Inelastic Diffraction Scattering*

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The mechanism of inelastic diffraction scattering, introduced by Drozdov and Inopin to discuss scattering from nuclei with quadrupole surface deformation, has been extended for scattering amplitude linear in the deformation to arbitrary multipolarity. The resulting angular distributions and gamma-ray angular correlations correspond closely to Born approximation predictions. The model provides a natural explanation for why the inelastic angular distributions of strongly absorbed projectiles are characterized by sharp and persistent oscillations. Comparison is made to observed inelastic scattering of alpha particles and deuterons by light nuclei. Large quadrupole deformations are indicated in many nuclei. The systematic appearance in $4n$ nuclei of states corresponding to octupole excitation is noted.

I. INTRODUCTION

ANGULAR distributions of moderate-energy nuclear projectiles, particularly alpha particles, which excite discrete nuclear states in light and intermediate nuclei have been analyzed with some success in terms of simple direct-interaction theories. The simplest of these theories[1–5] share the common assumptions, irrespective of the model chosen to represent the target nucleus: (a) The reaction proceeds through some single-step interaction, usually described by the Born or impulse approximation. (b) There is only a slow variation of the direct interaction itself with angle and energy. (c) The incident and final unbound particles are well represented by plane waves in the nuclear surface region. (d) The interaction occurs at a well-defined radius, R, near the nuclear surface.

On the other hand, the corresponding elastic angular distributions have been successfully fitted by optical-model computations in which there is large nuclear attenuation,[6,7] or equivalently, have been approximated by "black nucleus" expressions.[8,9] We therefore expect the projectile wave functions to be considerably distorted from plane waves. It is then paradoxical that direct inelastic theories based on assumption (c) above lead to angular distributions which correspond to experiment.

This paradox has been strikingly illustrated by the recent semiclassical model of Butler, Austern, and Pearson.[10] It is there shown that the scattered amplitude for a direct surface interaction, in the plane wave approximation, is essentially the sum of contributions from two circles of radius $\rho = l/K$, centered about the $\mathbf{K}$ axis. (l is the orbital angular momentum transfer and $\mathbf{K}\equiv\mathbf{k}_i-\mathbf{k}_f$, the linear momentum transfer.) The interference of these two contributions leads to the characteristic maxima and minima in the angular distributions. Assume now that the nucleus is strongly absorbing. In a classical approximation, the scattered amplitude at angle Θ is primarily composed of projectiles which have inelastically scattered on the unshaded area of Fig. 1; i.e., a small observer stationed at angle Θ would "see" the crescent moon presented by this area. For Θ larger than $\Theta'\equiv l/kR$, the angle at which the first maximum in the plane wave approximation occurs, and for inelastic scattering such that $k_f \cong k_i$, one of the circles will be within this area while the other circle is on the darker side of the nucleus. The amplitude from this latter circle is then greatly reduced, with concomitant destruction of the characteristic interference pattern in the angular distribution.

The above authors have explicitly computed angular distributions due to a direct interaction within the nuclear volume where the incident and final wave functions are attenuated in a semiclassical fashion and, indeed, find that as the mean free path decreases the

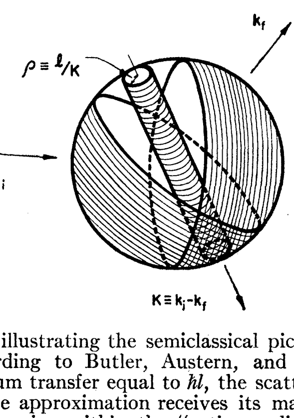

FIG. 1. Sketch illustrating the semiclassical picture of a direct interaction according to Butler, Austern, and Pearson.[10] For angular momentum transfer equal to $\hbar l$, the scattered amplitude in the plane-wave approximation receives its main contribution from events occurring within the "active cylinder" of radius $\rho = l/K$ centered about the $\mathbf{K}$ axis. If there is strong nuclear absorption, only portions of the "active cylinder" near the unshaded surface will contribute to the scattered amplitude in the $\mathbf{k}_f$ direction and thus the interference pattern in the angular distribution will be destroyed.

* Supported in part by the U. S. Atomic Energy Commission.
[1] Austern, Butler, and McManus, Phys. Rev. 92, 350 (1953).
[2] S. T. Butler, Phys. Rev. 106, 272 (1957).
[3] R. Huby and H. C. Newns, Phil. Mag. 42, 1442 (1951).
[4] S. Hayakawa and S. Yoshida, Progr. Theoret. Phys. (Kyoto) 14, 1 (1955), Proc. Phys. Soc. (London) A68, 656 (1955).
[5] J. S. Blair and E. M. Henley, Phys. Rev. 112, 2029 (1958).
[6] G. Igo and R. M. Thaler, Phys. Rev. 106, 126 (1957).
[7] W. B. Cheston and A. E. Glassgold, Phys. Rev. 106, 1215 (1957).
[8] A. I. Yavin and G. W. Farwell, Nuclear Phys. 12, 1 (1959).
[9] J. S. Blair, Phys. Rev. 108, 827 (1957).
[10] Butler, Austern, and Pearson, Phys. Rev. 112, 1227 (1958).

interference pattern is smoothed out and the magnitude of the cross section decreases. This prediction appears to be in qualitative disagreement with experiment, since the observed angular distributions of the strongly attenuated alpha particles show sharp interference patterns persisting to high order[11] while those of the less attenuated high-energy protons tend to be smoother.[12]

We believe that a way out of this dilemma is provided by the inelastic diffraction scattering mechanism, introduced by Drozdov to discuss inelastic scattering of neutrons from ellipsoidally deformed nuclei[13] and extended by Inopin to the excitation of quadrupole vibrational states.[14] In Drozdov's treatment one assumes, in contrast to the transparency requirement (c), that the nucleus is strongly absorbing within a radius R which is a function of collective deformation coordinates, α, of the nuclear surface. With the adiabatic approximation, that the "collision time" be short compared to the period associated with α motion, one finds approximate expressions for the inelastically scattered amplitudes which are very similar to the analogous Born approximations.

The physical picture for the process of inelastic diffraction scattering in the above approximation is the following: For given α, a noncircular hole is cut in the incident wave. The scattered amplitude, $f(\alpha,\Theta)$, when folded into the initial nuclear wave function, projects onto excited states involving the collective coordinates as well as the ground state. The sharper the strong absorption radius, the more pronounced will be the interference structure in both elastic and inelastic angular distributions.

In addition to avoiding the attenuation dilemma, the Drozdov mechanism has the following virtues: (a) It is experimentally observed that elastic and inelastic alpha-particle angular distributions are closely related. In particular, the oscillations of inelastic scattering angular distributions, where the excitation involves no change in parity, are typically 90° out of phase with those of the elastic scattering angular distribution. Stated equivalently, the radii deduced from Born approximation analyses of inelastic scattering angular distributions, R_{in}, are similar to the elastic scattering radii, R_{el}. This connection is naturally explained when the same mechanism gives rise to both elastic and inelastic scattering. In some other direct models, such as the independent-particle model[15] and alpha-particle model[5] in plane wave approximation, R_{in} is expected to be slightly less than R_{el}. It is also clear why R_{in} is so large, larger even than the mean radius of the nuclear potential, since $R_{\text{in}}\cong R_{\text{el}}$, which is the "sharp cutoff" radius.[9] (b) Large inelastic scattering cross sections are obtained for reasonable values of the collective coordinate matrix elements. (c) Those states which are strongly excited are reached by means of collective excitation. We will discuss later some of the evidence for this in light nuclei. The recent inelastic scattering experiments of Cohen and Rubin on intermediate nuclei have been similarly interpreted.[16,17]

In Sec. II we review and extend the calculations of the angular distributions for the Drozdov model. Comparison to the analogous Born approximation angular distributions is made in Sec. III. Angular correlations of gamma rays from excited states with the scattered projectile are treated in Sec. IV. Section V is devoted to a discussion of the approximations and limitations of the calculations. Inelastic alpha-particle and deuteron experiments are discussed in terms of the diffraction model in Sec. VI.

