

## Low-Temperature Specific Heat of Liquid He<sup>3</sup> near the Saturated Vapor Pressure and at Higher Pressures\*

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The specific heat of liquid He<sup>3</sup> has been measured near the saturation vapor pressure in the range 0.085°K to 0.75°K, and at pressures up to the melting pressure between 0.12°K and 0.6°K. There is no evidence of a specific heat transition in the range of measurements. Near 0.1°K the specific heat near the saturated vapor pressure appears to tend linearly to zero at 0°K with a slope of  $4.0 \pm 0.1$  cal/mole-deg<sup>2</sup>, providing evidence for regarding He<sup>3</sup> as a Fermi liquid near this temperature with an effective excitation mass of  $(2.00 \pm 0.05) \times$  the mass of a He<sup>3</sup> atom. Entropies calculated with this linear extrapolation and with the assumption that no transition occurs below 0.085°K agree well with previous evaluations. Measurements at higher pressures show a negative dependence of specific heat on pressure above 0.16°K, but a positive dependence below this temperature, in contradistinction to the case of an ideal Fermi gas.

### I. INTRODUCTION

THE first measurements of the specific heat of liquid He<sup>3</sup> were made by de Vries and Daunt<sup>1</sup> in the temperature range 0.5°K to 2.3°K. They were followed by the measurements of Roberts and Sydoriak<sup>2</sup> and of Abraham, Osborne, and Weinstock<sup>3</sup>, the lowest temperature reached being 0.23°K. Two main conclusions could be drawn from the experiments: (i) There was no evidence of a specific heat anomaly down to 0.23°K, and (ii) the thermal excitations in the liquid at these temperatures were not describable in terms of a perfect Fermi-Dirac gas. The first is of interest in connection with the possibility of superfluidity in He<sup>3</sup>, and the second indicated the need for measurements to be made at much lower temperatures in order to allow a possible extrapolation of the specific heat to 0°K, and hence to allow some interpretation to be made concerning the ground state of the liquid.

The aim of the present work was to extend the range of temperature of measurement to just below 0.1°K, and to determine the pressure dependence of the specific heat. In Secs. II and III we describe the apparatus and procedure used in the measurements, and the results are presented in Sec. IV, which gives our evaluations of the specific heat of liquid He<sup>3</sup> at a pressure of 6 to 14 cm Hg in the temperature range 0.085°K to 0.75°K and at pressures of 1.0, 2.5, 5.0, 10.0, 15.0, 20.0, 25.0, and 29.0 atmospheres in the temperature range 0.12°K to 0.60°K. Preliminary reports of this work have already been published.<sup>4,5</sup> As is pointed out in the discussion in

Sec. V, the results at the lowest pressure indicate that  $C_{sat}$  can be reasonably extrapolated as a linear function of temperature to 0°K, providing evidence for regarding He<sup>3</sup> as a Fermi liquid. It has been possible from these results to compute the entropy at the saturated vapor pressure,  $S_{sat}$ , and to compare it with the entropy determined at higher temperatures by other methods. It was also found that, below about 0.16°K,  $C_p$  increased with increasing pressure, indicating a negative temperature coefficient of expansion; whereas at higher temperatures  $C_p$  decreased with increasing pressure. The measurements of  $C_p$  under pressure, although carried out to 0.12°K, did not allow extrapolation to 0°K and hence  $S_p$  could not immediately be computed. However we describe in another paper some subsidiary experiments,<sup>6</sup> from which we can calculate the thermal expansion coefficient under pressure, and hence  $S_p$  as a function of  $T$  and  $p$ .

### II. APPARATUS

The measurements were made in an adiabatic calorimeter, containing approximately 0.5 cm<sup>3</sup> liquid He<sup>3</sup> under pressure, using techniques similar to those previously described.<sup>1</sup> The He<sup>3</sup>, of purity better than 99.9 per cent, was kindly loaned to us by the U. S. Atomic Energy Commission. The calorimeter, together with its electrical heater and magnetic thermometer, was cooled by a paramagnetic salt, the thermal connection being through a superconducting thermal valve which could be opened and closed magnetically, as has been described elsewhere.<sup>7</sup>

The He<sup>3</sup> calorimeter, together with the cooling salt and the magnetic thermometer, was mounted as shown in Fig. 1. The cooling salt, consisting of about 90 g of powdered iron ammonium alum of particle size approximately 1 mm, was inserted in a container,  $C$ , made of thin-walled nickel-silver tubing which had inside a copper fin system similar to that used by Heer, Barnes,

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<sup>1</sup> G. de Vries and J. G. Daunt, *Phys. Rev.* **92**, 1572 (1953); **93**, 631 (1954).

<sup>2</sup> T. R. Roberts and S. G. Sydoriak, *Phys. Rev.* **93**, 1418 (1954); **98**, 1672 (1955).

<sup>3</sup> Osborne, Abraham, and Weinstock, *Phys. Rev.* **94**, 202 (1954); Abraham, Osborne, and Weinstock, *Phys. Rev.* **98**, 551 (1955).

<sup>4</sup> Brewer, Sreedhar, Kramers, and Daunt, *Phys. Rev.* **110**, 282 (1958); *Bull. Am. Phys. Soc.* **3**, 133 (1958).

