Expansion Coefficient and Entropy of Liquid He³ under Pressure below 1°K*

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Measurements have been made of the change of temperature on adiabatic expansion of liquid He³ in the temperature range 0.15°K to 1.15°K, at average pressures up to 22 atmospheres. From these measurements, evaluations have been made of the expansion coefficient, α_p, at constant pressure, and of the change in entropy on compression from the saturated vapor pressure to higher pressures. The latter calculations, combined with the previously determined entropy at the saturated vapor pressure, give values of S_p, the absolute entropy under pressure. It is found that at all pressures α_p is negative up to a temperature T_0 which is a monotonically increasing function of pressure. Correspondingly, at sufficiently low temperatures $(\partial S_p/\partial p)_T$ is positive for all pressures, in agreement with theory. The S_p isobars allow plausible extrapolations to the absolute zero, from which the limiting slope at 0°K has been estimated. Comparison of these slopes with the very-low-temperature nuclear magnetic susceptibility yield some information concerning the variation with pressure of the spin-dependent interactions in liquid He³.

I. INTRODUCTION

IN a previous paper,[1] which we now refer to as I, measurements were reported of the specific heat of liquid He³ under a small pressure of 6 to 14 cm Hg down to a temperature of 0.085°K, and under higher pressures up to 29 atmospheres at temperatures between 0.12°K and 0.6°K. The measurements at the lowest pressure indicated that the specific heat tended to become a linear function of temperature in its approach to 0°K, with a slope of 4.00 cal/mole-deg². By extrapolation of this linear relation to 0°K, it was possible to compute from our results the entropy, S_{sat}, of liquid He³ under its saturated vapor pressure. The measurements at higher pressures, however, did not reach to low enough temperatures to allow reliable extrapolation of C_p to 0°K, and hence the entropy under pressure, S_p, could not be computed. It was in order to compute S_p by other methods that the present experiments were undertaken and they have yielded not only evaluations of S_p but also preliminary data for the thermal expansion coefficient under constant pressure, α_p.

The experiments consisted in measurement of the temperature changes, ΔT, occurring in liquid He³ during adiabatic expansions. If the adiabatic expansions are reversible, the effect is at constant entropy and $(\partial T/\partial p)_S$ is related thermodynamically to the thermal expansion coefficient at constant pressure, α_p, as follows:

$$\alpha_p = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p = \frac{C_p}{VT}\left(\frac{\partial T}{\partial p}\right)_S. \qquad (1)$$

As is discussed in Sec. III, our measurements of $\Delta T/\Delta p$ yield dT/dp within an error of at most 0.1 millideg/atmos, and from various considerations, also discussed in Sec. III, it is concluded that the irreversibilities in the experiments were small. We therefore have calcu-

lated α_p from our observed values of $\Delta T/\Delta p$ and from our C_p data of I by means of the following modification of Eq. (1):

$$\alpha_p \approx \frac{C_p}{VT}\left(\frac{\Delta T}{\Delta p}\right)_{obs}. \qquad (2)$$

The allowances made for changes in T, V and C_p during expansions are discussed in Sec. III, where we give evaluations of α_p at various pressures between 1.70 and 22.0 atmospheres in the temperature range 0.15°K to 0.6°K. It is interesting to note that $\Delta T/\Delta p$ and α_p are negative from the lowest temperature of measurement up to a temperature T_0 which is a monotonically increasing function of p. Below T_0, adiabatic compression produces a cooling, but the effect is small, as is detailed in Sec. III, and consequently would not offer an effective method of reaching low temperatures.

From our values of α_p as a function of p and T we can now obtain the entropy under pressure, S_p, using the equation

$$S_p - S_{sat} = -\int_{SVP}^{p} V\alpha_p dp, \qquad (3)$$

where S_{sat} is the entropy at the saturated vapor pressure, determined in I. We have evaluated $(S_p - S_{sat})$ at 0.6°K using our α_p values together with the value of α_p at the saturated vapor pressure calculated from the results of Taylor and Kerr[2]; and then have computed S_p at other temperatures by use of the values of C_p reported in I. In Sec. III the results of this computation are presented, giving S_p at 5.0, 10.0, 15.0, and 22.0 atmospheres in the temperature range 0.12°K to 0.6°K. In accordance with the negative values of α_p, it is found that at sufficiently low temperatures S_p increases with increasing pressure, as expected from the C_p data given in I.

We have also calculated $(S_p - S_{sat})$ more directly, without going through the procedure of integrating the

[1] Brewer, Daunt, and Sreedhar, preceding paper [Phys. Rev. 115, 836 (1959)].