II. ANGULAR DISTRIBUTIONS

In the adiabatic approximation the scattered amplitude from initial state, a, to final state, b, for a nucleus characterized by deformation parameters α_{lm}, is $\langle b|f(\alpha)|a\rangle$, where $f(\alpha)$ is the scattered amplitude for fixed α. Consider the nucleus to be strongly absorbing within the sharp radius,

$$R(\theta,\phi)=R_0(1+\sum_{l,m}\alpha_{lm}Y_{lm});$$

further, assume that the scattered amplitude from this strongly absorbing nucleus may be calculated for small-angle scattering in the Fraunhofer approximation[18]:

$$f(\alpha)\cong\frac{ik}{2\pi}\int\int dA\, e^{-i\mathbf{k}_f\cdot\mathbf{r}},\tag{2.1}$$

where the magnitude of the final momentum, k_f, is considered approximately equal to the magnitude of the initial momentum, $k_i\equiv k$, and where the integral is taken over the projection of the nuclear surface on a plane perpendicular to the incident beam (z axis). For the case of small deformations, such that only terms linear in α need be kept in the scattered amplitude, the projection is the same as the area within $R(\pi/2,\phi)$ on the x-y plane.

With the convention that $\mathbf{k}_f$ lies in the x-z plane and makes an angle, Θ, to the incident beam, the amplitude

[11] For example, 42-Mev alpha particles on Mg, P. C. Gugelot and M. Rickey, Phys. Rev. **101**, 1613 (1956).

[12] For example, 17-Mev protons on Mg, P. C. Gugelot and P. R. Phillips, Phys. Rev. **101**, 1614 (1956); 96-Mev protons on C, K. Strauch and F. Titus, Phys. Rev. **103**, 200 (1956).

[13] S. I. Drozdov, J. Exptl. Theoret. Phys. (U.S.S.R.) **28**, 734 (1955); **28**, 736 (1955) [translation: Soviet Phys. JETP **1**, 591, 588 (1955)].

[14] E. V. Inopin, J. Exptl. Theoret. Phys. (U.S.S.R.) **31**, 901 (1956) [translation: Soviet Phys. JETP **4**, 764 (1957)].

[15] C. A. Levinson and M. K. Banerjee, Ann. Phys. **2**, 471 (1957).

[16] B. L. Cohen, Phys. Rev. **105**, 1549 (1957).

[17] B. L. Cohen and A. G. Rubin, Phys. Rev. **111**, 1568 (1958).

[18] See for example P. M. Morse and H. Feshbach, *Methods of Theoretical Physics* (McGraw-Hill Book Company, Inc., New York, 1953), p. 1552.

to terms linear in α becomes

$$f(\alpha,\Theta)\cong\frac{ik}{2\pi}\int_0^{2\pi}d\phi\left\{\int_0^{R_0}e^{-ik\Theta r\cos\phi}rdr + e^{-ikR_0\Theta\cos\phi}R_0^2\sum_{l,m}\alpha_{lm}Y_{lm}(\pi/2,\phi)\right\}. \quad (2.2)$$

The first integral gives the familiar result for scattering from a circular black disk:

$$ikR_0^2J_1(kR_0\Theta)/(kR_0\Theta). \quad (2.3)$$

To calculate the second integral, we note that[19]

$$Y_{l,m}(\pi/2,\phi)=\left(\frac{2l+1}{4\pi}\right)^{\frac{1}{2}}\frac{[(l-m)!(l+m)!]^{\frac{1}{2}}}{(l-m)!!(l+m)!!}\times(-)^{\frac{1}{2}(l+m)}e^{im\phi},\quad (l+m)\text{ even};$$
$$=0,\quad (l+m)\text{ odd}; \quad (2.4)$$

and[20]

$$2\pi J_m(z)i^m=\int_0^{2\pi}e^{iz\cos\phi}\cos(m\phi)d\phi. \quad (2.5)$$

Thus

$$f(\alpha,\Theta)\cong ikR_0^2\left\{\frac{J_1(kR_0\Theta)}{(kR_0\Theta)}+\sum_{\substack{l,m\\(l+m)\text{ even}}}\left(\frac{2l+1}{4\pi}\right)^{\frac{1}{2}}i^l\frac{[(l-m)!(l+m)!]^{\frac{1}{2}}}{(l-m)!!(l+m)!!}\times\alpha_{lm}J_{|m|}(kR_0\Theta)\right\}. \quad (2.6)$$

For the small shape oscillations of a spherical nucleus, the vibrational model of Bohr and Mottelson is appropriate. In an even-even nucleus, the ground state contains no surface phonons, $|a\rangle = |I=0\rangle$, and the occurrence of α's in the scattered amplitude makes it possible to excite final states containing a single surface phonon, $|b\rangle=|I'M'\rangle=|lm\rangle$. Since $|\langle b|\alpha_{lm}|a\rangle|^2=(\hbar\omega_l/2C_l)$ in the harmonic approximation,[21] where $\hbar\omega_l$ is energy of one surface phonon and C_l is the corresponding surface tension, the cross section for excitation of a one-phonon state is

$$\frac{d\sigma}{d\Omega}(0\to l)=(kR_0^2)^2\sum_{m=-l,\,-l+2\cdots}^{l}\left(\frac{2l+1}{4\pi}\right)\left(\frac{\hbar\omega_l}{2C_l}\right)\times\frac{[(l-m)!(l+m)!]}{[(l-m)!!(l+m)!!]^2}J_{|m|}^2(kR_0\Theta). \quad (2.7)$$

For quadrupole surface vibrations ($l=2$), the cross section has been previously derived by Inopin[14] in a form equivalent to the following equation:

$$\frac{d\sigma}{d\Omega}(0\to 2)=(kR_0^2)^2\left(\frac{5}{8\pi}\right)\left(\frac{\hbar\omega_2}{C_2}\right)\{\tfrac{1}{4}J_0^2+\tfrac{3}{4}J_2^2\}. \quad (2.8)$$

Beyond their first maxima, the Bessel functions rather quickly approach their asymptotic form so that Eq. (2.8) is well approximated by

$$\frac{d\sigma}{d\Omega}(0\to 2)\cong(kR_0^2)^2\left(\frac{5}{8\pi}\right)\left(\frac{\hbar\omega_2}{C_2}\right)\times\left\{\frac{2}{\pi kR_0\Theta}\sin^2(kR_0\Theta+\tfrac{1}{4}\pi)\right\} \quad (2.9)$$

for $kR_0\Theta>6$.