<sup>5</sup> Brewer, Sreedhar, and Daunt, *Bull. Am. Phys. Soc. Ser. II*, **3**, 339 (1958).

<sup>6</sup> D. F. Brewer and J. G. Daunt, following paper [*Phys. Rev.* **115**, 843 (1959)].

<sup>7</sup> C. V. Heer and J. G. Daunt, *Phys. Rev.* **76**, 854 (1949).

and Daunt<sup>8</sup> in their magnetic refrigerator. Following a suggestion of Heer,<sup>9</sup> the interstices between the salt particles were filled with toluene, to act as a thermal bond between the salt and the metal, and the container was then sealed. Fiber spacers, *S S*, prevented the assembly from touching the brass vacuum case, *V*, and a copper shield, *R*, protected it from room temperature radiation from above, while allowing the vacuum case to be pumped through the holes, *W W*. The calorimeter, *P*, could be filled with  $\text{He}^3$  through the 0.056-cm bore stainless steel tube, *t*, which passed through the middle of the fin assembly in *C*. For the first experiments *P* consisted of a hollow copper pot of 0.508 cm<sup>3</sup> volume. For later experiments, including all those at high pressures, it was replaced by a modified calorimeter of approximately the same capacity because it was found that the relaxation time for thermal equilibrium increased appreciably with pressure, due apparently to an increased thermal boundary resistance between the liquid  $\text{He}^3$  and the calorimeter. The modification consisted in soldering to the base of the new calorimeter on its inner surface a scroll of thin copper, thereby increasing the surface area between the liquid  $\text{He}^3$  and the calorimeter by 122 cm<sup>2</sup> and correspondingly decreasing the relaxation time to a negligible value. The calorimeter was positioned rigidly by means of German-silver supports between it and the lower end of *C*. To establish thermal contact between the calorimeter and the cooling salt, a lead thermal valve, *Pb*, was used (approximate dimensions: 0.07 cm $\times$ 0.35 cm $\times$ 3.5 cm). The ends of the *Pb* valve were soldered to the calorimeter and to the central shank of the copper fin assembly projecting through the end cap of the container *C*. The valve was operated by a 1-kilogauss field of a permanent magnet located outside the liquid helium and liquid nitrogen Dewars which surrounded the apparatus.

Temperatures were determined by susceptibility measurements on a cylindrical specimen, *T*, of powdered cerium magnesium nitrate using an electronic mutual inductance bridge.<sup>10</sup> This salt was chosen because it is known<sup>11</sup> to obey Curie's law down to about 0.006°K and because of its small heat capacity and shape factor correction. Using a technique previously noted,<sup>12</sup> the powdered salt was cast in a cold-setting plastic (Castolite) in a nylon mold in which 6 copper strips (dimensions: 1 mm wide, 0.25 mm thick) were prelocated. When hardened this unit was mounted below the calorimeter, to which the ends of the copper strips were soldered. It was found that this gave satisfactory thermal contact between the thermometric salt and the

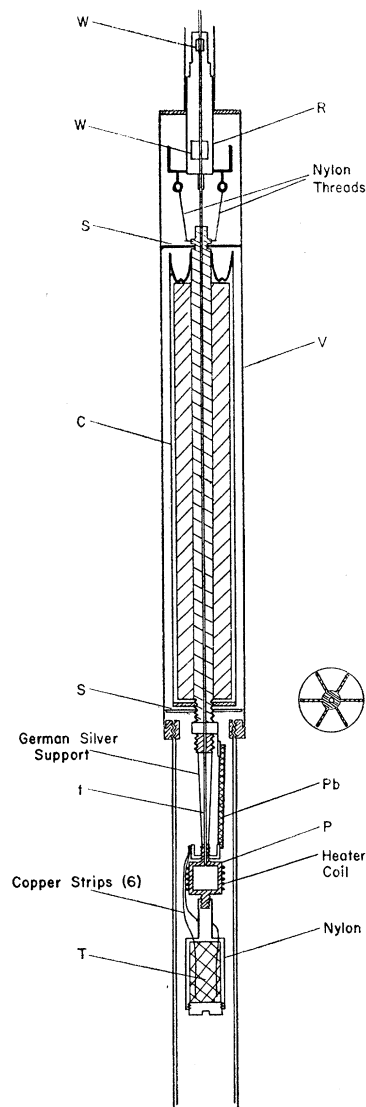


FIG. 1. Apparatus for measuring the specific heat of liquid  $\text{He}^3$ .

calorimeter down to the lowest temperatures of measurement (0.085°K). The heater consisted of a manganin coil of 100 ohms resistance noninductively wound around the outside of the calorimeter. Leads to the heater were of superconducting Nb wire of diameter 0.003 inch.

### III. PROCEDURE

After cooling the apparatus to helium temperatures, the magnetic thermometer was calibrated between 4.2°K and 1.2°K with a small pressure of exchange gas in the vacuum case. The relationship between the observed susceptibility and the reciprocal of the temperature was found to be linear in this temperature range and was used directly for determining the temperature below 1°K; corrections for demagnetizing effects and departure from Curie's law were negligible for this salt under the conditions used down to the lowest tempera-

<sup>8</sup> Heer, Barnes, and Daunt, *Rev. Sci. Instr.* **25**, 1088 (1954).

