

It is therefore of importance to study the permanence properties of the null field, that is, to find under which conditions Maxwell's equations will keep null a field initially null on a space-like hypersurface. Mariot¹¹ has in this connection proved the interesting theorem that this is so if the ξ -field is integrable, i.e., if it is orthogonal to a family of hypersurfaces. The geometrical meaning of the conditions imposed on the actual

¹¹ L. Mariot, thesis, Paris, 1957 (unpublished).

geometry by the existence of a null field is, however, not yet clear.¹²

ACKNOWLEDGMENT

I thank Dr. G. Rosen and Professor J. A. Wheeler for useful discussions. I am grateful to Professor R. Oppenheimer for the kind hospitality extended to me at the Institute for Advanced Study.

¹² Note added in proof.—Professor V. Hlavatý has kindly informed me of a forthcoming paper of his on this problem.

Some Consequences of Symmetries of the Strong Interactions*

G. FEINBERG AND R. E. BEHREND
Brookhaven National Laboratory, Upton, New York
 (Received March 5, 1959)

The consequences for the electromagnetic interactions of strange particles of some proposed symmetries of the strong interactions are discussed. A minimal electromagnetic coupling is assumed. It is shown that if the interactions with both π and K mesons of the nucleon and cascade particle are identical, if one neglects the $n-\Xi$ mass difference, and if charge independence is satisfied, then the electromagnetic form factors of the Λ^0 and Σ^0 , and in particular their magnetic moments, vanish. This is independent of any assumed relation between the Λ and Σ interactions, such as global symmetry. It is also shown that the same conditions, together with charge conjugation invariance, imply a generalized form of the Pais-Jost-Pugh theorem about processes involving only meson or photon external lines. If there is still more symmetry for the strong interactions, as in the case where the π interactions are globally symmetric and the K interactions also have this doublet structure, it is shown that the K^0 acts as a completely neutral particle insofar as electromagnetic interactions are concerned.

I. INTRODUCTION

THERE have been a number of proposals that the strong interactions of baryons with π mesons and K mesons possess more internal symmetry than is implied by invariance under conventional isotopic spin rotations.^{1,2} In this paper we will examine some of the consequences of these proposed new symmetries for the interaction of these particles with electromagnetic fields. Some of these results have been stated before under more restrictive assumptions than we will make here, and part of our purpose is to indicate how strong the symmetry required to derive a given result may be.

We shall assume the "principle of minimal electromagnetic coupling,"³ i.e., that the fundamental interaction of the electromagnetic field is only with the current 4-vector of the charged particles. All other interactions of the electromagnetic field are assumed to arise through the combined effects of the strong interactions and these minimal interactions. There is some

evidence for this assumption in π -nucleon physics, where the program of calculating the nucleon magnetic moments and form factors without introducing non-minimal terms has been moderately successful.⁴ In strange-particle physics, there is very little evidence about this, as the electromagnetic properties of the strange particles, other than their charges, have not been measured. There is, however, the original argument of Gell-Mann and Pais³ that the minimal principle is sufficient to eliminate strangeness-violating electromagnetic decays like $\Lambda^0 \rightarrow n + \gamma$. In this paper we use the minimal expression for the electromagnetic interaction to determine the transformation properties of the electromagnetic interaction under certain permutations of the baryons and mesons. Any terms which transform the same way could be added to the minimal interaction without changing the conclusions.

In Sec. II of the paper, we discuss the electromagnetic vertex operators, and in particular the magnetic moments, of the baryons. In Sec. III, a theorem proven by Pugh⁵ is generalized and proven under different hypotheses than he used. In Sec. IV the electromagnetic vertex of the neutral K mesons is discussed.

* Work performed under the auspices of the U. S. Atomic Energy Commission.

¹ M. Gell-Mann, Phys. Rev. **106**, 1296 (1957); A. Pais, Phys. Rev. **110**, 574 (1958); J. Schwinger, Ann. Phys. **2**, 407 (1957).

² G. Feinberg and F. Gürsey, Phys. Rev. **114**, 1153 (1959). This paper will be referred to as I. J. Sakurai, Phys. Rev. **113**, 1679 (1959).

³ M. Gell-Mann, *Proceedings of the Sixth Annual Rochester Conference on High-Energy Physics* (Interscience Publishers, Inc., New York, 1956); A. Pais, Phys. Rev. **86**, 663 (1952).

