and polarization for 2.6-Mev (lab) deuterons. The experimental points are taken from reference 14. The experimental points are arbitrarily normalized to fit the theoretical curves since the magnitude of the cross section was not measured. The curves a, b, c, d, and e were calculated with $l=1, j=3/2$, $R=6.18\times10^{-13}$ cm. (a) The Butler theory. (b) Butler theory modified to include Coulomb effects. (c) The cutoff distorted-wave Born approximation. (d) The distorted-wave Born approximation. (e) The distorted-wave Born approximation for various choices of the captured neutron wave function. The neutron wave functions used are shown in Fig. 14. The various cross sections are multiplied by different scale factors to facilitate drawing. (f) The distorted wave Born approximation for $l=0(j=0)$; $l=1(j=3/2)$; $l=2(j=3/2)$.

The Ti^{48} case is of particular interest because it shows that with the help of the DWBA analysis it is possible, even with a small accelerator, to use stripping reactions to study the properties of the heavier nuclei. In Fig. 13(f) we see that the angular distribution of the cross section still has enough structure to fix the orbital angular momentum of the captured neutron. In Figs. 13(e) and 14 we see that the cross section is

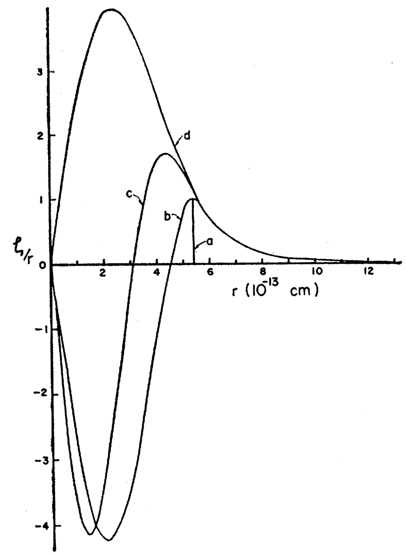


FIG. 14. The neutron radial wave functions used in the calculation of the curves shown in Fig. 13(e).

extremely sensitive to the inside contribution. Thus the stripping cross section in this case can be used to test the wave functions proposed by the shell model theory.

DISCUSSION

We have found that the distorted-wave Born approximation (DWBA) gives a better fit to stripping experiments than the Butler theory in four particular cases. This is not surprising since the Butler theory can be regarded as an approximation to the DWBA. It also was not unexpected that a cutoff would be unnecessary in the DWBA. It had been thought that the large imaginary part and the large radius of the deuteron optical potential would make the deuteron wave function so small inside the target nucleus as to render the inside contribution negligible. This turns out to be false—the inside contribution is not negligible. Thus the meaning of the cutoff in Butler theory remains obscure.

The degree of validity of the DWBA treatment for nuclear reactions remains to be ascertained. We will need more data and more calculations to compare with the data. The type of measurement that will be most useful in this task is one in which the elastic scattering cross sections associated with a given stripping reaction are determined. Related polarization and angular correlation measurements are also most useful. The point is that the theory for each of these processes involves the same set of optical model parameters so that each measurement gives a new set of conditions to be satisfied.

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