[2] R. D. Taylor and E. C. Kerr, Physica 24, 133 (1958); and private communication.

expansion coefficients, from the relation

$$-\left(\frac{\Delta S}{\Delta p}\right)_T = \frac{C_p}{T}\left(\frac{\Delta T}{\Delta p}\right)_{obs}, \qquad (4)$$

using average values for C_p and T. This gives directly the finite change in entropy during the experimentally observed expansions, which were through finite pressure differences of approximately 2 atmospheres. The resulting values of S_p as a function of p and T agree very well with those obtained from Eq. (3) using the procedure outlined in the preceding paragraph. This affords a check on the error involved in replacing $(\partial T/\partial p)_S$ by $(\Delta T/\Delta p)_{obs}$, and the error is found to be very small.

The S_p isobars allow a plausible extrapolation to 0°K, and the slopes, γ, given by

$$\gamma = \lim_{T \to 0}(\partial S/\partial T)_p, \qquad (5)$$

have been estimated. Some discussion of the relation between these γ values and the very low-temperature nuclear magnetic susceptibility of liquid He[3] under pressure, as determined by Fairbank and Walters,[3] is given in Sec. IV, together with a comparison with theoretical work, notably that of Brueckner and Gammel.[4]
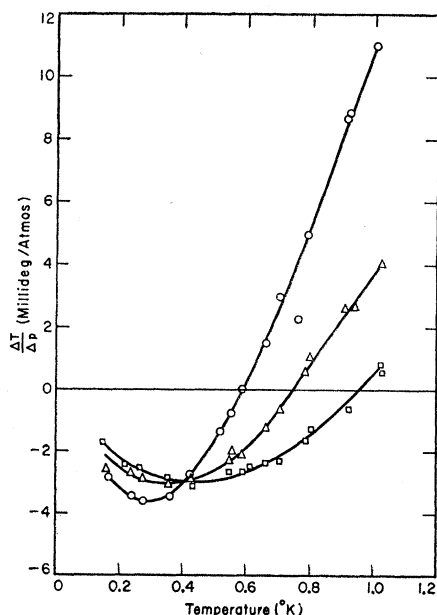

FIG. 1. Typical curves of $(\Delta T/\Delta p)$ *versus* temperature. $\bigcirc$, average pressure $p_{Av}=1.70$ atmos; $\triangle$, $p_{Av}=5.60$ atmos; $\square$, $p_{Av}=11.54$ atmos.

[3] G. K. Walters and W. M. Fairbank, Phys. Rev. 103, 263 (1956); W. M. Fairbank and G. K. Walters, *Symposium on Liquid and Solid He[3]* (Ohio State University Press, Columbus, 1957), p. 205; *Low Temperature Physics and Chemistry* (University of Wisconsin Press, Madison, 1958), p. 86.

[4] K. A. Brueckner and J. L. Gammel, Phys. Rev. 109, 1040 (1958).

II. APPARATUS AND METHOD

The apparatus employed for these experiments was the same as that used for measuring the specific heat of liquid He[3] under pressure and is described in I. Essentially, it consists of an adiabatic calorimeter of about 0.5 cm[3] capacity with a large internal surface area to avoid thermal boundary resistance effects, connected by a superconducting Pb thermal valve to the paramagnetic salt (iron ammonium alum) used for cooling below 1°K. The salt and calorimeter were enclosed in a vacuum jacket surrounded by liquid helium. Temperatures were measured with a magnetic thermometer of cerium magnesium nitrate, thermally attached to the calorimeter. Pressures over the liquid He[3] were measured with a calibrated Bourdon gauge with an accuracy of better than 0.05 atmospheres. The He[3], of purity better than 99.9%, was kindly loaned to us by the U. S. Atomic Energy Commission.

Most experiments were carried out by first condensing He[3] at a pressure just greater than 14.5 atmospheres into the calorimeter at about 1°K, and then demagnetizing the salt from a selected initial field to some temperature between 0.15°K and 1.0°K. The calorimeter and liquid were cooled to approximately the same temperature as the salt by maintaining the Pb thermal valve "open" by means of a one-kilogauss magnetic field, and then were thermally isolated by removing the field from the Pb valve. After adjustment of its pressure to 14.5 atmospheres, the temperature of the liquid was measured at half-minute intervals. On reaching steady conditions, the liquid was expanded from the pressure of 14.5 atmospheres to 12.5 atmospheres. During and immediately after the expansion, temperatures were read at 15-second intervals. When steady conditions were again reached, so that the net change in temperature, ΔT, could be accurately assessed, a further expansion was carried out to 10.5 atmospheres, and this procedure was continued for subsequent expansions of approximately 2 atmospheres each down to a final pressure of 0.5 atmosphere, which was the pressure of the helium gas reservoir. During the period after an expansion, the pressure rose slowly by a few tenths of an atmosphere, so that the net pressure change, ΔT, was generally not exactly 2 atmospheres.