⁴ Federbush, Goldberger, and Treiman, Phys. Rev. **112**, 642 (1958).

⁵ R. Pugh, Phys. Rev. **109**, 989 (1958).

II. ELECTROMAGNETIC PROPERTIES OF THE HYPERONS

The minimal electromagnetic interactions of the baryons and mesons are

$$H_{em} = ie \left[\frac{1}{2} \bar{\psi}_N \gamma_\mu (1 + \tau_3) \psi_N + \frac{1}{2} \bar{\psi}_\Xi \gamma_\mu (\tau_3 - 1) \psi_\Xi + \bar{\psi}_\Sigma \gamma_\mu T_3 \psi_\Sigma \right] A_\mu + ie \left[\frac{1}{2} \bar{\phi}_K (1 + \tau_3) \partial_\mu \phi_K - \frac{1}{2} (\partial_\mu \bar{\phi}_K) (1 + \tau_3) \phi_K \right] A_\mu + \frac{1}{2} e^2 \bar{\phi}_K (1 + \tau_3) \phi_K A_\mu^2 + ie \left[\bar{\phi}_\pi T_3 \partial_\mu \phi_\pi \right] A_\mu + e^2 \bar{\phi}_\pi T_3^2 \phi_\pi A_\mu^2. \quad (1)$$

Here ψ_N , ψ_Ξ , and ϕ_K are 2-component isotopic spinors, for the nucleon, cascade particle, and K meson; ψ_Σ and ϕ_π are isotopic vectors for the Σ hyperon and π meson. τ_3 is the Pauli matrix, and T_3 is the isotopic spin-one matrix.

This Hamiltonian will be invariant if one changes each positively charged baryon into a negatively charged baryon, each neutral baryon into another neutral baryon, and similarly for the mesons, and also changes the sign of the electromagnetic field. A transformation of this kind can be performed by first making a charge symmetry rotation for all the particles with isotopic spin, and next interchanging nucleon with cascade particle and the K^+ with \bar{K}^0 , and finally changing the sign of the electromagnetic field. To discuss this formally, we consider two transformations. The first is

$$\begin{aligned} \psi_N &\rightarrow \psi_\Xi & \text{or } p &\rightarrow \Xi^0, \quad n \rightarrow \Xi^-, \\ \psi_\Xi &\rightarrow -\psi_N & \text{or } \Xi^0 &\rightarrow -p, \quad \Xi^- \rightarrow -n, \\ \phi_K &\rightarrow \phi_K^G & \text{or } K^+ &\rightarrow \bar{K}^0, \quad K^0 \rightarrow -\bar{K}^+, \\ \phi_K^G &\rightarrow -\phi_K & \text{or } \bar{K}^0 &\rightarrow -K^+, \quad \bar{K}^+ \rightarrow +K^0. \end{aligned} \quad (A)$$

Here ϕ_K^G is the 2-component spinor

$$\begin{pmatrix} \bar{K}^0 \\ -\bar{K}^+ \end{pmatrix}.$$

This transformation is a rotation among particles with opposite hypercharge, and is a special case of the rotations called ζ rotations in I. The second transformation we consider is the conventional isospin

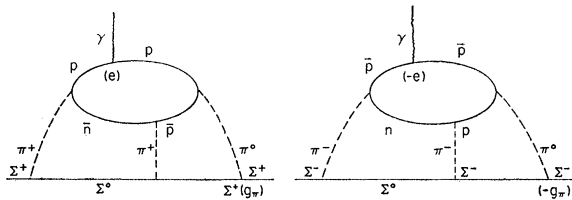


FIG. 1. Two diagrams contributing to the Σ^+ and Σ^- electromagnetic interactions which have the same sign. The vertices which have opposite coupling constants in the two diagrams are labelled. It may be seen that the opposite charge of the p and \bar{p} are compensated by the opposite sign of the Σ^+ and Σ^- couplings with the π^0 .