<sup>9</sup> We are indebted to Dr. C. V. Heer for suggesting this technique to us.

<sup>10</sup> Pillinger, Jastram, and Daunt, *Rev. Sci. Instr.* **2**, 159 (1958).

<sup>11</sup> Cooke, Duffus, and Wolf, *Phil. Mag.* **44**, 623 (1953); J. M. Daniels and F. N. H. Robinson, *Phil. Mag.* **44**, 630 (1954).

<sup>12</sup> J. G. Daunt and W. L. Pillinger, *Suppl. Bull. inst. intern. froid Annexe* 1955-3, p. 158.

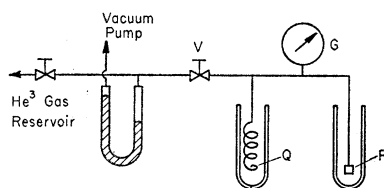


FIG. 2. System for condensing liquid He<sup>3</sup> at high pressures into the calorimeter.

ture of operation (0.085°K). The helium bath was then pumped down to the lowest temperature possible (about 1°K) and He<sup>3</sup> condensed into the calorimeter to a pressure slightly higher than the vapor pressure; the liquid-vapor interface was thus situated at some point in the capillary tube, *t*. Exchange gas was pumped out of the vacuum case and demagnetization carried out slowly (10 to 15 min) from an initial field of 18 kilogauss down to about 3 kilogauss at which value the fringing field at the Pb thermal valve was still sufficient to keep it "open." The 3-kilogauss field was then reduced to zero in about 20 seconds and the Pb thermal valve re-opened by means of a 1-kilogauss permanent magnet for a period of one-half to one hour to allow the He<sup>3</sup> to cool to the temperature of the paramagnetic salt. After slowly removing the 1-kilogauss field, thereby isolating the calorimeter from the cooling salt, susceptibility observations on the magnetic thermometer were taken as a function of time. At approximately ten-minute intervals, measured quantities of heat were put into the calorimeter through the 100-ohm heater, current and voltage being measured potentiometrically. Temperature rises of approximately 10 millidegrees were used in the measurement range 0.085° to 0.2°K and up to 100 millidegrees in the range 0.2 to 0.75°K.

For measurements at higher pressures, the simple pressurizing system shown in Fig. 2 was used. He<sup>3</sup> gas from the supply reservoir was condensed both into the calorimeter, *P*, at about 1°K and into a copper spiral, *Q*, of volume approximately 1 cm<sup>3</sup> which was immersed in a separate He<sup>4</sup> bath. The valve *V* was then turned off and when the He<sup>4</sup> Dewar around *Q* was removed, the liquid He<sup>3</sup> in the spiral vaporized, increasing the pressure in the calorimeter to a value which was meas-

ured on the Heise bourdon gauge *G*. This arrangement was both simple and convenient, and avoided difficulties associated with the use of a mercury Toepler pump in a closed metal system.

#### IV. RESULTS

In computation of the results allowance was made for the shrinkage of the calorimeter between the temperature at which its volume was measured (~300°K) and liquid helium temperature (1.0%), and for the additional effective volume of the filling tube (0.8%). Due to difficulty in accurate direct measurement of the volume of the second calorimeter used, it was computed indirectly by normalizing the results obtained with it to those obtained with the first calorimeter. In calculating the mass of He<sup>3</sup> in the calorimeter, the molar volumes measured by Sherman and Edeskuty<sup>13</sup> at 1.2°K were used, with a correction of approximately 1% to allow for thermal contraction between this temperature and the temperatures of measurement. The amount of contraction was calculated from the expansion coefficients at the vapor pressure measured by Taylor and Kerr,<sup>14</sup> and at higher pressures from our own measurements<sup>6</sup> extrapolated to 1.2°K. The variation in mass throughout an experimental run, which is less than 1/3%, has not been allowed for. A run made with the calorimeter empty showed that the correction for its heat capacity, including that of the magnetic thermometer, was negligible.

In the temperature range of measurement it can be shown that<sup>15</sup> the specific heats at constant pressure, under saturated vapor pressure, and at constant volume are equal (i.e.  $C_p = C_{sat} = C_v$ ) to within approximately 0.2%. The value of the mechanical equivalent of heat, *J*, was taken to be 4.186 joules/calorie.

The results are given in Table I which are taken from the smoothed curves drawn through the observed individual data. A total of 280 individual data points were taken. In Fig. 3 typical smoothed  $C_p$  versus *T* isobars are shown for three pressures: 6 to 14 cm Hg, 5 atmos, and 25 atmos. Individual data for the measurements at 6 to 14 cm Hg are shown in Fig. 4, in order to give an indication of the scatter and reproducibility; different symbols refer to experimental runs made on different days. At other pressures at which measurements were made, i.e., at 1.5, 2.5, 5.0, 10.0, 15.6, 20.0, 25.0, and 28.8 atmos, the individual data points used to obtain the smoothed curves, although not so numerous, showed similar reproducibility and scatter. It is estimated that the error in any one specific heat determination is

TABLE I. Specific heat of liquid He<sup>3</sup> in cal/mole-deg as a function of *p* (atmospheres) and *T* (°K).