In the case of monopole excitation

$$\frac{d\sigma}{d\Omega}(0\to 0)=(kR_0^2)^2\frac{1}{4\pi}|\langle b|\alpha_0|a\rangle|^2J_0^2(kR_0\Theta), \quad (2.10)$$

which is well approximated beyond $kR_0\Theta\sim 4$ by

$$\frac{d\sigma}{d\Omega}(0\to 0)\cong(kR_0^2)^2\frac{1}{4\pi}|\langle b|\alpha_0|a\rangle|^2\times\left\{\frac{2}{\pi kR_0\Theta}\sin^2(kR_0\Theta+\tfrac{1}{4}\pi)\right\}. \quad (2.11)$$

The matrix elements for monopole excitation cannot be estimated in the same manner as those of the other surface vibrations since they must be associated with a change in nuclear density. One possible treatment is to discuss dilational vibrations in C^{12} and O^{16} in terms of the alpha-particle model.[22] Let us assume that the change in surface radius, $R_0\alpha_0(4\pi)^{-\frac{1}{2}}$, is approximately the same as the radial displacement from the equilibrium position of the alpha particles, δr. The matrix element of δr has been estimated by Schiff:

$$|\langle b|\delta r|a\rangle|^2=\left(\frac{\hbar^2/2MA}{\hbar\omega}\right), \quad (2.12)$$

where M is the mass of the proton, A is the atomic number, and $\hbar\omega$ is the excitation energy for the first vibrational state. With the assumption that the 7.65-Mev state of C^{12} corresponds to a dilational vibration, the alpha-model prediction for the differential cross section to that state is then

$$\frac{d\sigma}{d\Omega}(0\to 0)=(kR_0)^2(0.23)J_0^2\times 10^{-26}\text{ cm}^2. \quad (2.13)$$

Comparison with inelastic electron scattering measure-

[19] Alder, Bohr, Huus, Mottelson, and Winther, Revs. Modern Phys. 28, 432 (1956), Eq. (II A.25). Here $(2n)!!\equiv 2\cdot 4\cdot 6\cdots 2n$ and $(2n+1)!!\equiv 1\cdot 3\cdot 5\cdots(2n+1)$.

[20] Reference 18, p. 1323.

[21] Reference 19, Eqs. (V. 26, 27, and 29).

[22] L. Schiff, Phys. Rev. 98, 1281 (1955).

ments indicates that these collective estimates of $\langle b|\delta r|a\rangle$ are too large by a factor 3 to 5.

For octupole surface vibrations,

$$\frac{d\sigma}{d\Omega}(0\rightarrow 3)=(kR_0^2)^2\left(\frac{7}{8\pi}\right)\left(\frac{\hbar\omega_3}{C_3}\right)\times\{\tfrac{3}{8}J_1^2(kR_0\Theta)+\tfrac{5}{8}J_3^2(kR_0\Theta)\},\quad(2.14)$$

which for $kR_0\Theta>8$ is well represented by

$$\frac{d\sigma}{d\Omega}(0\rightarrow 3)\cong(kR_0^2)^2\left(\frac{7}{8\pi}\right)\left(\frac{\hbar\omega_3}{C_3}\right)\times\left\{\frac{2}{\pi kR_0\Theta}\sin^2(kR_0\Theta-\tfrac{1}{4}\pi)\right\}.\quad(2.15)$$

Similarly, the cross section for $l=4$ vibrations would be

$$\frac{d\sigma}{d\Omega}(0\rightarrow 4)=(kR_0^2)^2\left(\frac{9}{8\pi}\right)\left(\frac{\hbar\omega_4}{C_4}\right)\times\left\{\frac{9}{64}J_0^2+\frac{20}{64}J_2^2+\frac{35}{64}J_4^2\right\}\quad(2.16)$$

which asymptotically becomes

$$\frac{d\sigma}{d\Omega}(0\rightarrow 4)\cong(kR_0^2)^2\left(\frac{9}{8\pi}\right)\left(\frac{\hbar\omega_4}{C_4}\right)\times\left\{\frac{2}{\pi kR_0\Theta}\sin^2(kR_0\Theta+\tfrac{1}{4}\pi)\right\}.\quad(2.17)$$

The cross sections have the following general characteristics: (1) The differential cross sections for even-parity excitations are out of phase with the elastic cross section; those for odd-parity excitations are in phase with the elastic cross section. This is a consequence of the restriction, $(l+m)$ even, in Eq. (2.6). This property also results from Born approximation calculations. (2) There is not the correspondence between the location of the first maximum and the order of the transition which appears in Born approximation calculations. Thus it is easy to determine the parity change of a transition but much more difficult to assign the angular momentum transfer from inspection of the experimental differential cross sections.

The rotational transitions of a nucleus with a small, but fixed, axially symmetric deformation lead to the same angular distributions.[13,23] Consider a nucleus whose radius is given by

$$R=R_0[1+\beta_l Y_{l0}(\bar{\theta})],\quad(2.18)$$

where $\bar{\theta}$ is the angle with respect to the symmetry axis. From the addition theorem for Legendre polynomials, one obtains

$$\alpha_{lm}=\left(\frac{4\pi}{2l+1}\right)^{\frac{1}{2}}\beta_l Y_{lm}(\omega),\quad(2.19)$$

where ω represents the orientation of the symmetry axis. The differential cross section for rotational excitation of an even-even target is then given to order β^2 by Eq. (2.7) with the familiar replacement

$$(2l+1)\hbar\omega_l/2C_l\rightarrow\beta_l^2.\quad(2.20)$$

These approximate formulas are easily generalized to odd-A nuclei for which the collective model is relevant. Consider first the case of vibrational excitation when (a) the ground-state wave function contains no surfon admixture and (b) excited states exist which result from the simple product of ground-state and one-surfon configurations. This is rarely a good approximation for $l=2$ surfons,[24] but should be more satisfactory for $l=3$; because the octupole surfons have odd parity, the ground state cannot contain one-surfon configurations (with no change of parity of particle configuration) while the lowest excited states have no zero-surfon configurations. In the above "extremely weak coupling" approximation, $|a\rangle=|jm\rangle$ while

$$|b\rangle=\textstyle\sum_{m'}(j,l,m',M'-m'|I',M')|j,m'\rangle|l,M'-m'\rangle,\quad(2.21)$$

where $|l,M'-m'\rangle$ is a one-surfon state and $(j,l,m',M'-m'|I',M')$ is a Clebsch-Gordan coefficient. The unitary property of the Clebsch-Gordan coefficients enables one to show that the cross section to a definite state I' with excitation of surfons of order l is related to the previous cross sections for an even-even nucleus, Eq. (2.7), by a simple statistical factor:

$$\frac{d\sigma}{d\Omega}(l;I\rightarrow I')=\frac{(2I'+1)}{(2l+1)(2I+1)}\frac{d\sigma}{d\Omega}(0\rightarrow l).\quad(2.22)$$

Further the statistical factor summed over $|I-l|\leqslant I'\leqslant(I+l)$ equals unity. Thus the sum of the cross sections over all "single-surfon" final states in an odd-A nucleus is predicted to be the same as the corresponding vibrational cross section in a neighboring even-even nucleus, provided the surface parameters are essentially unchanged.

Consider next the case of rotational excitation of an odd-A nucleus whose states are well described by the strong-coupling wave functions

$$|IMK\rangle=\left[\frac{2I+1}{16\pi^2}\right]^{\frac{1}{2}}\left\{\phi_\Omega D_{M,K}^I(\omega)+(-)^{I-j}\phi_{-\Omega}D_{M,-K}^I(\omega)\right\}.\quad(2.23)$$

[23] S. I. Drozdov, J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 1288 (1958) [translation: Soviet Phys. JETP **7**, 889 (1958)].

[24] See for example the discussion of Alder *et al.*, reference 19, Sec. V C. 5.

One again finds[25] that the cross section to a definite state I' is related by a statistical factor to the cross section previously computed for rotational excitation of an even-even nucleus, Eqs. (2.7) and (2.20):

$$\frac{d\sigma}{d\Omega}(l; I\to I')=(I,l,K,0|I',K')^2\frac{d\sigma}{d\Omega}(0\to l). \quad (2.24)$$

We note that Eq. (2.24) also contributes to elastic scattering when $I\neq 0$. For scattered amplitude linear in α,

$$\frac{d\sigma_{\mathrm{el}}}{d\Omega}=(kR_0^2)^2\frac{J_1^2(kR_0\Theta)}{(kR_0\Theta)^2}+(I,l,K,0|I,K)^2\frac{d\sigma}{d\Omega}(0\to l). \quad (2.25)$$

This implies that the elastic cross sections will flatten out and get in phase with even-parity change inelastic cross sections at the larger angles. Collective transitions are restricted within the rotational band built on the ground state.