rotation $e^{i\pi T_2(\text{total})}$,

$$\begin{aligned} \psi_N &\rightarrow e^{\frac{1}{2}i\pi\tau_2} \psi_N & \text{or } p &\rightarrow n, \quad n \rightarrow -p, \\ \psi_\Xi &\rightarrow e^{\frac{1}{2}i\pi\tau_2} \psi_\Xi & \text{or } \Xi^0 &\rightarrow \Xi^-, \quad \Xi^- \rightarrow -\Xi^0, \\ \psi_\Sigma &\rightarrow e^{i\pi T_2} \psi_\Sigma & \text{or } \Sigma^+ &\rightarrow -\Sigma^-, \quad \Sigma^- \rightarrow -\Sigma^+, \\ & & & \Sigma^0 \rightarrow -\Sigma^0, \\ \Lambda^0 &\rightarrow \Lambda^0, \\ \phi_\pi &\rightarrow e^{i\pi T_2} \phi_\pi & \text{or } \pi^+ &\rightarrow -\pi^-, \quad \pi^- \rightarrow -\pi^+, \\ & & & \pi^0 \rightarrow -\pi^0, \\ \phi_K &\rightarrow e^{\frac{1}{2}i\pi\tau_2} \phi_K & \text{or } K^+ &\rightarrow K^0, \quad K^0 \rightarrow -K^+. \end{aligned} \quad (B)$$

Under the product transformation AB , one has

$$\begin{aligned} \psi_N &\rightarrow e^{\frac{1}{2}i\pi\tau_2} \psi_\Xi & \text{or } p &\rightarrow \Xi^-, \quad n \rightarrow -\Xi^0, \\ \psi_\Xi &\rightarrow -e^{\frac{1}{2}i\pi\tau_2} \psi_N & \text{or } \Xi^0 &\rightarrow -n, \quad \Xi^- \rightarrow p, \\ \psi_\Sigma &\rightarrow e^{i\pi T_2} \psi_\Sigma & \text{or } \Sigma^+ &\rightarrow -\Sigma^-, \quad \Sigma^- \rightarrow -\Sigma^+, \\ & & & \Sigma^0 \rightarrow -\Sigma^0, \\ \Lambda^0 &\rightarrow \Lambda^0, \\ \phi_\pi &\rightarrow e^{i\pi T_2} \phi_\pi & \text{or } \pi^+ &\rightarrow -\pi^-, \quad \pi^- \rightarrow -\pi^+, \\ & & & \pi^0 \rightarrow -\pi^0, \\ \phi_K &\rightarrow e^{\frac{1}{2}i\pi\tau_2} \phi_K^G = -\bar{\phi}_K & \text{or } K^+ &\rightarrow -\bar{K}^+, \\ & & & K^0 \rightarrow -\bar{K}^0. \end{aligned} \quad (AB)$$

Thus under the product transformation AB , each positive baryon goes into a negative baryon and the neutral baryons go into neutral baryons, and similarly for the mesons. Therefore if one changes the sign of A_μ , H_{em} will be invariant. It should be noted the H_{em} is not invariant under A and B separately.

It is important to note that this transformation does not mix Σ with Λ , or with nucleon and cascade particle. This distinguishes it from the transformation to be considered in Sec. IV, which also leaves H_{em} invariant. However, since this transformation interchanges nucleon and cascade particle, it will only be useful if one neglects the mass difference between these particles, and also certain mass splittings within each multiplet. When this is done, the free-particle Lagrangian is invariant under AB . It is not necessary to neglect the Σ - Λ mass difference or the Σ -nucleon mass difference, for example.

We assume that the strong interactions are charge independent, and so invariant under B . Then in order to get invariance under the product, it is necessary that they also are invariant under A . The strong interactions with π and K mesons which are charge independent are

$$\begin{aligned} H_\pi &= g_{NN\pi} \bar{\psi}_N \tau_3 \psi_N \phi_\pi + g_{\Sigma\Sigma\pi} \bar{\psi}_\Sigma T_3 \psi_\Sigma \phi_\pi + g_{\Sigma\Lambda\pi} \bar{\psi}_\Sigma \psi_\Lambda \phi_\pi \\ &\quad + g_{\Xi\Xi\pi} \bar{\psi}_\Xi T_3 \psi_\Xi \phi_\pi + \text{H.c.}, \\ H_K &= g_{NAK} \bar{\psi}_N \psi_\Lambda \phi_K + g_{N\Sigma K} \bar{\psi}_N T_3 \psi_\Sigma \phi_K + g_{\Sigma\Lambda K} \bar{\psi}_\Sigma \psi_\Lambda \phi_K^G \\ &\quad + g_{\Sigma\Sigma K} \bar{\psi}_\Sigma T_3 \psi_\Sigma \phi_K^G + \text{H.c.}, \end{aligned} \quad (2)$$

where Dirac matrices are omitted. Invariance under A now implies the following relations among the coupling

constants:

$$\begin{aligned} g_{NN\pi} &= g_{\Xi\Xi\pi}, \\ g_{NAK} &= g_{\Xi AK}, \\ g_{N\Sigma K} &= g_{\Xi\Sigma K}. \end{aligned} \quad (3)$$