The experiments were repeated for fourteen different temperatures between 0.15°K and 1.15°K by choosing suitable initial fields for demagnetization and at average pressures up to 22.0 atmospheres.

III. RESULTS

(a) The Expansion Coefficient

Figure 1 shows typical curves of the measured $\Delta T/\Delta p$ *versus* temperature for three average pressures of 1.70, 5.60, and 11.54 atmospheres. Similar curves have been drawn for our results at 3.68, 7.58, 9.54, 11.54, 13.50, and 22.0 atmospheres, and smoothed values of $\Delta T/\Delta p$ have been obtained from them. The
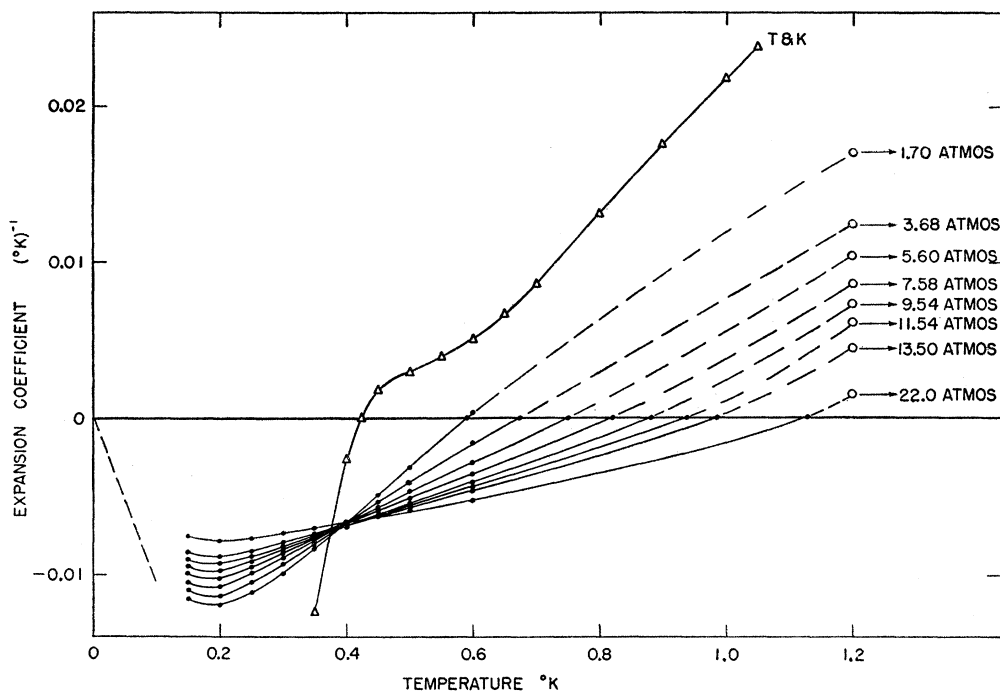
FIG. 2. The expansion coefficient α_p as a function of temperature for various pressures. The curve labelled T and K gives α_p at the saturated vapor pressure as calculated from Taylor and Kerr's values[2] of α_{sat}.

expansion coefficient α_p was then calculated as a function of temperature at various pressures by use of Eq. (2). Values of C_p have been taken by interpolation from our own measurements reported in I, and the molar volumes from the pVT data of Sherman and Edeskuty[5] at 1.2°K. The error introduced by using the molar volumes at 1.2°K instead of at the temperature of measurement is in all cases less than 1%. We have only been able to use Eq. (2) up to 0.6°K, which is the highest temperature to which C_p is known.

In using Eq. (2) one may introduce systematic errors due (1) to approximating a differential term by one involving finite differences, and (2) to neglecting changes in C_p, V, and T during the expansions. The first error is unavoidable in the method used, but as shown in Sec. III (b) (see also Fig. 4) it contributes an error in α_p which is probably less than 0.0001/°K. In assessing the second error (2) we note that the maximum changes in C_p, V, and T during 2 atmosphere expansions in the temperature range 0.15°K to 0.6°K are, respectively, 4, 6, and 3%. These maxima however do not occur at the same temperature, $\Delta C_p/C_p$ being greatest when $\Delta T/T$ is least. In order to allow for these changes we have used in Eq. (2) always *average* values of T, V, and C_p occurring in each expansion, and it is estimated that the errors in α_p introduced in this way are less than 0.0001/°K.