III. COMPARISON TO BORN APPROXIMATION ANGULAR DISTRIBUTIONS

Assume that the potential for a projectile with mass m has a sharp boundary which is amenable to deformation:

$$\begin{aligned} V&=V_0, \quad r<R=R_0(1+\sum_{l,m}\alpha_{l,m}Y_{lm});\\ &=0, \quad r>R. \end{aligned} \quad (3.1)$$

To lowest order in the collective coordinates, the inelastic scattering cross section on a spin-zero target in Born approximation, with plane-wave initial and final wave functions over the entire nuclear surface,[4] is

$$\frac{d\sigma}{d\Omega}(0\to l)=\frac{1}{4\pi}\left[\frac{2m}{\hbar^2}R_0^3V_0\right]^2\frac{k_f}{k_0}j_l^2(KR_0)\times\{\sum_{m,M'}|\langle I'M'|\alpha_{lm}|0\ 0\rangle|^2\}, \quad (3.2)$$

where j_l is a spherical Bessel function. Similarly the elastic scattering cross section is

$$\frac{d\sigma_{\mathrm{el}}}{d\Omega}=\left[\frac{2m}{\hbar^2}R_0^3V_0\right]^2\left[\frac{j_1(KR_0)}{KR_0}\right]^2. \quad (3.3)$$

It is well known that the above Born approximation formula for the elastic scattering cross section from a square well is equivalent to the circular-black-disk cross section with the replacements

$$\left[\frac{2m}{\hbar^2}R_0^3V_0\right]\to(kR_0^2), \quad (3.4)$$

and $j\to J$. Comparison between Eq. (3.2) and Eqs. (2.9), (2.11), (2.15), and (2.17) shows that the same correspondence holds for the asymptotic form of the inelastic cross sections.

This correspondence is hardly surprising since both the Fraunhofer estimate for diffractional scattered amplitude and the Born approximation amplitude are suitable averages, over the nuclear volume or surface, of the phase factor $\exp(i\mathbf{K}\cdot\mathbf{r})$. In view of the large attenuation of nuclear projectiles indicated by elastic scattering analyses, we feel, however, that the strong-absorption diffraction model is the more plausible. An essentially equivalent statement is that the Born approximation estimates have proved successful only because they so closely correspond to the results of the diffraction model.

IV. ANGULAR CORRELATION OF GAMMA RAYS

Correlation functions are obtained from combining the nuclear statistical matrix with suitable gamma statistical matrices. The nuclear statistical matrix is defined as

$$A_{M',M''}{}^{I'}\equiv\frac{1}{2I+1}\sum_M\langle M'|f(\alpha,\Theta)|M\rangle^*\times\langle M''|f(\alpha,\Theta)|M\rangle. \quad (4.1)$$

Let us here restrict ourselves to the case of even-even nuclei, so that $I=0$, with a subsequent pure multipole transition back to the ground state. Then the corresponding statistical matrix for unpolarized gamma rays is

$$\begin{aligned} a_{M',M''}{}^{I}=&(2I'+1)(4\pi)^{\frac{1}{2}}(-)^{M'+1}\\ &\times\sum_{\nu=0,\ \mathrm{even}}^{2I'}Y_{\nu,\ M'-M''}(\theta_\gamma,\phi_\gamma)\frac{1}{(2\nu+1)^{\frac{1}{2}}}\\ &\times(I',I',-M',M''|\nu,M''-M')\\ &\times(I',I',-1,1|\nu,0). \end{aligned} \quad (4.2)$$

The joint probability for finding the inelastically scattered projectile at angle Θ and gamma radiation at angle θ_γ, ϕ_γ with respect to beam ($\phi_\gamma=0$ corresponds to the gamma ray in the scattering plane) is proportional to the correlation function

$$W(\Theta,\theta_\gamma,\phi_\gamma)=\sum_{M',M''}A_{M',M''}{}^{I'}a_{M',M''}{}^{I'}. \quad (4.3)$$

Let us consider first, the quadrupole excitation ($l=2$). Since the scattered amplitudes have the symmetry property $\langle M'|f|0\rangle=\langle -M'|f|0\rangle$, we may use the result of Banerjee and Levinson[26] for correlation in the scattering plane:

$$W(\Theta,\theta_\gamma,\phi_\gamma=0)=a+b\sin^2[2(\theta_\gamma-\theta_0)], \quad (4.4)$$

[25] S. I. Drozdov, J. Exptl. Theoret. Phys. (U.S.S.R.) **30**, 786 (1956) [translation: Soviet Phys. JETP **3**, 759 (1956)].

[26] M. K. Banerjee and C. A. Levinson, Ann. Phys. **2**, 499 (1957). We correct some misprints in Eqs. (38), (39), and (40) of this paper.

where

$$a+\tfrac{1}{2}b=|T_0{}^2|^2+(8/3)|T_1{}^2|^2+\tfrac{2}{3}|T_2{}^2|^2 -\tfrac{1}{3}(\sqrt{6})(T_0{}^2T_2{}^{2*}+\text{c.c.}), \quad (4.5)$$

$$\tfrac{1}{2}b\sin 4\theta_0=\tfrac{4}{3}(T_2{}^2T_1{}^{2*}+\text{c.c.}) -\tfrac{2}{3}(\sqrt{6})(T_1{}^2T_0{}^{2*}+\text{c.c.}), \quad (4.6)$$

$$\tfrac{1}{2}b\cos 4\theta_0=a+\tfrac{1}{2}b-(16/3)|T_1{}^2|^2, \quad (4.7)$$

and $T_{M'}{}^2\equiv\langle M'|f|0\rangle$. Further, since $\langle 1|f|0\rangle=0$, one obtains $a=0$ and $\sin 4\theta_0=0$ so that[27]

$$W(\Theta,\theta_\gamma,\phi_\gamma=0)=b\sin^2(2\theta_\gamma). \quad (4.8)$$

This simple result may be understood from consideration of the scattered amplitude for the case of rotational excitation where $\alpha_{2m}=(4\pi/5)^{\frac{1}{2}}\beta_2Y_{2m}(\omega)$. For very small Θ, only $m=0$ contributes and the correlation function is the radiation pattern resulting from a quadrupole oscillator symmetric about the z axis. For $kR_0\Theta>6$, it is reasonable to approximate the Bessel functions by their asymptotic forms, in which case the scattered amplitude for $l=2$ can be shown to be proportional to $Y_{20}(\omega')$, where ω' gives orientation of the ellipsoid with respect to the x axis, i.e., the direction in the scattering plane perpendicular to the beam. Thus in the asymptotic region, the x axis becomes the symmetry axis and the correlation function in the scattering plane remains proportional to $\sin^2(2\theta_\gamma)$.

The correlation function following octupole excitation is readily obtained in the asymptotic region since one then finds that the scattered amplitude is proportional to $Y_{30}(\omega')$; i.e., the x axis is again a symmetry axis. The gamma correlation function is thus proportional to $[P_3{}^1(\theta_\gamma')]^2$, where θ_γ' is the angle between the gamma-ray direction and the x axis and $P_l{}^m$ is an associated Legendre function.