It also implies that the Dirac covariants occurring in the nucleon terms must be the same as those occurring in the Ξ terms, so that N and Ξ have the same parity. It may also be noted that if one had instead of (3) the relation

$$\begin{aligned} g_{NAK} &= -g_{\Xi AK}, \\ g_{N\Sigma K} &= -g_{\Xi\Sigma K}, \end{aligned} \quad (3')$$

it would be possible to modify transformation A by inserting an additional minus sign in the transformation of ψ_N and ψ_{Ξ} , which would maintain invariance of H_{em} and give invariance of the strong interactions.

These conditions (3) on the coupling constants are independent of any over-all doublet structure of the $\Sigma\Lambda$ system. Thus they would be satisfied in the model proposed by Sakurai² and in I in which the π and K couplings involve different doublets. They could also be satisfied in a theory in which the Λ does not interact directly with π mesons at all, or even if the Σ and Λ have opposite parity.⁶ Of course, if the conditions (3) are satisfied one cannot account for the nucleon-cascade mass difference by either the π or K interactions, and must look elsewhere for this effect.

Let us examine the consequences of invariance under the transformation AB . This implies that the electromagnetic vertex of the Λ^0 and Σ^0 , and in particular their magnetic moments, vanish identically, whereas those of the Σ^+ and Σ^- are equal in magnitude and opposite in sign. To prove this, consider the quantity

$$\langle T[\bar{\psi}_\Lambda(x)\psi_\Lambda(y)A_\mu(z)] \rangle_0,$$

where T is the time-ordering operator and $\langle \ \rangle_0$ means expectation value in the physical vacuum. This quantity is related⁷ by a simple linear integral transform to the electromagnetic vertex operator for the Λ^0 , and we will therefore symbolize it by $\Gamma_{\Lambda^0\mu}$.

Let U be the unitary transformation which generates the permutations in AB . Since this commutes with the total Hamiltonian when the above coupling constant relations are satisfied and $m_N - m_{\Xi}$ is neglected, it will leave the vacuum invariant. Then

$$\begin{aligned} \Gamma_{\Lambda^0\mu} &= T\langle \bar{\psi}_\Lambda \psi_\Lambda A_\mu \rangle_0 \\ &= T\langle U \bar{\psi}_\Lambda \psi_\Lambda A_\mu U^\dagger \rangle_0. \end{aligned}$$

By the definition of what U does to the different fields,

$$\begin{aligned} \Gamma_{\Lambda^0\mu} &= -T\langle \bar{\psi}_\Lambda \psi_\Lambda A_\mu \rangle_0 \\ &= -\Gamma_{\Lambda^0\mu} \\ &= 0. \end{aligned}$$

Therefore the electromagnetic vertex, and hence the magnetic moment, of the Λ^0 vanish. Similarly one can prove the following theorems, if one has invariance under AB .

1. $\Gamma_{\Sigma^0\mu} = -\Gamma_{\Sigma^0\mu} = 0$, so the magnetic moment of the Σ^0 vanishes.
2. $\Gamma_{\Sigma^+\mu} = -\Gamma_{\Sigma^-\mu}$, so the magnetic moments of the Σ^+ and Σ^- are opposite.
3. $\Gamma_{p\mu} = -\Gamma_{n\mu}$, so the magnetic moments of the proton and neutron are opposite.
4. $\Gamma_{n\mu} = -\Gamma_{\Xi^0\mu}$, so the magnetic moments of the neutron and Ξ^0 are opposite.
5. Since $\Sigma^0 \rightarrow -\Sigma^0$, $\Lambda^0 \rightarrow \Lambda^0$ and $A_\mu \rightarrow -A_\mu$, the decay $\Sigma^0 \rightarrow \Lambda^0 + 2$ photons is forbidden. This decay might be expected to occur $\sim 1/300$ of all Σ^0 decays in the absence of a selection rule.

