PHYSICAL REVIEW VOLUME 115, NUMBER 3 AUGUST 1, 1959

Absorption of 1-Bev Photons*†

E. MALAMUD
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York

(Received March 12, 1959)

The total cross section for the attenuation of high-energy gamma rays has been measured in various elements. The variation with atomic number, Z, has been observed by measuring the absorption of 1-Bev gamma rays in 12 different elements ranging from hydrogen to uranium. Additional measurements were made in copper at 400 and 700 Mev to show the energy dependence of the absorption processes. These results are then combined with those of other investigators at lower energies and show that the theory of pair production in the nuclear field correctly predicts the cross section as a function of atomic number and photon energy. The measurements in low-Z elements give information on pair production in the field of the electron. The results are in closer agreement to the calculations of Wheeler and Lamb than to the estimate of Joseph and Rohrlich. In addition a short experiment was performed to measure the symmetry of the energy distribution between the electron and positron members of the pair. The results show that the energy-sharing curve is symmetrical as predicted by theory.

I. INTRODUCTION

BY measuring the transmission of photons through known thicknesses of various materials, an absorption coefficient is obtained which is the sum of the absorption coefficients for the various interaction processes. Photons near the peak energy of the bremsstrahlung beam from the Cornell synchrotron are used. High-energy gamma rays are absorbed almost entirely by Compton scattering and pair production with a small contribution from photonuclear effects.

Pair production can occur in two ways: the pair can be produced in the field of the nucleus. This process is coherent pair production and will be called nuclear-field or N.F. pair production in this paper. On the other hand, a pair can be produced in the field of an atomic electron. This is incoherent pair production and will be called electron-field or E.F. pair production.

Essentially then, this experiment is a study of the total cross section at high energy for nuclear-field pair production, electron-field pair production, and Compton scattering. The high-Z absorbers provide information on N.F. pair production. The well known breakdown of the Born approximation in high-Z materials can be observed, and the results checked against the exact theory of Davies, Bethe, and Maximon.[1] In this theory a Coulomb correction to the Born approximation result is calculated.

In low-Z absorbers, E.F. pair production contributes an appreciable fraction of the total cross section. Measurements of sufficient accuracy in low-Z elements give information on the electron field pair production and can be compared with various predictions.

II. EXPERIMENTAL ARRANGEMENT AND PROCEDURE

The experimental arrangement is shown in Fig. 1. The bremsstrahlung beam from the Cornell synchrotron passes through the monitor, a thin ionization chamber which responds to photons of all energies and consists of eleven $\frac{1}{32}$-in. thick aluminum plates spaced $\frac{1}{2}$ in. apart. This chamber attenuates 1-Bev photons about 8%. Alternate plates of the monitor are connected to

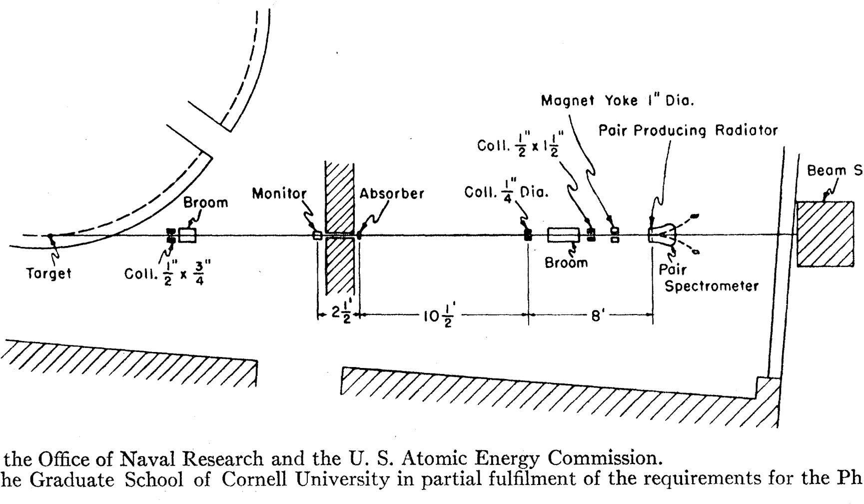


FIG. 1. Experimental arrangement for the total absorption measurements.

---

* Assisted by the joint program of the Office of Naval Research and the U. S. Atomic Energy Commission.

† Based on a thesis submitted to the Graduate School of Cornell University in partial fulfilment of the requirements for the Ph.D. degree.

[1] Davies, Bethe, and Maximon, Phys. Rev. **93**, 788 (1954).

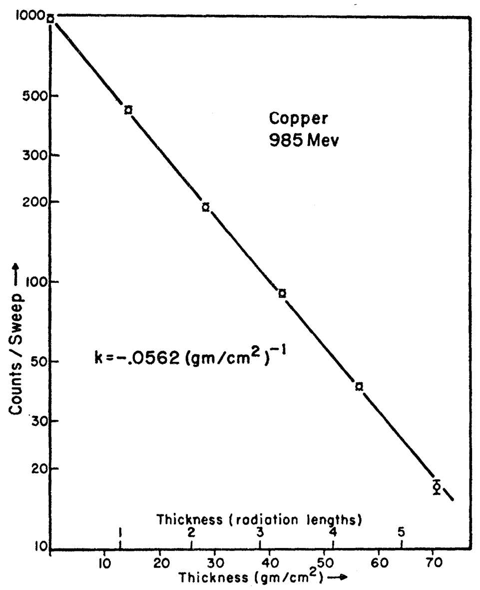


FIG. 2. Absorption curve for copper at 985 Mev. The straight line drawn was obtained by least-squares analysis of the data.