The above predictions are in only partial agreement with the Born approximation prediction[28] and experiment.[29,30] The Born approximation correlation functions have the same simple form but θ_γ is taken with respect to the direction of momentum transfer, $\mathbf{K}$. The angle between the $\mathbf{K}$ direction and the z axis is approximately given by $\theta_K\cong\pi/2-\Theta/2$, so that only for small Θ and negligible excitation energy is $\mathbf{K}$ essentially parallel to the x axis.

We believe that the failure of our correlation function to track the changing $\mathbf{K}$ direction at larger scattering angles is due to the use of an approximation which is valid only at small scattering angles. Specifically, the assumption that the appropriate "shadow line" is the projection of the nuclear surface on the x-y plane neglects the symmetry of the scattered amplitude with respect to the initial and final momenta. A more appropriate choice of shadow line for the larger Θ might be the projection on to the $\mathbf{K}$-y plane, since this plane is symmetric to both the incident and final beams. Such a choice leads to all the previous results in Secs. II and IV except that the argument $kR_0\sin\Theta$ is replaced by $2kR_0\sin(\Theta/2)$ and θ_γ is the angle between the gamma ray and the $\mathbf{K}$ axis.

The correlation function for circularly polarized gamma rays is predicted to be zero for the Drozdov model. From Eq. (2.6) one sees the nuclear statistical matrix is unchanged under the exchange $M'\leftrightarrow -M''$. The statistical matrix for circularly polarized gamma rays, $\hat{a}_{M',M''}{}^{I'}$, is the same as Eq. (4.2) except that the ν sum runs over only odd ν. From the fact that $l+m$ is even and the symmetry of the Clebsch-Gordan coefficient $(I',I',-M',M''|\nu,M''-M')$, one finds that $\hat{a}_{M',M''}{}^{I'}$ is odd under this exchange and hence the correlation function vanishes.

V. LIMITATIONS AND APPROXIMATIONS OF THE MODEL

Many approximations of marginal validity have been employed in this paper.

(a) The adiabatic approximation. The Hamiltonian for the quantum mechanical system, projectile plus deformed or deformable nucleus is

$$H=K+V(\mathbf{r},\alpha)+H(\alpha) \quad (5.1)$$

where K is the kinetic energy of the projectile, V is the optical potential, and $H(\alpha)$ is the Hamiltonian of the collective coordinates.[31] The formal solution of the scattered amplitude is[32]

$$\langle b|T|a\rangle=\left\langle b\left|V+V\frac{1}{E-K-V-H(\alpha)+i\epsilon}V\right|a\right\rangle, \quad (5.2)$$

where $|a\rangle$ and $|b\rangle$ are solutions of the free Hamiltonian, $H_0=K+H(\alpha)$. The adiabatic approximation amounts to neglect of $H(\alpha)$ in the denominator of the second term.[33,34] This neglect seems plausible since it occurs in a propagator which is sandwiched between the V's; in other words, the intermediate states which contribute to the matrix element will generally have energies considerably different from E so that $H(\alpha)$ is small compared to typical values of the energy difference $E-K-V$.

The criterion for the validity of the adiabatic approximation is usually given[13,34] to be that the classical time for projectile to pass the nucleus should be short com-

[27] Equation (4.8) appears to be in disagreement with the correlation function obtained by E. V. Inopin, reference 14.
[28] G. R. Satchler, Proc. Phys. Soc. (London) **A68**, 1037 (1955).
[29] R. Sherr and W. F. Hornyak, Bull. Am. Phys. Soc. Ser. II, **1**, 197 (1956).
[30] G. B. Shook, Phys. Rev. **114**, 310 (1959).
[31] See reference 19, Eq. (V.2) and subsequent.
[32] M. Gell-Mann and M. L. Goldberger, Phys. Rev. **91**, 398 (1953).
[33] G. F. Chew and G. C. Wick, Phys. Rev. **85**, 636 (1952); G. F. Chew and M. L. Goldberger, Phys. Rev. **87**, 778 (1952).
[34] D. M. Chase, Phys. Rev. **104**, 838 (1956).

pared to the period for collective motion, i.e.,

$$\frac{R}{v}\frac{1}{(1/\omega)}=(kR)\frac{(\hbar\omega)}{(\hbar^2k^2/m)}\ll 1. \qquad (5.3)$$

Since $kR\sim 10$ for 20-Mev alpha particles and $\hbar\omega$ is typically in the range 1–10 Mev, this criterion is rarely comfortably satisfied. A criterion suggested by inspection of Eq. (5.2), $\hbar\omega/E\ll 1$, is more easily satisfied.

A related approximation, that $k_f\cong k_i$ in Eq. (2.2), will be justified[34] when

$$(k_i-k_f)R\Theta\cong(kR\Theta)\hbar\omega/[(\hbar^2k^2/m)]\ll 1. \qquad (5.4)$$

This restriction is not easily satisfied for $\Theta\sim 1$ but we shall find this to be but one of many difficulties which occur at large Θ.

(b) The scattered amplitude is expanded only to terms first order in α. One sees from Eq. (2.2) that the most relevant expansion parameter is not α itself but rather $kR_0\Theta\alpha$.[14] For quadrupole vibrational excitations, $|\langle b|\alpha|a\rangle|^2=\hbar\omega_2/2C_2\sim 1$ Mev/2(50 Mev), so that the predicted cross sections are not expected to be reliable for $kR_0\Theta\gtrsim 10$. The convergence is worse for rotational excitation where the deformations are larger. One finds, by explicitly expanding Drozdov's scattered amplitude from an ellipsoid of arbitrary deformation, that the correction terms tend to wash out the interference structure.

(c) Complete absorption is assumed within the sharp radius $R(\pi/2,\phi)$. Both the statement that there is a sharp boundary and that the far side of the nucleus is "black" are approximations, since it is well known that the nuclear density is tapered near the surface and that even energetic alpha particles have a nonzero mean free path.[6] Recently McCarthy[35] has computed the alpha current flux in the nuclear region resulting from optical-model calculations with parameters which fitted elastic scattering from argon; he finds that the alpha particles do penetrate beyond the surface and that there is a small but finite flux on the "dark" side of the nucleus. Nevertheless, this flux is much closer to the zero flux assumed in deriving the diffraction formula Eq. (2.1), than to an undamped plane-wave current. Further, the lack of sensitivity of elastic differential cross sections to changes in the internal optical potential[7,36] and the qualitative agreement between strong-absorption calculations and alpha elastic scattering cross sections, for both heavy[9,37] and light[8] targets, suggest that the scattered amplitude at forward angles is not very sensitive to small deviations from the "black" condition on the far side of the nucleus.