Some of those theorems have been stated before under more special conditions,^{8,9} such as global symmetry for the pion interactions and neglect of the K interactions. We emphasize that they have been proven here under the assumptions of charge independence and symmetry between the nucleon and cascade particle alone, even in the presence of K interactions, and that no global symmetry is required.

It is interesting to note that while in low-order perturbation calculations there is no contribution to $\Gamma_{\Sigma^+\mu} + \Gamma_{\Sigma^-\mu}$ or to $\Gamma_{\Lambda^0\mu}$ from pion interactions alone, this is not true in general. In particular, diagrams in which the π mesons that are virtually present in the Σ create nucleon pairs, one of which then emits a photon, and in which also a π^0 is exchanged between the nucleon and Σ , will not give opposite contributions to the Σ^+ and Σ^- vertex. Examples of such diagrams are given in Fig. 1. It is remarkable that such effects occur for the first time in such a high order of perturbation theory. We note that when invariance under AB is satisfied, such diagrams are cancelled by corresponding diagrams in which cascade-particle pairs occur in the intermediate state. Also, in theories where pair effects are neglected such as the Chew model, there will be no pion contribution to the quantities $\Gamma_{\Lambda^0\mu}$, $\Gamma_{\Sigma^0\mu}$, $\Gamma_{\Sigma^+\mu} + \Gamma_{\Sigma^-\mu}$ provided that charge symmetry is satisfied. This follows because in such theories one need not consider the nucleon and cascade particle at all because they cannot be reached starting from a state with Σ or Λ and only utilizing π and electromagnetic interactions. In this case the interaction of the Σ and Λ with π and photon is invariant under B , and this implies the result stated.

One may next ask how these results will be affected by whatever mechanism is responsible for the splitting of the mass of the nucleon and cascade particle. Some speculations as to the source of such a splitting within

⁶ It is also possible to have invariance under the combined transformation AB if the K^+ and K^0 have odd relative parity, although then neither A nor B are valid symmetries separately.

⁷ Y. Takahashi, *Nuovo cimento* **6**, 371 (1957).

⁸ M. Gell-Mann, *1958 Annual International Conference on High-Energy Physics at CERN*, edited by B. Ferretti (CERN, Geneva, 1958).

⁹ Theorems 1 and 2 have been stated under essentially the same conditions by D. C. Peaslee, *Nuovo cimento* **6**, 1 (1957).

the framework of symmetric π and K interactions will be published elsewhere. In those processes where the nucleon and cascade particle occur only virtually, one possibility is to introduce the experimental values of the masses of Ξ and N for the virtual particles, while maintaining the coupling constant equalities given above. Then in a lowest-order perturbation calculation of the K contribution it is found that the Σ^0 and Λ^0 moments and the sum of the Σ^+ and Σ^- moments are still very small compared to the Σ^+ moment, being perhaps $\sim 5\%$ of the latter.¹⁰ We therefore think that a measurement of the Λ^0 magnetic moment in particular would provide a good test of the symmetry between interactions of nucleon and Ξ with π and K mesons which is implied by invariance under A . Such a measurement will perhaps be done in the near future at Brookhaven and elsewhere.

III. A GENERALIZATION OF THE PUGH THEOREM

Let us consider the transformation obtained by taking the product of AB with the charge-conjugation transformation C . This will act on the particles as follows:

$$\begin{aligned} \psi_N &\rightarrow e^{\frac{1}{2}i\pi\tau_2}\psi_{\Xi^c}, \\ \psi_{\Xi} &\rightarrow -e^{\frac{1}{2}i\pi\tau_2}\psi_{N^c}, \\ \psi_{\Sigma} &\rightarrow e^{i\pi T_2}\psi_{\Sigma^c}, \\ \psi_{\Lambda} &\rightarrow \psi_{\Lambda^c}, \\ \phi_{\pi} &\rightarrow -\phi_{\pi}, \\ \phi_K &\rightarrow -\phi_K, \\ A_{\mu} &\rightarrow A_{\mu}. \end{aligned} \quad (ABC)$$

(The label ψ^c means the charge-conjugate field.)