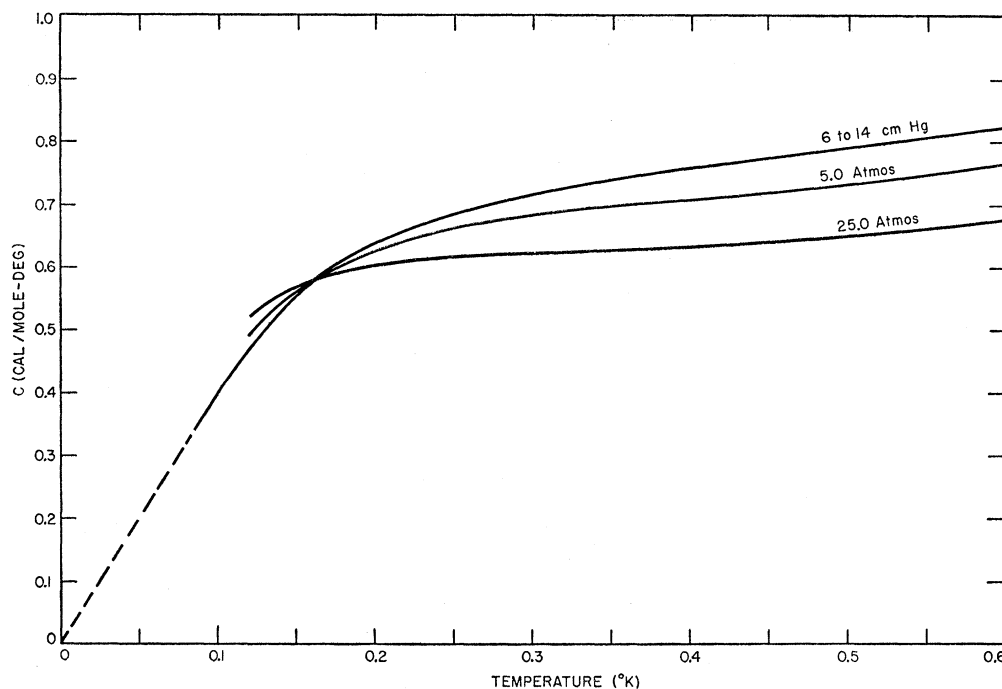
<i>T</i> \ <i>p</i>	V.P.	1.0	2.5	5.0	10.0	15.0	20.0	25.0	29.0
0.100	0.400								
0.120	0.472								
0.150	0.555	0.558	0.559	0.560	0.562	0.565	0.567	0.569	0.571
0.200	0.640	0.637	0.632	0.626	0.615	0.609	0.606	0.603	0.602
0.250	0.684	0.680	0.673	0.662	0.644	0.630	0.621	0.616	0.612
0.300	0.714	0.709	0.700	0.685	0.661	0.640	0.629	0.621	0.617
0.350	0.737	0.728	0.716	0.698	0.670	0.648	0.636	0.627	0.622
0.400	0.757	0.746	0.728	0.706	0.679	0.654	0.642	0.633	0.627
0.450	0.777	0.764	0.742	0.717	0.687	0.662	0.649	0.640	0.637
0.500	0.793	0.779	0.758	0.732	0.696	0.671	0.657	0.650	0.647
0.550	0.807	0.794	0.774	0.748	0.710	0.683	0.667	0.661	0.659
0.600	0.823	0.810	0.790	0.764	0.724	0.693	0.676	0.674	0.673
0.650	0.845								
0.700	0.867								
0.750	0.890								

<sup>13</sup> R. H. Sherman and F. J. Edeskuty, *Symposium on Liquid and Solid He<sup>3</sup>* (Ohio State University Press, Columbus, 1957), p. 44; *Proceedings of the Fifth International Conference on Low-Temperature Physics and Chemistry, Madison, Wisconsin, 1957*, edited by J. R. Dillinger (University of Wisconsin Press, Madison, 1958), p. 102.

<sup>14</sup> R. Dean Taylor and E. C. Kerr, *Physica* 24, 133 (1958).

<sup>15</sup> This requires a knowledge of the expansion coefficient, measurements of which in the temperature range 0.1 to 0.6°K are reported by us in a subsequent paper.

FIG. 3. Smoothed curves of the specific heat of liquid He<sup>3</sup> versus temperature for three pressures.



within  $\pm 0.02$  cal/mole-deg and that the probable error in the smoothed results is  $\pm 0.01$  cal/mole-deg.

## V. DISCUSSION

### (a) Specific heat at the saturated vapor pressure

It can be shown<sup>15</sup> that the possible difference between the value of  $C_p$  at 6 to 14 cm Hg and  $C_{sat}$  is within the experimental error of our measurements. Direct comparisons can be made therefore between our values<sup>16</sup> of  $C_p$  for  $p=6$  to 14 cm Hg and previous observations on  $C_{sat}$ , which have been made in the temperature range above  $0.23^\circ\text{K}$ .<sup>3</sup> It is found, as is evident in Fig. 4, that the agreement is satisfactory. We shall refer to our results at the lowest pressures therefore as  $C_{sat}$ .