(d) The Fraunhofer approximation for the scattered amplitude. This is basically a small-scattering-angle approximation. Indicative of this is the fact that one can derive formulas for the scattering from a black sphere where the arguments variously contain $\sin\Theta$, $2\sin(\Theta/2)$, or even $2\tan(\Theta/2)$, corresponding to different choices for the shadow line; in the last two cases the shadow line is chosen to be a circle or ellipse, respectively, in the $\mathbf{K}$-y plane. Clearly all forms are equivalent at small Θ; the extension of the simple Fraunhofer formulas to larger angles however, is more a matter of recipe than clean derivation (see the discussion near the end of Sec. IV). It frequently has been observed[8,38,39] that the angular separation or the difference in $\sin(\Theta/2)$ between the maxima or minima in elastic angular distributions is constant; this suggests that the Fraunhofer formulas, with the argument $kR_0\Theta$ or $2kR_0\sin(\Theta/2)$, may have some relevance at larger angles, but we recognize that this is an extrapolation beyond their intended range of validity.†

(e) Coulomb effects have been neglected.[40] These will manifest themselves in two ways: (1) distortion of the plane wave fronts near the nucleus; (2) specific Coulomb contributions to the scattered amplitude. Both of these effects will influence very-small-angle scattering from light nuclei but are probably safely ignored for larger angles. An upper limit on the specific Coulomb contributions to the scattered amplitude is provided by a Born approximation calculation in which the incident and final projectile wave functions are represented by plane waves and a uniform charge density is assumed with a charge radius approximately equal to the absorption radius $R(\theta,\phi)$. One obtains the results of Sec. III with the replacement

$$\left[\frac{3R_{180}}{4\sin^2(\Theta/2)}\right]\to\left[\frac{2m}{\hbar^2}R_0^3V_0\right], \qquad (5.5)$$

where

$$R_{180}\equiv ZZ'e^2/E=ZZ'(1.44)\times(10^{-13})\text{ cm}/E\text{ (in Mev)},$$

the distance of closest approach for a head-on collision between point charges Z and Z' (target and projectile, respectively). At moderate scattering angles, even were the plane-wave approximation valid for treating Coulomb scattering and excitation, the distance $[3R_{180}/$

[35] I. E. McCarthy, Nuclear Phys. **10**, 583 (1959).
[36] G. Igo, Phys. Rev. Letters **1**, 72 (1958).
[37] Kerlee, Blair, and Farwell, Phys. Rev. **107**, 1343 (1957).
[38] Igo, Wegner, and Eisberg, Phys. Rev. **101**, 1508 (1956).
[39] Seidlitz, Bleuler, and Tendam, Phys. Rev. **110**, 682 (1958).
† *Note added in proof.*—Additional arguments may be given for choosing the argument in the Fraunhofer formulas to be $(kR_0\Theta)$ or $[2kR_0\sin(\Theta/2)]$: (a) R. J. Glauber, in unpublished notes entitled "High Energy Collision Theory," has pointed out that the necessary symmetry of the scattered amplitude, for fixed scatterer, with respect to interchange and sign reversal of the initial and final momenta is manifestly included if the shadow plane of Eq. (2.1) is taken to be perpendicular to the direction $\mathbf{k}_i+\mathbf{k}_f$. In this way the argument naturally occuring in the Fraunhofer expressions is $[2kR_0\sin(\Theta/2)]$. (b) In collaboration with D. Sharp and L. Wilets (to be published), we have recently derived cross sections for monopole and quadrupole excitation without use of the Fraunhofer approximation. At small angles and for large kR_0, these results are equivalent to the Fraunhofer formulas; the similarity is maintained out to $\Theta\sim 90°$ when the argument $kR_0\Theta$ is used in the Fraunhofer expressions.
[40] S. I. Drozdov, Atomnaya Energ. **2**, 501 (1957) [translation: J. Nuclear Energy **7**, 231 (1958)].

$4\sin^2(\Theta/2)]$ is generally small compared to its equivalent in diffraction expressions, $[kR_0^2]$.

For very small angles such that $kR\Theta \ll 1$ and for small Coulomb distortion (i.e., $n \equiv ZZ'e^2/\hbar v < 1$), the Born approximation estimates for Coulomb scattering and excitation are realistic; the Coulomb matrix elements then receive their essential contributions from a large enough volume, $(1/K)^3 = (1/k\Theta)^3$, so that they are not greatly altered by the hole in the incident and final plane waves which is taken out by the absorbing nucleus. The Coulomb and diffraction contributions are out of phase so that the cross sections may be added. For the above restrictions,[41]

$$\frac{d\sigma_{\text{el}}}{d\Omega} = \frac{9R_{180}^2}{16\sin^4(\Theta/2)}\left[\frac{j_1(KR)}{KR}\right]^2 + [kR_0^2]^2 \times \left[\frac{J_1(kR_0\Theta)}{(kR_0\Theta)}\right]^2 \quad (5.6)$$

$$\rightarrow \frac{R_{180}^2}{\Theta^4} + \tfrac{1}{4}[kR_0^2]^2, \quad (5.7)$$

and

$$\frac{d\sigma}{d\Omega}(0\rightarrow 2) = \frac{9R_{180}^2}{16\sin^4(\Theta/2)}\left(\frac{5}{4\pi}\right)\left(\frac{\hbar\omega_2}{2C_2}\right) j_2^2(kR_0\Theta) + [kR_0^2]^2\left(\frac{5}{4\pi}\right)\left(\frac{\hbar\omega_2}{2C_2}\right)\{\tfrac{1}{4}J_0^2(kR_0\Theta) + \tfrac{3}{4}J_2^2(kR_0\Theta)\} \quad (5.8)$$

$$\rightarrow [kR_0^2]^2\left(\frac{5}{4\pi}\right)\left(\frac{\hbar\omega_2}{2C_2}\right) \times \left[\left(\frac{kR_{180}}{5}\right)^2 + \tfrac{1}{4}\right]. \quad (5.9)$$

Thus Coulomb quadrupole excitation will be competitive with the corresponding diffractional inelastic scattering at very forward angles. On the other hand, both Coulomb and diffraction contributions to the cross section for octupole excitation are here predicted to approach zero at forward angles.

One general criticism which previously has been made of simple Born or impulse approximation models also applies to the present calculation[5]: Some of the desired features of the angular distributions, such as regularly spaced and sharply defined oscillations, and large effective radii, are essentially guaranteed by our drastic approximations. It will be interesting to see if these features stlll emerge when the rotational optical model of Chase, Wilets, and Edmonds[42] is extended to medium-energy projectiles, since the Drozdov model should be merely an approximation to that model.

[41] A. Akhieser and I. Pomeranchuk, Uspekhi Fiz. Nauk **65**, 593 (1958), Eq. (30).

[42] Chase, Wilets, and Edmonds, Phys. Rev. **110**, 1080 (1958).

VI. COMPARISON TO EXPERIMENT

If the present calculations are appropriate to nature, not only must they yield the observed elastic and inelastic scattering angular distributions but also lead to reasonable values for the collective matrix elements. There has been accumulating evidence that collective concepts are applicable to the light nuclei, not only in the *s-d* shell[43] but also in the *p* shell.[44]

Concerning angular distributions we generally find that inelastic scattering data which previously have been fitted with Born approximation expressions are either equally well or better fitted with the formulas of Sec. II in which the radii are those already determined from a fit to the corresponding elastic scattering angular distributions. The squares of spherical Bessel functions occurring in the Born approximation formulas have maxima and minima at arguments which are approximately 45° larger than those at corresponding maxima and minima in the ordinary Bessel functions; this has led to interaction radii, determined from fitting the small-angle structure of inelastic scattering angular distributions to Born approximation expressions, which tend to be larger than the corresponding black-disk radii from analysis of the elastic scattering.[8] This discrepancy is removed when both the elastic and inelastic scattering are analyzed with the diffraction model.

Comparisons will be here restricted to medium-energy alpha particle or deuteron scattering experiments, since it is here found that the strong-absorption approximations have their greatest validity and further the wavelengths are short enough so that many orders of a diffraction pattern may possibly occur in a typical angular distribution. As an example, we show in Figs. 2 and 3 angular distributions for alpha elastic scattering and inelastic scattering to the 2.24-Mev (2+) and 4.98-Mev levels, respectively, of S^{32}.[45] The maxima and minima of the elastic pattern have been well fitted to the black-disk prediction with $R_0 = 6.19 \times 10^{-13}$ cm. [The Fraunhofer formula has been extended to large angles with the recipe that the argument is taken as $2kR_0 \sin(\Theta/2)$, corresponding to the shadow line in the **K**-y plane; see Sec. V(d).] The inelastic scattering angular distributions are clearly out of phase and in phase, respectively, with the elastic pattern; they are compared to Eqs. (2.8) and (2.14), respectively, arbitrarily matched at small angles, where the approximations are most valid. [Again, the inelastic scattering diffraction formulas have been extrapolated to large angles with the argument $2kR_0 \sin(\Theta/2)$.] This normalization leads to $|\beta_2| = 0.4$ or $C_2 = 40$ Mev [2.24-Mev (2+) level] and $|\beta_3| = 0.2$ or $C_3 = 350$ Mev [4.98-Mev level]. Both elastic and inelastic scattering experimental cross sections rapidly fall below the diffraction formulas

[43] G. Rakavy, Nuclear Phys. **4**, 375 (1957).