This transformation therefore reverses the sign of the π and K fields, permutes the baryons and antibaryons as indicated, and leaves the electromagnetic field invariant.

Consider now a process (or a Feynman diagram) in which the external lines are all K mesons, π mesons, or photons. In such a process the number of K 's involved must be even by strangeness conservation. (One counts K 's in the initial and final states.) Thus the phase factor in the transformation of the K is irrelevant. The following theorem therefore holds, if one has invariance under ABC :

In any process only involving real π mesons, K mesons, and photons, the sum of the number of π 's in the initial and final states must be even.

This theorem is a generalization of the Pais-Jost theorems,¹¹ which are satisfied when one neglects electromagnetic interactions. A similar theorem has been proven by Pugh,⁵ for processes involving only π mesons and photons, assuming global symmetry for the π couplings and neglecting the K interactions. It

would appear that in any case the condition of global symmetry is unnecessary, and could be replaced by the weaker condition of symmetry between \bar{N} and Ξ . One also sees that the theorem can be extended to include K interactions if they also show a symmetry between N and Ξ .

This theorem has no obviously useful consequences, except that it forbids the π^0 decay into 2 photons, as discussed by Pugh and others. This decay therefore only occurs via the mechanism which splits the N and Ξ mass.

IV. ELECTROMAGNETIC VERTEX OF K^0

The discussion in the last two sections was independent of any doublet structure of the Σ - Λ system. The results to be given here will however require such a structure. It will therefore be useful to rewrite the baryon-electromagnetic interaction in terms of the four doublets N_1, N_2, N_3, N_4 :

$$H_{\text{em}}^{\text{baryon}} = ieA_{\mu} \left[\frac{1}{2}\bar{N}_1\gamma_{\mu}(1+\tau_3)N_1 + \frac{1}{2}\bar{N}_2\gamma_{\mu}(1+\tau_3)N_2 + \frac{1}{2}\bar{N}_3\gamma_{\mu}(\tau_3-1)N_3 + \frac{1}{2}\bar{N}_4\gamma_{\mu}(\tau_3-1)N_4 \right]. \quad (4)$$

We also express in terms of these doublets the completely symmetric π and K interactions with baryons discussed in references 1 and 2.

$$H_{\pi} = iG_{\pi} \sum_{i=1}^4 \bar{N}_i \gamma_5 \tau_{\alpha} N_i \phi_{\pi}^{\alpha}, \quad (5)$$

$$H_K = G_K [\bar{N}_1 N_2 K^0 + \bar{N}_1 N_3 K^+ + \bar{N}_3 N_4 K^0 - \bar{N}_2 N_4 K^+] + \text{H.c.} \quad (6)$$

Here the doublets are defined by

$$N_1 = \begin{pmatrix} p \\ n \end{pmatrix}; \quad N_3 = \begin{pmatrix} (\Lambda^0 + \Sigma^0)/\sqrt{2} \\ \Sigma^- \end{pmatrix};$$

$$N_2 = \begin{pmatrix} \Sigma^+ \\ (\Lambda^0 - \Sigma^0)/\sqrt{2} \end{pmatrix}; \quad N_4 = \begin{pmatrix} \Xi^0 \\ \Xi^- \end{pmatrix}.$$

Both of the strong interactions are invariant under the transformation I :

$$\begin{aligned} N_1 &\rightarrow e^{\frac{1}{2}i\pi\tau_2}N_3, \\ N_2 &\rightarrow e^{\frac{1}{2}i\pi\tau_2}N_4, \\ N_3 &\rightarrow e^{\frac{1}{2}i\pi\tau_2}N_1, \\ N_4 &\rightarrow e^{\frac{1}{2}i\pi\tau_2}N_2, \\ \phi_{\pi} &\rightarrow e^{i\pi T_2}\phi_{\pi}, \\ K^0 &\rightarrow K^0, \\ K^+ &\rightarrow \bar{K}^+. \end{aligned} \quad (I)$$

This transformation is the product of an " I rotation" $e^{i\pi T_2}$, in the notation of Pais,¹ with the reflection operation

$$\begin{aligned} N_1 &\leftrightarrow N_3, \\ N_2 &\leftrightarrow N_4, \\ K^+ &\leftrightarrow \bar{K}^+. \end{aligned}$$

¹⁰ H. Katsumori, Progr. Theoret. Phys. (Kyoto) **18**, 375 (1957).