Our results for  $C_{sat}$  show (a) that at least down to  $0.085^\circ\text{K}$  there is no sign of a  $\lambda$ -transition or other anomaly and (b) that it appears that a linear extrapolation of the curve can be made to zero specific heat at  $0^\circ\text{K}$  with a slope,  $\gamma$ , of  $4.00$  cal/mole-deg<sup>2</sup>. Such a linear temperature dependence of the specific heat near absolute zero, characteristic of a perfect Fermi gas, may not necessarily occur in a system of Fermi particles in the presence of interactions, as recently emphasized by Landau.<sup>17</sup> The experimental confirmation of a linear region thus provides some evidence for regarding He<sup>3</sup>

near  $0.1^\circ\text{K}$  as a degenerate Fermi liquid with an approximately constant effective mass, in agreement with the theories of, for example, Landau<sup>17</sup> and Brueckner and Gammel.<sup>18</sup> This result also agrees with the observation of Fairbank *et al.*,<sup>19</sup> who found that the nuclear magnetic susceptibility tended to become constant, independent of temperature at temperatures from about  $0.18^\circ\text{K}$  to the lowest temperature of their measurements ( $\sim 0.10^\circ\text{K}$ ).

Although our results, taken together with the considerations noted above, indicate that the linear extrapolation of  $C_{sat}$  to  $0^\circ\text{K}$  can be made with some confidence, it should be pointed out that our accuracy does not entirely preclude the existence of a small amount of curvature even at our lowest temperature of measurement. This would result in an increase in the limiting slope of the specific heat at  $0^\circ\text{K}$  by an amount which we estimate would be less than  $0.4$  cal/mole-deg<sup>2</sup> even if the specific heat continued to be curved right down to absolute zero. Moreover, even with the existence of a strictly linear region, an extrapolation to  $0^\circ\text{K}$  cannot be completely unambiguous. As we have already pointed out,<sup>4</sup> there remains the possibility that a transition may yet occur at a lower temperature than that of the lowest

<sup>18</sup> K. A. Brueckner and J. L. Gammel, *Phys. Rev.* **109**, 1040 (1958).

<sup>19</sup> Fairbank, Ard, and Walters, *Phys. Rev.* **95**, 566 (1954); W. M. Fairbank and G. K. Walters, *Symposium on Liquid and Solid He<sup>3</sup>* (Ohio State University Press, Columbus, 1957), p. 205; *Proceedings of the Fifth International Conference on Low-Temperature Physics and Chemistry, Madison, Wisconsin, 1957*, edited by J. R. Dillinger (University of Wisconsin Press, Madison, 1958), p. 86.

<sup>16</sup> The results given here differ slightly from and supersede those reported by us earlier [Brewer, Sreedhar, Kramers, and Daunt, *Phys. Rev.* **110**, 282 (1958)]. The revision is due to the subsequent greatly increased number of points measured and their reduced scatter, and to a more accurate evaluation of the calorimeter volume.

<sup>17</sup> L. Landau, *Zhur. Eksptl. i Teoret. Fiz.* **30**, 1058 (1956) [translation: *Soviet Phys. JETP*, **3**, 920 (1957)].

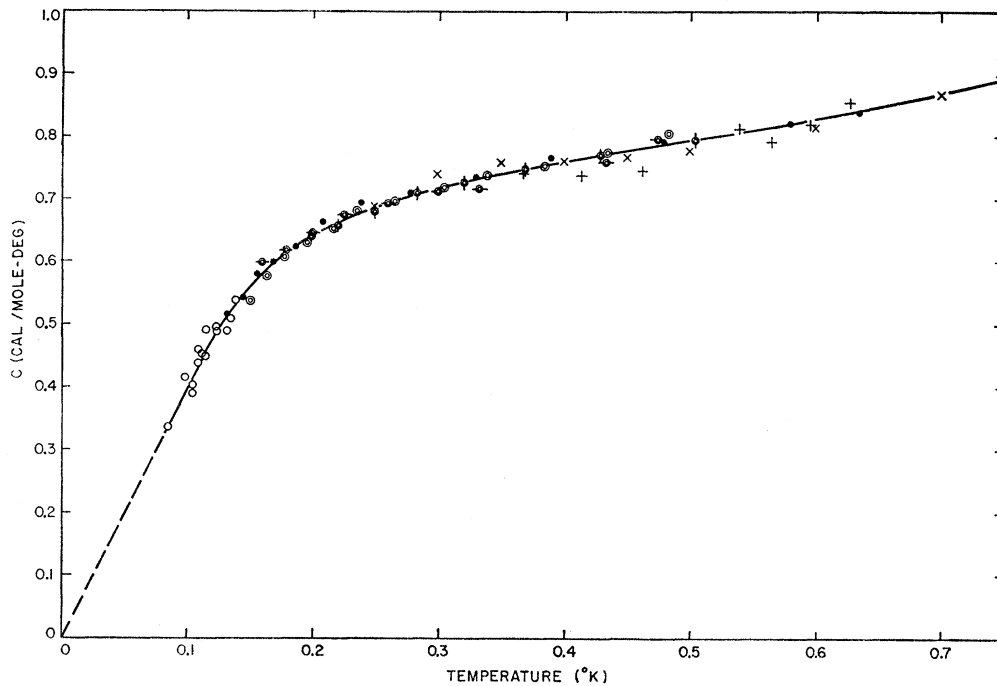


FIG. 4.  $C_{\text{sat}}$  for liquid  $\text{He}^3$  versus temperature. ■, de Vries and Daunt.<sup>1</sup> +, Roberts and Sydoriak.<sup>2</sup> ×, Abraham, Osborne, and Weinstock.<sup>3</sup> Our experimental points obtained on different days are given by the various circled markings. The open circles below 0.15°K refer to our previous measurements<sup>4</sup> with the first calorimeter.

observation,<sup>20</sup> and such a transition, as for example in superconductors, might be one which leaves the high-temperature entropy unaffected. It would still be of interest and value therefore to extend measurements of  $C_{\text{sat}}$  to yet lower temperatures.