[5] R. H. Sherman and F. J. Edeskuty, *Symposium on Liquid and Solid He³* (Ohio State University Press, Columbus, 1957), p. 44; *Low Temperature Physics and Chemistry* (University of Wisconsin Press, Madison, 1958), p. 102.

Smoothed curves of α_p are shown in Fig. 2. In this figure the temperatures, T_0, where α_p is zero at various pressures have also been inserted since these are known directly from experiment, being given when $(\Delta T/\Delta p)_{obs}$ is zero. Figure 3 shows T_0 as a function of average pressure. The curve of T_0 *versus* p has been extrapolated to the vapor pressure (≈ 0 atmos) giving a value of 0.50°K for the temperature at which α_p for liquid He³ at the saturated vapor pressure becomes zero.

Some comments should be made concerning the precision of our determinations of $\Delta T/\Delta p$ and the problem of the reversibility of the expansions. The susceptibility measurements on the thermometer salt allowed temperature differences in individual ex-
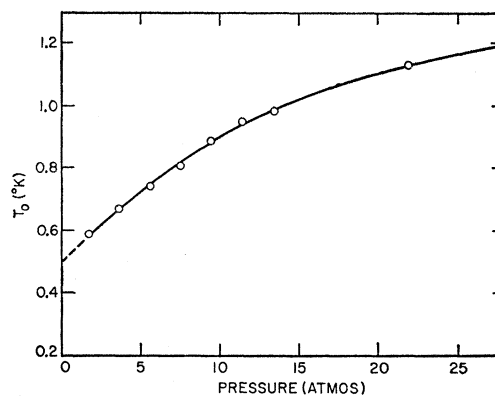

FIG. 3. Variation with pressure of the temperature, T_0, at which the expansion coefficient is zero.

pansions to be assessed to within 0.1 millidegrees at the lowest temperatures, 0.3 millidegrees at 0.6°K, and 0.7 millidegrees at 1°K. The pressure gauge readings contribute a constant percentage error within ±2 percent. By drawing smoothed curves the random errors are reduced, and we estimate that the $\Delta T/\Delta p$ curves are accurate to within ±0.2 millideg/atmos up to 0.6°K, and ±0.4 millideg/atmos near 1°K. The over-all random error in the α_p data up to 0.6°K is probably within ±0.0004/°K.

Possible systematic errors due to irreversibilities in the expansions might be serious, but are difficult to assess. Our α_p values given in Fig. 2 are computed neglecting possible irreversible heating and therefore may require a positive correction added to them. Such a positive correction would reduce the quoted T_0 values and bring the extrapolated value at $p=0$ into better agreement with that found by Taylor and Kerr[2] at the saturated vapor pressure.

However, there are several reasons for supposing that irreversible heating is not large enough to account for this discrepancy. (a) In order to shift our results so that T_0 at $p=0$ agrees with Taylor and Kerr's value (see Sec. IV), we should have to allow the possibility of about 1600 ergs of irreversible heating during an expansion, which is rather large. This would have to arise from irreversible effects associated with turbulent or viscous flow of the liquid, and with heat conduction due to temperature inhomogeneties in the liquid. The possibility of turbulence in the capillary tube is ruled out since the Reynolds' number for the flow of liquid is about 100; and if turbulence does not occur in the tube it will not occur in the liquid at the entrance of the tube. The heat developed during viscous flow of

the liquid is also quite negligible. The third possibility is difficult to estimate but must be extremely small since the temperature changes are themselves small and should occur uniformly throughout the liquid. (b) Some preliminary experiments were carried out in which it was possible both to expand and to compress the liquid. It was found that the values of $\Delta T/\Delta p$ were the same within the experimental error for expansion and compression. (c) Finally, the rate of expansion was varied in the experiments from about 3 seconds per atmospere to 10 seconds per atmosphere, without producing any systematic differences in the results.