300 volts and to a condenser which integrates the charge. This charge is measured by an integrator circuit.[2] The unit of beam is the "sweep."

Shielding between the absorber and the monitor keeps back-scattered radiation out of the monitor.

The defining collimator for the pair spectrometer is $\frac{1}{4}$ in. in diameter. It subtends an angle of about 2 milliradians at the absorber.

A broom magnet with a field of about 7000 gauss sweeps out the electrons and positrons made in the absorber and collimator.

The pair spectrometer consists of a magnet with pole tips spaced 1 in. apart, a copper pair-producing radiator, and two $\frac{1}{2}$-in. thick plastic scintillation counters. These are sensitive to electrons and positrons of the same energy.

The resolution function of the pair spectrometer is a triangle whose full width at half maximum, ΔE, is 4.7% of the photon energy. This fraction, $\Delta E/E$, is approximately constant as the magnetic field in the pair spectrometer is varied. The total attenuation cross sections at high energy are slowly varying functions of the energy so that for this experiment the spectrometer can be considered a monochromatic detector. The energy was calibrated by the stretched wire technique and is known to an accuracy of $\lesssim 1\%$.

Helium fills the space between the poles of the broom magnet and the pair spectrometer to reduce the background. The background is also reduced by a secondary collimator following the broom magnet which is not "seen" by the beam. The background is measured by removing the copper pair producing radiator, and is $\lesssim 5\%$ of the rate with it in place.

The two counters are hooked in direct coincidence and also in delayed coincidence to monitor accidentals. The electronics are standard circuits and have a resolving time of about 0.6 μsec. If N is the number of real coincidences, A is the number of accidental coincidences, s is the number of "sweeps" all with the radiator in place, and M, B, and t are the corresponding quantities with the radiator removed, then the number of counts in a run, C, is given by

$$C=\frac{N-A}{s}-\frac{M-B}{t}.$$

From runs with absorber and without absorber the cross section can be obtained. In practice runs were made with absorbers of various thicknesses and the data plotted in the form $\ln C$ *vs* x where x = thickness in g/cm^2. The best straight line through the points is found by a least-squares analysis. The slope of the line is the mass absorption coefficient.

The absorber thicknesses used ranged from 0.70 mean free paths (m.f.p.) of lithium to copper absorbers of almost 4 m.f.p. The hydrogen measurement was done with carbon and polyethylene absorbers of about 2.8 m.f.p. The hydrogen thickness was 0.26 m.f.p. When working with thick absorbers it is essential that the absorption occurs exponentially. The absorption in copper was measured with several different thicknesses and the result is shown in Fig. 2.

Statistical errors were computed by two methods: from the statistical accuracy of each run (internal error), and also from the deviations of the measurements from the absorption curve found by least-squares analysis (external error). The larger of the two is the one quoted. In most cases these two errors agreed closely.

The results of the experiment are shown in Table I.

III. CORRECTIONS AND SYSTEMATIC ERRORS

The transmission method is capable of quite high accuracy because only the ratio of counts in the two detectors is used to obtain the total cross section. Since it is not necessary to know detection efficiencies many of the usual uncertainties are absent. The systematic errors that are present are small and in many cases a correction can be calculated. Following is a summary of the corrections and errors that are important in this experiment.

A. Corrections

1. The method of taking data in this experiment was to vary the beam intensity for absorber in and out runs to keep roughly the same counting rate at the pair spectrometer. Thus it is essential that the monitor be independent of incident beam intensity. When the beam becomes too intense recombination of ions in the chamber will reduce the amount of charge collected.

---

[2] R. Littauer, Rev. Sci. Instr. **25**, 148 (1954).

TABLE I. Experimental total cross sections.

Absorber	Energy (Mev)	Corrected k (Mass abs. coeff.) cm^2/g	Statistical error (%)	Other errors (%)	Total error (%)	Total cross section (per atom)
H (CH_2-C)	1000	0.01160	2.9	0.8	3.0	19.4 ±0.6 mb
Li	995	0.008666	2.0	1.1	2.3	99.9 ±2.3 mb
Be	985	0.01117	1.0	0.3	1.1	167.1 ±1.8 mb
C	995	0.01697	0.8	0.4	0.9	338.4 ±3.1 mb
Al	985	0.03044	1.3	0.2	1.3	1.363±0.018 barns
Ti	1002	0.04632	1.3	0.4	1.4	3.683±0.052 barns
Cu	412	0.05322	0.8	0.3	0.9	5.614±0.051 barns
Cu	700	0.05535	3.1	0.3	3.1	5.839±0.181 barns
Cu	985	0.05622	1.4	0.3	1.4	5.930±0.083 barns
Mo	1002	0.07638	1.3	0.3	1.3	12.17 ±0.16 barns
Sn	985	0.08298	0.7	0.3	0.8	16.35 ±0.13 barns
Ta	1002	0.1067	1.9	0.3	1.9	32.04 ±0.61 barns
Pb	1003	0.1143	1.0	0.3	1.0	39.32 ±0.39 barns
U	1003	0.1200	1.1	0.3	1.1	47.43 ±0.52 barns