It is of interest to note that the value of  $\gamma$  computed by Brueckner and Gammel for  $\text{He}^3$  as a Fermi liquid is 3.67 cal/mole-deg<sup>2</sup>, differing only by 7% from our experimental value. As far as we are aware, this is the only theoretical evaluation of  $\gamma$  from first principles. Previous considerations of the specific heat of liquid  $\text{He}^3$  as that of a Fermi system, for example by Pomeranchuk,<sup>21</sup> Singwi,<sup>22</sup> and London,<sup>23</sup> and by Temperley<sup>24</sup> treated the system as a gas. Recent work by Goldstein<sup>25</sup> allow, as is described in the following paper,<sup>6</sup> numerical values of  $\gamma$  to be computed provided the low temperature susceptibility is known. The liquid-like treatments of Landau<sup>17</sup> (see also Khalatnikov and Abrikosov<sup>26</sup>), Penrose,<sup>27</sup> and Huang and Yang<sup>28</sup> do not

<sup>20</sup> For example, Cooper, Mills, and Sessler [Physica 24, 183 (1958)] have discussed the possibility of a transition to superfluidity at very low temperatures. See also O. K. Rice, Phys. Rev. 97, 558 (1955).

<sup>21</sup> I. Pomeranchuk, Zhur. Exptl. i Teoret. Fiz. 20, 919 (1950).

<sup>22</sup> K. S. Singwi, Phys. Rev. 87, 540 (1952).

<sup>23</sup> T. C. Chen and F. London, Phys. Rev. 89, 1038 (1953).

<sup>24</sup> H. N. V. Temperley, Proc. Phys. Soc. (London) A67, 495 (1954).

<sup>25</sup> L. Goldstein, Phys. Rev. 96, 1455 (1954) and 112, 465 (1958).

<sup>26</sup> I. M. Khalatnikov and A. A. Abrikosov, Zhur. Exptl. i Teoret. Fiz. 32, 915 (1957); [translation: Soviet Phys. JETP 5, 745 (1957)].

<sup>27</sup> O. Penrose, Symposium on Liquid and Solid  $\text{He}^3$  (Ohio State University Press, Columbus, 1957), p. 85.

<sup>28</sup> K. Huang and C. N. Yang, Phys. Rev. 105, 787 (1957).

permit an immediate theoretical numerical evaluation of the term  $\gamma$ . It should again be remarked that the theory of Brueckner and Gammel, like our linear extrapolation, assumes the absence of a transition near absolute zero. If a transition were to occur, it would be possible that the theory, although one describing the ground state, would be a good approximation near 0.1°K but incorrect at 0°K.

One may characterize the Fermi liquid in the linear region near 0°K by a constant effective mass  $m^*$  given by

$$m^*/m = C/C_F, \quad (1)$$

where  $m$  is the mass of the free  $\text{He}^3$  atom,  $C$  the observed specific heat, and  $C_F$  the specific heat of an ideal Fermi-Dirac gas of particles of constant mass  $m$  and density equal to that of the liquid. In Fig. 5 we show  $C/C_F$  plotted as a function of temperature for four different pressures. Although the curve of  $C_{\text{sat}}/C_F$  has been drawn horizontal below 0.1°K, this again depends on the validity of our linear extrapolation to absolute zero. A residual curvature in the specific heat would result in an upward trend of the graph to a higher value of  $C_{\text{sat}}/C_F$  at 0°K which we estimate would not exceed 2.2. If the linear extrapolation is correct, we obtain a value of  $m^*/m$  at the saturated vapor pressure of  $2.00 \pm 0.05$  between 0°K and 0.1°K. At higher temperatures, the observed specific heat departs from a simple Fermi function with constant effective mass, as shown by the rapid drop of  $C/C_F$  in Fig. 5. Equation (1) no

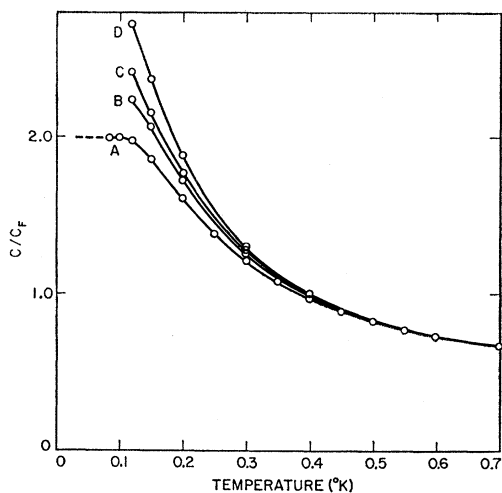


FIG. 5. The ratio  $C/C_F$  versus temperature, where  $C$  is the observed specific heat,  $C_F$  the specific heat of an ideal Fermi-Dirac gas of the same density as the liquid. Curve A,  $p=6-14$  cm Hg. Curve B,  $p=5$  atmospheres. Curve C,  $p=10$  atmospheres. Curve D,  $p=25$  atmospheres.

longer holds in this region, since, apart from theoretical difficulties, there are terms in  $dm^*/dT$  appearing in the expression for a Fermi-Dirac gas with a temperature-dependent effective mass. Thus we cannot conclude from Fig. 5 without further examination that  $m^*/m$  becomes less than unity at the higher temperatures.