(b) S_p, The Entropy at Constant Pressure

We have calculated the entropy change, (S_p-S_{sat}), on compression at various temperatures from Eq. (3) using our α_p values, together with the values of α_p at the saturated vapor pressure calculated from Taylor and Kerr's data. For the latter calculation, we used the relation

$$\alpha_{sat}-\alpha_p=\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T\left(\frac{dp}{dT}\right)_{sat},\qquad(6)$$

where α_{sat} is the expansion coefficient along the vapor pressure line and $(dp/dT)_{sat}$ the slope of the vapor pressure curve. Values of the bulk modulus, $-(1/V)(\partial V/\partial p)_T$, used in Eq. (6) were in turn calculated from Sherman and Edeskuty's pVT data extended to lower temperatures with the help of the expansion coefficient measurements. The results are given in Fig. 4 by the solid lines, the curves for temperatures above 0.6°K being obtained by integration of the extrapolated expansion coefficients. It is difficult to assess the accuracy of these higher temperature results, but they cannot be regarded as very reliable. We include them here since no other values are at present available. Calculations have not been carried out at temperatures lower then 0.4°K, since α_p at the saturated vapor pressure is not known below 0.35°K.

As explained in Sec. I, we have also been able to calculate values of ΔS in the expansions [Eq. (4)] directly from the $\Delta T/\Delta p$ measurements. The results of these computations are shown in Fig. 4 by the circled points. It is clear that the entropies of compression evaluated by the two different methods agree closely. The error introduced by replacing $(\partial T/\partial p)_S$ by $(\Delta T/\Delta p)$ is estimated to be less than 0.0015 cal/mole-deg.

Using these (S_p-S_{sat}) results at 0.6°K and our S_{sat} values given in I, we have calculated S_p as a function of temperature at 5.0, 10.0, 15.0, and 22.0 atmos by integration of the measured C_p values[1] between 0.6°K and 0.12°K. The values of S_p, together with some evaluations above 0.6°K obtained as described below, are shown in Fig. 5 and Table I.
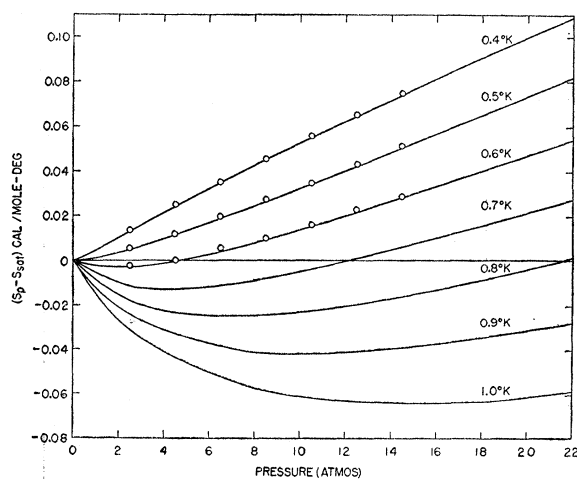

FIG. 4. The change in entropy, (S_p-S_{sat}), on compression from the saturated vapor pressure to higher pressures. The solid lines are calculated from the expansion coefficients, using extrapolated values of α_p at the higher temperatures. The circles represent values calculated directly from the $(\Delta T/\Delta p)$ measurements (see text).
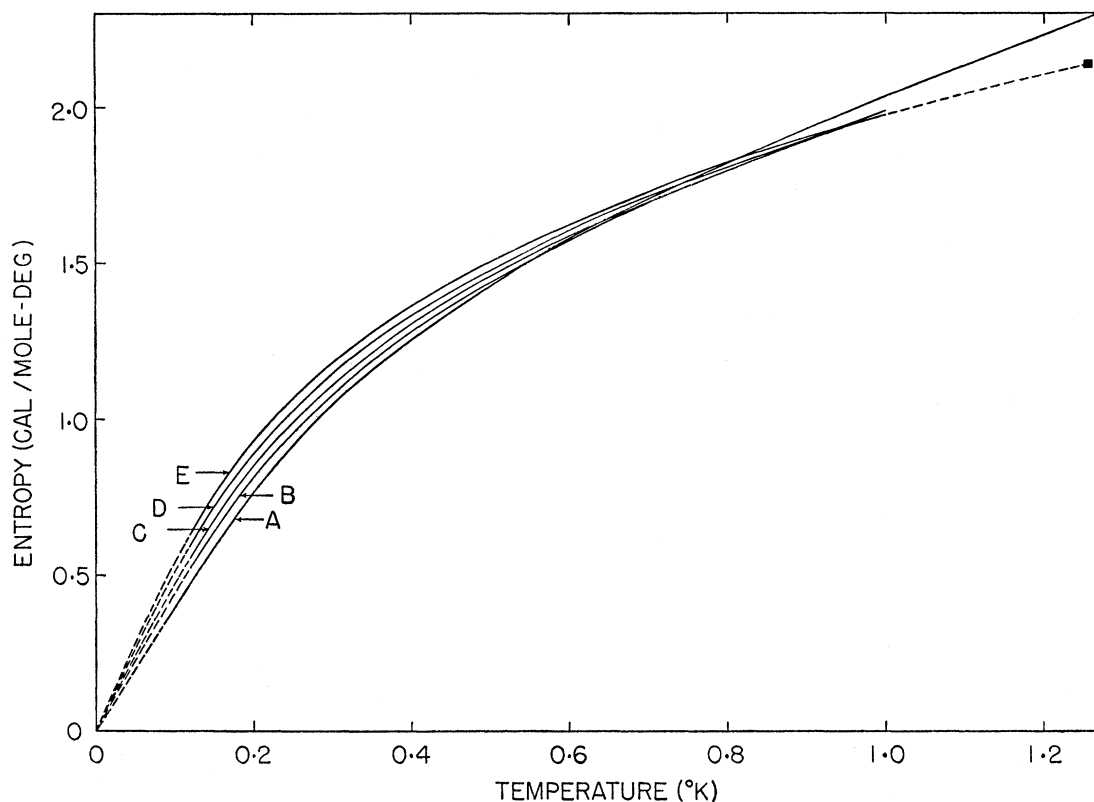
FIG. 5. Entropy of liquid He³ as a function of temperature at various pressures. Curve A, saturated vapor pressure; B, 5 atmos; C, 10 atmos; D, 15 atmos; E, 22 atmos. The value given by Hammel *et al.* at 1.248°K and 10 atmos is represented by ■.