¹¹ A. Pais and R. Jost, Phys. Rev. **87**, 871 (1952).

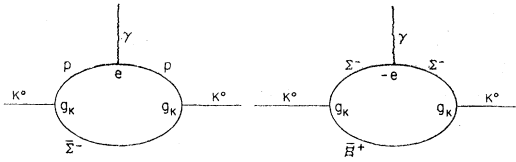


FIG. 2. Example of diagrams which cancel in the electromagnetic interaction of the K^0 . The diagrams are the same when invariance under the transformation I is satisfied, except for the opposite sign of the electromagnetic interactions.

It may also be seen that if one simultaneously changes the sign of A_μ , the minimal electromagnetic interactions both of the baryons and of the π and K mesons are invariant under this transformation. It follows from this that the electromagnetic vertex of the K^0 , which is essentially defined by

$$\Gamma_{K^0\mu} = T\langle K^0 \bar{K}^0 A_\mu \rangle_0,$$

satisfies

$$\Gamma_{K^0\mu} = T\langle U_1 K^0 \bar{K}^0 A_\mu U_1^\dagger \rangle_0 = -\Gamma_{K^0\mu} = 0, \quad (7)$$

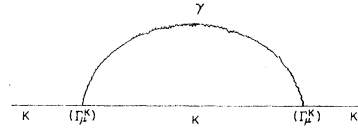
where U_1 is the unitary transformation generating the transformation I .

Thus if the symmetry corresponding to transformation I is satisfied and if we neglect the mass differences of the baryons, the K mesons will be electromagnetically completely neutral. One of the authors (G.F.) has previously pointed out¹² that since the K^0 is represented by a complex field, it could have a linear electromagnetic interaction, coming from its dissociation into charged baryons. What we have now shown is that if the symmetry given by transformation I holds, the K^0 nevertheless acts like a real field insofar as its linear electromagnetic interaction is concerned. By examining the transformation I , it may be seen that what is happening is a cancellation of the intermediate proton- $\bar{\Sigma}^-$ pairs against intermediate Σ^- - \bar{p} pairs, as shown in Fig. 2.

To the extent that transformation I is a valid symmetry, we therefore expect that the charge form factor of the K^0 will vanish. This may have an interesting

¹² G. Feinberg, Phys. Rev. **109**, 1381 (1958).

FIG. 3. Feynman-Speisman type diagram for the K -meson electromagnetic self-mass.



consequence concerning the mass difference of the K mesons. One of the terms in the electromagnetic K -meson self-mass is the one considered by Feynman and Speisman,¹³ in which one inserts the two electromagnetic vertex functions for K^0 and K^+ into the lowest-order Feynman diagrams (See Fig. 3).

By the above theorem, the contribution for the K^0 vanishes. The result for the K^+ is positive, if the bare K^+ vertex is used. Thus within this approximation it may be difficult to account for the recently observed¹⁴ mass difference $m_{K^0} - m_{K^+} = 4$ Mev.

Of course there are other terms in the expressions for the self-mass, and so one need not yet conclude either that the symmetry I is strongly violated, or that there are nonminimal electromagnetic contributions to the self mass. It may be noted that the symmetry AB considered in Sec. II implies that the masses of the Σ^+ and Σ^- are equal even in the presence of electromagnetic interactions. One may then consider the possibility that both the $K^0 - K^+$ mass difference and the $\Sigma^+ - \Sigma^-$ mass difference have a common nonminimal electromagnetic origin.

It may finally be mentioned that if one requires invariance under transformation I , and also charge independence, one is led automatically to the interactions (5), (6) for H_π, H_K with the usual ambiguity in the sign of the K -cascade interactions.

ACKNOWLEDGMENTS

The authors would like to thank Dr. F. Gürsey for many discussions and suggestions. They would also like to thank Dr. L. Landovitz, Dr. L. Marshall, and Dr. A. Pais for helpful discussions.

¹³ R. Feynman and G. Speisman, Phys. Rev. **94**, 500 (1954).

¹⁴ A. H. Rosenfeld *et al.*, Phys. Rev. Letters **2**, 110 (1959); F. S. Crawford *et al.*, Phys. Rev. Letters **2**, 112 (1959).