Various attempts have been made to account for the behavior of the observed specific heat in terms of the elementary excitations in the liquid. Khalatnikov and Abrikosov,<sup>26</sup> using the Fermi-liquid model of Landau,<sup>17</sup> have computed  $C_{sat}$  using the following excitation spectrum:

$$E(p) = (p - p_0)^2 / 2\mu, \quad (2)$$

where  $E(p)$  is the energy of an excitation of momentum  $p$ ,  $\mu$  its effective mass, and  $p_0$  is a constant. This yields a value of  $C_{sat}$  which is temperature independent and equal to  $\frac{1}{2}R$  cal/mole-deg at temperatures above the degeneracy temperature (assessed by them to be about 1°K), and which varies linearly with  $T$  in its approach to 0°K. Such a temperature dependence of the specific heat resembles in very rough outline the observed behavior of  $C_{sat}$ , but clearly not in detail. Other spectra, such as that proposed by Usui,<sup>29</sup> have yet to be thoroughly investigated.

If it is permissible to speak of a contribution,  $C_{ph}$ , to the specific heat due to phonon excitations, an assumption which is open to doubt,<sup>30</sup> then  $C_{ph} = 0.059 T^3$  cal/mole-deg as computed from the sound velocity<sup>31,32</sup>

<sup>29</sup> T. Usui (private communication).

<sup>30</sup> For example see report of *Symposium on Liquid and Solid He<sup>3</sup>*, (Ohio State University Press, Columbus, 1957); R. P. Feynman, *Phys. Rev.* **91**, 1301 (1953); L. D. Landau, *J. Exptl. Theoret. Phys. U.S.S.R.* **32**, 59 (1957) [translation: *Soviet Phys. JETP* **5**, 101 (1957)].

<sup>31</sup> H. Flicker and K. R. Atkins, *Symposium on Liquid and Solid He<sup>3</sup>* (Ohio State University Press, Columbus, 1957), p. 11; *Proceedings of the Fifth International Conference on Low-Tempera-*

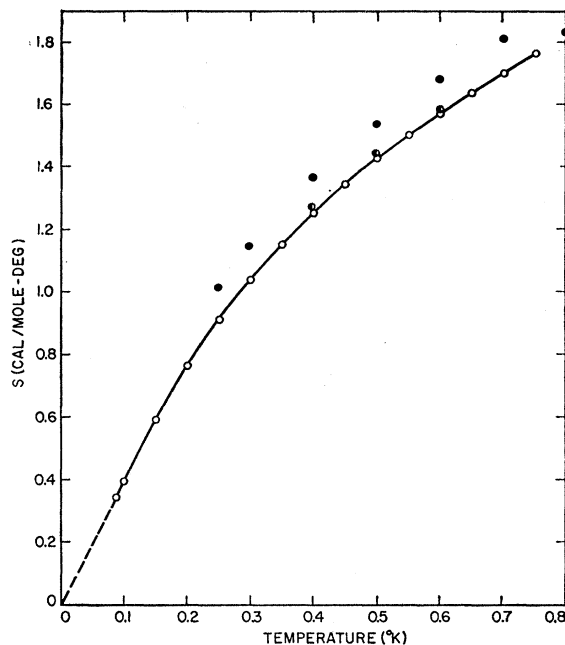


FIG. 6. The entropy of liquid He<sup>3</sup> at the saturated vapor pressure versus temperature. ●, Roberts and Sydoriak.<sup>2</sup> ●, Abraham, Osborne, and Weinstock.<sup>3</sup> ○, present results.

near 0°K. At 1°K therefore  $C_{ph}$  would amount only to approximately 6% of the observed value of  $C_{sat}$ , and to much less at lower temperatures.

Many workers, for example de Boer and Cohen,<sup>33</sup> Rice,<sup>34</sup> Price,<sup>35</sup> and Temperley,<sup>36</sup> have considered the thermal excitations in terms of a cell model of the liquid with two or more He<sup>3</sup> atoms per cell. By suitable choice of numerical values of the parameters involved, the cell model can be made to reproduce the observed specific heat curve above about 0.25°K adequately well. However, unless one further assumes that the energy levels are appropriately broadened, this approach must lead to an exponential temperature dependence of the specific heat as  $T \rightarrow 0^\circ\text{K}$ , rather than a linear variation.

### (b) Entropy

The entropy at the saturated vapor pressure,  $S_{sat}$ , has been calculated from our observed specific heats with the linear extrapolation of  $C_{sat}$  to 0°K referred to in Sec. V (a) above. The results are shown in Fig. 6 and

*ture Physics and Chemistry, Madison, Wisconsin, 1957*, edited by J. R. Dillinger (University of Wisconsin Press, Madison, 1958), p. 95.