For temperatures above 0.6°K it is only possible to obtain S_p from $(S_p - S_{\text{sat}})$ given in Fig. 4, since the specific heat at higher pressures is not known above this temperature. Values of S_{sat} above 0.75°K were taken from the data of Abraham, Osborne, and Weinstock[6] diminished by 0.105 cal/mole-deg to bring them into agreement with our own value[1] at this temperature. The curve for 10 atmospheres shown in Fig. 5 can be extrapolated reasonably to the value of 2.144 cal/mole-deg at 1.248°K given by Hammel *et al.*[7]

Although the measurements of the expansion coefficients are not as accurate as one might desire, the resulting values of the entropy under pressure are not subject to the same fractional error since, for example, the total change in entropy on compression between 0 and 22 atmospheres at 0.6°K is less than 4% of S_{sat}. The accuracy of the values of S_p is therefore mainly determined by the accuracy of our S_{sat} data, assessed in I to be ±0.01 cal/mole-deg due to random errors and a possible systematic error which would tend to increase S_p by an amount less than 0.03 cal/mole-deg.

[6] Abraham, Osborne, and Weinstock, Phys. Rev. **98**, 551 (1955); and private communication.
[7] Hammel, Sherman, Kilpatrick, and Edeskuty, Physica **24**, 1 (1958); and private communication.

IV. DISCUSSION

(a) The Expansion Coefficient

The pVT data of Sherman and Edeskuty[5] allow α_p to be calculated roughly at various pressures at 1.2°K. We have made these calculations for pressures of 1.70, 3.68, 5.60, 7.58, 9.54, 11.54, 13.50, and 22.0 atmospheres and have plotted the results in Fig. 2. As is evident in

TABLE I. Entropy of liquid He³ in cal/mole-deg as a function of p (atmospheres) and T (°K).

T ＼ p	V.P.	5.0	10.0	15.0	22.0
0 → 0.1	4.00T				
0.120	0.476	0.522	0.560	0.599	0.639
0.150	0.594	0.641	0.679	0.719	0.760
0.200	0.766	0.813	0.849	0.889	0.930
0.250	0.914	0.957	0.990	1.028	1.066
0.300	1.042	1.079	1.110	1.143	1.181
0.350	1.153	1.186	1.213	1.243	1.278
0.400	1.253	1.279	1.303	1.330	1.362
0.450	1.344	1.363	1.383	1.407	1.437
0.500	1.426	1.438	1.456	1.478	1.507
0.550	1.503	1.509	1.524	1.542	1.569
0.600	1.573	1.574	1.587	1.603	1.627
0.700	1.703	1.691	1.699	1.711	1.730
0.800	1.822	1.798	1.799	1.807	1.823
0.900	1.933	1.899	1.891	1.894	1.905
1.000	2.036	1.990	1.975	1.972	1.977

TABLE II. Values of γ, defined by $\gamma = \lim_{T \to 0}(\partial S/\partial T)_p$ units in cal/mole-deg².

p atmos	γ observed	γ theor (B and G)	$(\gamma_p/\gamma_{\text{sat}})_{\text{obs}}$	$(\gamma_p/\gamma_{\text{sat}})_{\text{theor}}$	$\gamma(1 - \frac{1}{2}\epsilon n_0/N)^{-1}$
0	4.00	3.68	1.00	1.00	20[a]
5.0	4.50	4.14	1.13	1.12	...
10.0	4.90	4.80	1.23	1.30	...
11.2	...	...	...	...	25[a]
15.0	5.30	...	1.33	...	...
22.0	5.75	...	1.44	...	...
27.6	...	...	...	...	29[a]

[a] Calculated from susceptibility data.

the figure, these data, within the limits of accuracy of their computation, fall on a reasonable extrapolation of our results.