[44] D. Kurath, Phys. Rev. **106**, 975 (1957).

[45] P. C. Robison and G. W. Farwell (to be published).

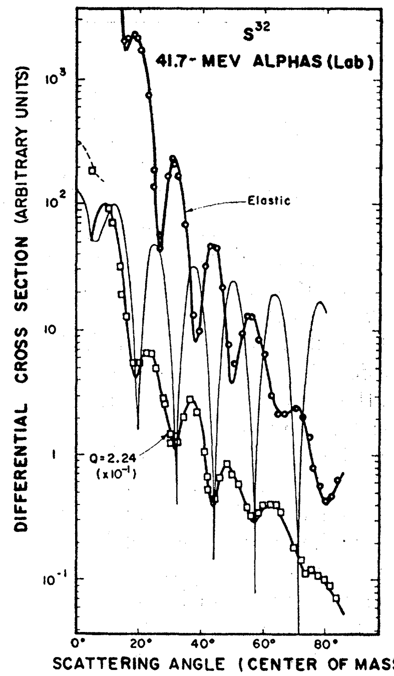

FIG. 2. Angular distribution of alpha particles from S^{32} with incident laboratory energy of 43 Mev. The open circles are the elastic scattering cross sections and the open squares are the inelastic scattering cross sections to the 2.24-Mev (2+) state (multiplied by a factor 10^{-1}) as measured by Robison and Farwell.[45] The solid curve gives the theoretical cross section for a collective quadrupole transition where the radius, R_0, is that deduced from a fit to the elastic angular distribution, 6.19×10^{-13} cm. The dashed curve is the theoretical cross section including Coulomb excitation, Eq. (5.8); this estimate is realistic only for very small angles.

as the scattering angle increases while spacing continues to be regular; this is another demonstration[9] of the phenomenon that a crude physical optical model satisfactorily gives the location of the oscillations but fails to yield their magnitude for large orders in the pattern. We note that the envelopes of both inelastic scattering curves have the same angular dependence which strengthens the interpretation that both involve the same sort of nuclear excitation, differing only with regard to multipolarity.

Other scattering experiments are discussed below, in order of increasing Z, in terms of naive application of the simple diffraction formulas of Sec. II.

Be^9—48-Mev α[46]

The location of the minima in the elastic scattering angular distribution leads to $R_0=5.0\times10^{-13}$ cm. The black-disk elastic scattering cross section at the first maximum is then 119 millibarns, in good agreement with the observed cross section, 118 millibarns. The predicted cross section at the second maximum is 28 millibarns, to be compared with the observed 16.5 millibarns. The inelastic scattering cross sections to the 2.43-Mev level are then correctly predicted to have minima at 29° and 47°. With the assumption that this state has $I=\frac{5}{2}$, $K=\frac{3}{2}$, application of Eqs. (2.24) and (2.8) to the first maximum of the inelastic scattering cross section yields $|\beta_2|=0.46$. We note that the Drozdov model avoids one of the difficulties which we encountered when the impulse approximation was applied to an alpha-particle model for Be^9, namely that the calculated interaction radius was far too small[5]; at the same time it allows us to retain the alpha-particle model as a collective description which leads to large prolate ellipsoidal distortion.

Be^9—24-Mev deuterons[46]

If the interaction radius for deuterons is the same as that for alpha particles, corresponding points in the 24-Mev deuteron and 48-Mev alpha particle cross sections should be related by the ratio $k_d^2/k_\alpha^2=0.35$. Thus one expects 41 and 5.8 millibarns at the first two deuteron elastic scattering maxima while Summers-Gill finds 26.6 and 5.05 millibarns, respectively. Similarly, one anticipates 16.4 and 2.6 millibarns at the first two inelastic scattering maxima where 12.5 and 5.8 millibarns are observed. Actually, the spacing of the oscillations of the deuteron elastic scattering angular distri-

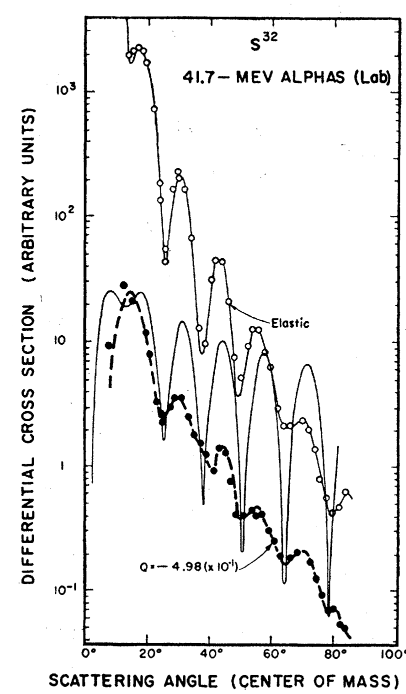

FIG. 3. Angular distribution of alpha particles scattered from S^{32} with incident laboratory energy of 43 Mev. The open circles are the elastic scattering cross sections and the closed circles are the inelastic scattering cross sections to the 4.98-Mev state (multiplied by a factor 10^{-1}) as measured by Robison and Farwell.[45] The solid curve gives the theoretical cross section for a collective octupole transition where the radius, R_0, is that deduced from the elastic scattering data, 6.19×10^{-13} cm.

[46] R. G. Summers-Gill, Phys. Rev. **109**, 1591 (1958).

bution suggest a somewhat larger interaction radius, 5.35×10^{-13} cm, which increases the discrepancy between the strong absorption cross sections and the observed experimental values.

C^{12}—41-Mev α[8,47]

Analysis of the elastic scattering yields $R_0=5.1\times10^{-13}$ cm. Application of Eq. (2.8) to the first maximum of the inelastic scattering cross section to the 4.43-Mev (2+) level gives $|\beta_2|=0.28$. The center-of-mass angular distribution to the 7.65-Mev level, thought to be 0+, is 90° out of phase with the elastic scattering angular distribution over three maxima and minima, in accordance with Eq. (2.10). Comparison between this formula and the experimental cross section at center-of-mass angle 38°, 1.6 millibarns, leads to $\langle b|\alpha_0|a\rangle=0.086$. The corresponding change in the surface radius, $R_0\alpha_0/(4\pi)^{-\frac{1}{2}}$, is then a factor 4 times smaller than radial displacement as estimated by the alpha-particle model, Eq. (2.12). It has been observed by Schiff[22] that the experimental value for the electric monopole matrix element is similarly a factor 3 to 5 times smaller than the alpha-particle model prediction, which is again proportional to $\langle b|\delta r|a\rangle$. When the dilational excitation of an alpha-particle model was considered in the Born or impulse approximation,[5] the angular distribution was predicted to be proportional to $K^2j_1^2(KR)$ which is at variance with experiment. It is gratifying to find that the failure of this calculation to match experiment is attributable to the use of the Born or impulse approximation rather than to the notion of collective excitation. The large excitation energy, 7.65 Mev, means that the adiabatic approximation is highly questionable.