<sup>32</sup> Laquer, Sydoriak, and Roberts, *Symposium on Liquid and Solid He<sup>3</sup>* (Ohio State University Press, Columbus, 1957), p. 15; *Proceedings of the Fifth International Conference on Low-Temperature Physics and Chemistry, Madison, Wisconsin, 1957*, edited by J. R. Dillinger (University of Wisconsin Press, Madison, 1958), p. 98.

<sup>33</sup> J. de Boer and E. G. D. Cohen, *Physica* **21**, 79 (1955).

<sup>34</sup> O. K. Rice, *Phys. Rev.* **97**, 263 and 1176 (1955); **98**, 847 (1955).

<sup>35</sup> P. J. Price, *Phys. Rev.* **97**, 259 (1955).

<sup>36</sup> H. N. V. Temperley, *Phys. Rev.* **97**, 835 (1955); *Proc. Phys. Soc. (London)* **A68**, 1136 (1955).

TABLE II. Entropy,  $S$ , of liquid  $\text{He}^3$  at the saturated vapor pressure as a function of temperature.  $S$  in cal/mole-deg;  $T$  in  $^\circ\text{K}$ .

$T$	0 $\rightarrow$ 0.1	0.150	0.200	0.250	0.300	0.350	0.400	0.450	0.500	0.550	0.600	0.650	0.700	0.750
$S$	$4.00T$	0.594	0.766	0.914	1.042	1.153	1.253	1.344	1.426	1.503	1.573	1.640	1.703	1.764

in Table II. It is estimated that the errors in these evaluations due to possible errors in  $C_{\text{sat}}$  are  $\pm 0.01$  cal/mole-deg and that the absolute values of  $S_{\text{sat}}$  may involve a systematic error due to a possible remaining curvature in the specific heat below  $0.1^\circ\text{K}$  which would tend to add an amount less than  $0.03$  cal/mole-deg. In Fig. 6 we include for comparison previous evaluations of  $S_{\text{sat}}$  made by Roberts and Sydoriak<sup>2,37</sup> and by Abraham *et al.*<sup>3,38</sup> Since at that time no extrapolation of  $C_{\text{sat}}$  to  $0^\circ\text{K}$  could be made, the absolute entropies were evaluated by computing entropy differences over the range of their specific heat measurements and by computing  $S_{\text{sat}}$  at one temperature either from vapor pressure data<sup>2</sup> or from heat of vaporization data.<sup>38</sup> It can be seen in Fig. 6 that the agreement between our results and those of Roberts and Sydoriak is remarkably good, whereas there appears to be an almost temperature-independent difference of  $0.10$  cal/mole-deg between our data and those of Abraham *et al.* The latter quote a possible error of  $\pm 0.03$  cal/mole-deg, which combined with our estimate of the maximum error could reduce the discrepancy to  $0.04$  cal/mole-deg. This discrepancy could be interpreted as an indication of a possible transition in liquid  $\text{He}^3$  below our lowest temperature of measurement of  $C_{\text{sat}}$ . We are inclined, however, to consider that it indicates a somewhat larger combined experimental error than has been estimated.

### (c) Specific heat under pressure

As is shown by the typical curves of Fig. 3 for  $C_p$  at 5.0 and at 25.0 atmospheres,  $C_p$  decreases with increas-

<sup>37</sup> See also E. F. Hammel, *Progress in Low-Temperature Physics* (North Holland Publishing Company, Amsterdam, 1955), Vol. 1, p. 78.

<sup>38</sup> Abraham, Osborne, and Weinstock, *Physica* **24**, 132 (1958); Weinstock, Abraham, and Osborne, *Suppl. Nuovo cimento* **9**, 310 (1958); and private communication.

ing pressure above about  $0.16^\circ\text{K}$ , whereas from this temperature down to  $0.12^\circ\text{K}$ , which was the lowest temperature of measurement,  $(\partial C_p/\partial p)_T$  is positive. A positive value of  $(\partial C_p/\partial p)_T$  was predicted by Brueckner and Gammel<sup>18</sup> for  $\text{He}^3$  as a Fermi liquid near  $0^\circ\text{K}$ , in marked contradistinction to a negative value which is expected for an ideal Fermi gas. Unfortunately our measurements of  $C_p$  do not allow satisfactory extrapolation to  $0^\circ\text{K}$ . In a subsequent paper, however, we describe additional measurements which allow a preliminary computation of  $S_p$  and therefrom a reasonable extrapolation of  $S_p$  and  $C_p$  to  $0^\circ\text{K}$ . Detailed discussion therefore is postponed to the subsequent paper.

The variation of  $C_p/C_F$ , where  $C_F$  is the specific heat of an ideal Fermi-Dirac gas of the appropriate density, is shown in Fig. 5. The shape of the curves is qualitatively similar to that for  $C_{\text{sat}}/C_F$ , but it is noteworthy that as the temperature is increased the value of  $C_p/C_F$  becomes independent of pressure. Thus above  $0.5^\circ\text{K}$  the observed specific heat has the same pressure dependence within about 1% as an ideal Fermi-Dirac gas. At the low-temperature end of the curves,  $C/C_F$  is still rising rapidly with decreasing temperature, suggesting that a linear region in the specific heat at higher pressures may only be observed at temperatures well below  $0.1^\circ\text{K}$ .

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