In Fig. 2 we also show values of α_p at the saturated vapor pressure, calculated, as explained in Sec. III (b), from the measurements by Taylor and Kerr[2] of α_{sat}. These results are generally consistent with ours, but at lower temperatures there is some disagreement. The temperature T_0 at which α_p is zero is found by Taylor and Kerr to be 0.42°K at the saturated vapor pressure. As is seen from Fig. 3, this value falls appreciably lower than the extrapolation of our curve to $p=0$, which gives $T_0 = 0.50$°K. The reason for this discrepancy is unknown. As already discussed in Sec. III (a), irreversible heating in our experiments is unlikely to be large enough to account for it.

The curves for α_p versus T all show a minimum, which occurs at about 0.2°K for all pressures. Thermodynamically one would expect this minimum to occur at a temperature higher than that where $(\partial C_p/\partial p)_T$ is zero and this is indeed observed, since in I it was found that the C_p versus T isobars all cross one another in the neighborhood of 0.16°K. This thermodynamic requirement follows from the general relation

$$\left(\frac{\partial \alpha}{\partial T}\right)_p = -\frac{1}{VT}\left(\frac{\partial C_p}{\partial p}\right)_T - \alpha_p^2. \qquad (7)$$

At the minimum, where $(\partial \alpha/\partial T)_p$ is zero, $(\partial C_p/\partial p)_T$ must be negative, a condition which holds above the cross-over region of the C_p isobars.

It appears that the α_p versus T curves of Fig. 2 also cross over each other at roughly the same temperature of 0.4°K, although the accuracy of the data does not allow this conclusion to be drawn with any certainty. At lower temperatures $(\partial \alpha/\partial p)_T$ is positive, in general agreement with the theory of Brueckner and Gammel.[4]

Below 0.1°K one would expect $(\partial \alpha/\partial T)_p$ to be practically constant, since as is discussed below, it appears that not only S_{sat} but also S_p is very nearly linearly dependent on T below this temperature. Unfortunately our measurements of α_p do not go to sufficiently low temperatures to show up this linear region. However we have calculated $\lim_{T \to 0}(\partial \alpha/\partial T)_p$ at $p=0$ from our values of $(\partial S/\partial p)_T$ near $T=0$ obtained from the extrapolations of S_p discussed in Sec. IV (b).

The resulting value of $-0.105T \pm 25\%$ (deg K)$^{-1}$ is shown in Fig. 2 by the broken line through the origin and may be compared with evaluations of this term based on the theory of Brueckner and Gammel,[4] which are $-0.076T$ (deg K)$^{-1}$ by Brueckner and Atkins[8] and $-0.08T$ (deg K)$^{-1}$ by de Boer.[9]

(b) The Entropy at Constant Pressure

In the lower temperature region of Fig. 5 the entropy S_p increases with increasing pressure, as predicted qualitatively in the theory of Brueckner and Gammel.[4] At higher temperatures the curves of S_p versus T cross one another (in the temperature range between 0.5°K and about 1.0°K) and $(\partial S/\partial p)_T$ becomes negative.

The curves for S_p versus T are still not linear at the lowest temperature of measurement, but the remaining curvature is rather small, and we have made extrapolations to 0°K which are linear within 3 to 4% below 0.1°K. The question of such extrapolations has already been discussed in I and will not be repeated here, except to add that in the present case they are not so reliable as in I. The estimated values of γ as defined by Eq. (5) are given in column 2 of Table II. The variation of γ with pressure can also be computed from the theoretical evaluation by Brueckner and Gammel of the variation of the effective mass, m^*, with interatomic distance r, by use of the relation

$$m^*/m = \gamma/\gamma_F,$$

where m is the mass of the He³ atom and γ_F the linear entropy term for a perfect Fermi gas of the same density as the liquid. This definition of m^* is identical with that used in I. Values of γ obtained in this way are given in column 3 of Table II. In making the calculations we have used Brueckner and Gammel's theoretical value of $m^*/m = 1.84$ for their theoretical interatomic distance $r_0 = 2.60$ A at the saturated vapor pressure. Since these values of m^*/m and r_0 do not agree with experiment, we also show in Table II the ratios $(\gamma_p/\gamma_{\text{sat}})_{\text{theoret}}$ and $(\gamma_p/\gamma_{\text{sat}})_{\text{obs}}$, where the meaning of the symbols is obvious. Although the theoretical values are not known for pressures greater than about 10 atmospheres, it appears that the theoretical variation of γ with pressure