O^{16}—40-Mev α[8]

Analysis of the elastic scattering cross section leads to $R_0=5.64\times10^{-13}$ cm. The inelastic scattering cross section to the 6.06-Mev (0+)–6.13-Mev (3−) doublet is compared to the theoretical cross section for octupole excitation, Eq. (2.14), in Fig. 4. The normalization chosen in this figure gives $|\beta_3|=0.18$ or $C_3=650$ Mev. The experimental cross section attributable to excitation of the 0+ state is unknown; if it is of the order of the cross section for excitation of the 7.65-Mev level in C^{12}, however, the contamination due to this level will only slightly decrease the estimates above.[48] The angular distributions of Fig. 4 have been cut off at 90°, for although there is structure in both elastic and inelastic scattering experimental cross sections beyond this angle, there is no correspondence to the simple diffraction formulas.

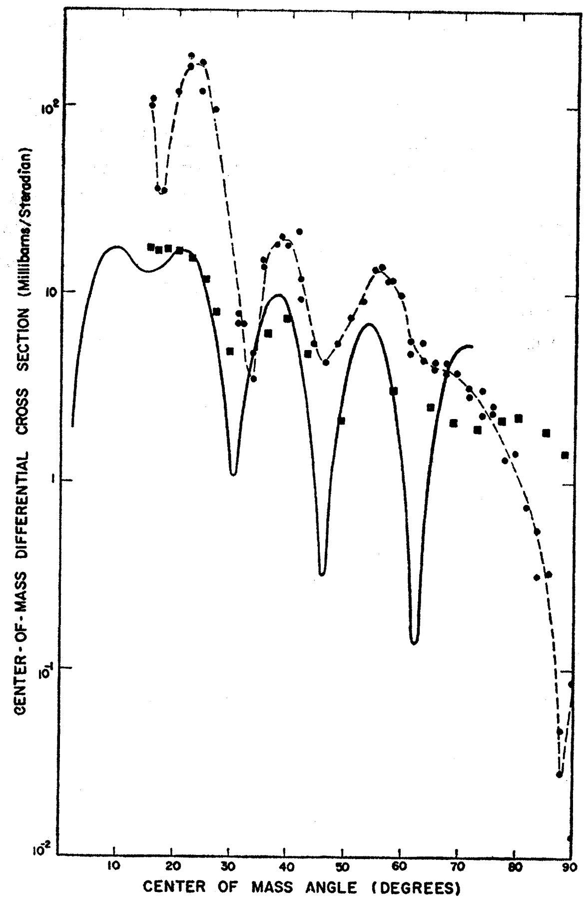

FIG. 4. Angular distribution of alpha particles scattered from O^{16} with incident laboratory energy of 41 Mev. The circles are the elastic scattering cross sections and the squares are the inelastic scattering cross sections to the 6.06-Mev (0+)–6.14-Mev (3−) unresolved doublet as measured by Yavin and Farwell.[8] The solid curve gives the theoretical cross section for a collective octupole transition where the radius is deduced from a fit to the elastic scattering angular distribution, 5.64×10^{-13} cm, and $|\beta_3|$ is equal to 0.18.

Ne^{20}—18.02-Mev α[39]

A fit to the second, third, and fourth minima of the elastic scattering cross section gives an average $R_0=6.16\times10^{-13}$ cm. The predicted elastic scattering cross section at the first maximum is then 149 millibarns while the observed value is 167 millibarns. The inelastic scattering angular distribution with excitation of the 1.63-Mev (2+) level is perfectly out of phase with the elastic scattering oscillations for scattering angle less than 90°. When 67 millibarns is taken as the inelastic scattering cross section at the first maximum (this disregards the high cross section at the first angle), one finds $|\beta_2|=0.35$ or $C_2=34$ Mev. The first two maxima for excitation of the 7.2-Mev group, which contains a 3− state, are clearly in phase with the elastic and out of phase with the 1.63-Mev inelastic scattering angular distributions. When interpreted in terms of octupole excitation, the observed cross section to the 7.2-Mev group at the

[47] S. F. Eccles and D. Bodansky, Phys. Rev. **113**, 608 (1959).
[48] See discussion of monopole matrix elements of O^{16} in J. J. Griffin, Phys. Rev. **108**, 328 (1957).

first maximum, 25 millibarns, leads to $|\beta_3|=0.26$ or $C_3=370$ Mev.

Mg^{24}—42-Mev α[11,30]

The oscillations of the elastic and 1.37-Mev (2+) inelastic scattering angular distributions are out of phase over five maxima. A match to the inelastic cross section at the first maximum yields $|\beta_2|=0.3$. Shook has also observed strong excitation of a level or group at 6.3 Mev. The second maximum near 30° is clearly in phase with the elastic scattering angular distribution. The estimated cross section at this point, 3 millibarns, when interpreted as octupole excitation leads to $|\beta_3|=0.08$ or $C_3=3400$ Mev. (The estimate of $|\beta_3|$ is probably an underestimate since the observed cross sections at second and higher maxima are generally lower than the predictions of the diffraction theory.)

A^{40}—40-Mev α[8]

The interaction radius from the elastic scattering angular distribution is $6.38\times(10^{-13})$ cm and the ratio of observed to black-disk cross section at the first maximum is 0.97. The inelastic scattering cross section to the 1.43-Mev state is roughly out of phase with the elastic pattern over a range of five minima. The inelastic scattering cross section at the second maximum gives $|\beta_2|=0.09$ or $C_2=490$ Mev, values which probably reflect the proximity of the shell closure at $Z=N=20$.

Ca^{40}—43-Mev α[30]

The inelastic scattering angular distribution to a state believed to be at 4.48 Mev is in phase with the elastic scattering pattern over a range of five maxima. Absolute cross sections were not measured; however, the relative elastic scattering cross section is almost identical to the elastic cross section from A^{40} at 41 Mev which suggests that the magnitudes also are the same. The inelastic scattering cross section at its first maximum is then estimated to be 6.0 millibarns which corresponds to $|\beta_3|=0.095$ or $C_3=1800$ Mev.

Low-lying levels of intermediate-mass nuclei ($Z>20$) have been strongly excited through inelastic scattering of protons.[16,17] The levels below or near 1 Mev are almost invariably those strongly excited by Coulomb excitation and are fairly well understood in terms of quadrupole surface vibrations. It has been suggested by Lane and Pendlebury[17] and independently by Levinson[49] that the "anomalous" excitation might be understood in terms of surface octupole excitation. Recent alpha-particle scattering experiments by Chen and McDaniels[50] have strengthened this interpretation; comparison of the elastic and inelastic 2+ and "anomalous" angular distributions show that the elastic and "anomalous" patterns are in phase, but out of phase with the 2+ inelastic oscillations. This work will be reported separately.

In summary, many of the alpha-particle and deuteron inelastic scattering angular distributions following excitation of low-lying levels of light nuclei are consistent with the predictions of the simple diffraction model. There is independent evidence for large quadrupole deformations for most of the nuclei (C^{12}, Ne^{20}, Mg^{24}, S^{32})[51] where a fit to the diffraction formulas also yields large $|\beta_2|$. A state associated with octupole deformation appears to be present in all the $4n$ nuclei examined; the energy of this state slowly decreases with increasing A [O^{16}, 6.13 Mev (3−); Ne^{20}, 7.2 Mev (3−); Mg^{24}, 6.3 Mev; S^{32}, 4.98 Mev; Ca^{40}, 4.48 Mev].

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[49] C. A. Levinson (private communication).
[50] S. W. Chen and D. McDaniels (private communication).
[51] R. H. Helm, Phys. Rev. **104**, 1466 (1956).