[8] K. A. Brueckner and K. R. Atkins, Phys. Rev. Letters 1, 315 (1958).
[9] J. de Boer (private communication).

is too large at the higher pressures. However, we do not attach great importance to a numerical comparison since the result is very sensitive to the value taken for r_0, and the theoretical value of r_0 does not agree well with experiment. To illustrate this, we show in Fig. 6 the variation of m^*/m with interatomic distance as given by theory and by experiment. Values of r were calculated from the molar volumes V by use of the equation

$$V = \tfrac{4}{3}\pi N r^3,$$

where N is Avogadro's number.

It is of interest to consider the relationship between the slope, γ, of the linear entropy term at 0°K and the nuclear magnetic susceptibility of the Fermi system.

If one considers a gas-like Fermi system without any spin dependent interactions, then its linear term γ_F is quite generally given by:

$$\lim_{T \to 0}(\partial S/\partial T)_p \equiv \gamma_F = \tfrac{1}{3}\pi^2 R(\chi/c), \qquad (8)$$

where R is the gas constant, χ the temperature-independent magnetic susceptibility at 0°K, and c the Curie constant. If, however, spin-dependent interactions are present, then the simple relationship given by (8) fails, as has been pointed out, for example, by Buckingham,[10] Brueckner and Gammel,[4] and Landau.[11] Following Buckingham's treatment one should include in the expression for the susceptibility an interaction term ϵ, so that:

$$\lim_{T \to 0}(\partial S/\partial T)_p \equiv \gamma = \tfrac{1}{3}\pi^2 R(\chi/c)\{1 - \tfrac{1}{2}\epsilon n_0/N\}, \qquad (9)$$

where γ is the observed slope of the entropy, n_0 the density of levels at the Fermi surface, and N is Avogadro's number.

Using Eq. (9), we have computed rough values for $\gamma(1 - \tfrac{1}{2}\epsilon n_0/N)^{-1}$ from the nuclear magnetic susceptibility data of Fairbank et al.,[3] and these data, at three pressures, are given in Table II. The possible errors are large at the higher pressures, due to difficulty in assessing accurately the slope of the $(\chi T/c)$ versus T curves. The values given probably represent lower limits. It will be seen that at all pressures
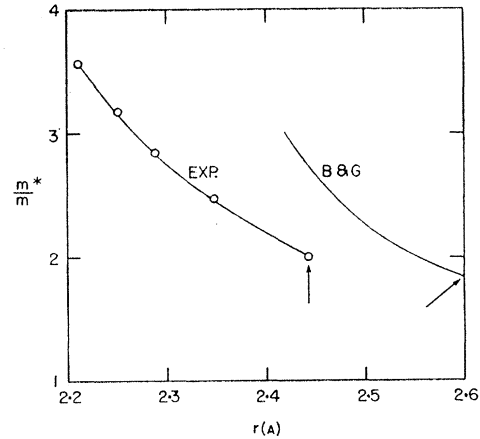

FIG. 6. Variation of m^*/m at 0°K with interatomic distance. B and G, theory of Brueckner and Gammel[4]; Exp, experiment. The arrows indicate the points corresponding to the saturated vapor pressure.

$\gamma(1 - \tfrac{1}{2}\epsilon n_0/N)^{-1}$ is about five times larger than the calorimetrically observed values of γ, indicating that the spin-dependent terms have a dominant effect on the relation between the susceptibility and the entropy, as has been shown in detail by Brueckner and Gammel.[4]

It is interesting to note also that since $(1 - \tfrac{1}{2}\epsilon n_0/N)$ equals approximately 0.2 at all pressures, the term ϵn_0 must depend weakly on pressure.

Goldstein[12] has suggested that the entropy of liquid He³ be separated into a spin entropy and a term independent of spin, the term γ_σ characterizing the "spin entropy" is given by

$$\gamma_\sigma = (\chi/c)R \ln 2.$$

Again using the susceptibility data of Fairbank et al., we have computed rough values of γ_σ and find values of approximately 4, 5 and 6 cal/mole-deg² at $p = 0$, 11.2 and 27.6 atmospheres, respectively, which fall close to the observed values.

ACKNOWLEDGMENTS

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