Boag[3] has worked out the theory of recombination in ion chambers in pulsed beams and verified this theory experimentally. He gives the following formula for the fraction of charge collected, f:

$$f=\frac{\ln(1+u)}{u},$$

where $u=ad^2/V_0$, d=spacing of the plates in the chamber, V_0=voltage across the plates, and a is a constant that depends on the gas filling the chamber (air in this case), the beam area, and the beam intensity. The constant a was determined for three different beam intensities in an auxiliary experiment. This was done by measuring the fraction of charge collected as a function of voltage on the chamber, V_0. The correction to the cross sections was $<0.1\%$ in most cases. The maximum correction was 0.6% to the lithium result.

2. Small corrections $\lesssim 1\%$ are made in the data for drift and dead time in the electronic integrating circuit.

3. The displacement of air from the beam path by the long absorbers, lithium, carbon, and polyethylene, caused a small correction of the order of 0.1%. The lithium target was placed in a long glass tube, filled with helium, with a 0.003-in. polyethylene window at each end, and the lithium correction includes the effect of the windows.

4. All the absorbers have been analyzed spectrographically for impurities. In three cases it was necessary to reduce the measured cross sections by the following amounts: lithium, 0.2%; beryllium, 1.6%; aluminum, 0.6%. The chemical analyses will be discussed in more detail in the next section.

B. Other Errors

1. The integrator circuit does not reset exactly to zero at the end of each sweep. This causes a small uncertainty in the cross section, not more than 0.1%.

2. Radiation can be back-scattered from the absorber into the monitor, causing a systematic error. This effect was checked by placing the absorber at different positions relative to the monitor, and causes less than 0.1% error in the cross section.

3. Second generation photons that can affect the measured cross section are caused by electrons and positrons created in the absorber, which in turn radiate photons that fall in the pair spectrometer energy "window." Also Compton-scattered photons can enter this "window." The number of degraded photons depends on the following parameters:

(1) Absorber thickness.

(2) E_m-E_0, where E_m is the peak bremsstrahlung energy and E_0 is the pair spectrometer energy.

(3) The width of the pair spectrometer window, ΔE.

(4) The angle subtended by the pair spectrometer at the absorber and the diameter of the beam traversing the absorber.

Of the above, (2) and (4) are the most important. The pair spectrometer energy was set about 10% below the peak bremsstrahlung energy. Under such conditions a calculation of the number of degraded photons showed that the error in the cross section was less than 0.5% in all cases.[4]

4. The uncertainty in measuring the g/cm^2 in the absorber has been calculated for each absorber and ranges from 0.05% to 0.20% except for lithium and hydrogen. The lithium absorber consisted of ten 2-in. diameter rods, each 6 in. long. These rods were not very uniform in cross section and this caused a 1% uncertainty in the lithium thickness. The uncertainty in the hydrogen thickness is 0.36% but most of this is due to the uncertainty in the C:H ratio in the polyethylene rather than in the thickness of the absorbers. This ratio was determined from the average of the analyses on four separate pieces of polyethylene.[5]

5. Chemical analysis of the absorbers was usually

---

[3] J. W. Boag, Brit. J. Radiobiology **23**, 601 (1950).

[4] I am grateful to Professor K. Greisen for his help with this calculation.

[5] The C:H ratio in the polyethylene absorbers was determined by the Schwarzkopf Microanalytical Laboratory, Woodside, New York.

TABLE II. Theoretical cross sections and "experimental" electron-field pair cross sections for 1-Bev photons.

Absorber and atomic number	Experimental cross section (mb/atom)	Theoretical cross sections Compton scattering (mb/atom)	Nuclear-field pairs (mb/atom)	Photomesons (mb/atom)	"Experimental" E.F. pairs (mb/atom)	Theoretical E.F. pairs (Wheeler-Lamb) (mb/atom)	$Z\sigma_{\mathrm{E.F.}}$("exp") / $\sigma_{\mathrm{N.F.}}$(theor)
H 1	19.4±0.6	1.12	8.70	0.13	9.5±0.6	9.51	1.09±0.07
Li 3	99.9±2.3	3.36	72.01	0.27	24.3±2.3	28.38	1.01±0.09
Be 4	167.1±1.8	4.52	125.97	0.33	36.0±1.8	37.15	1.14±0.05
C 6	338.4±3.1	6.72	276.74	0.43	54.5±3.1	54.47	1.18±0.07

TABLE III. Theoretical cross sections (in barns/atom) and experiment/theory ratios for 1-Bev photons.

Absorber and atomic number	Compton scattering (b/atom)	Nuclear-field pairs (Born approx.) (b/atom)	Coulomb correction (b/atom)	Electron-field pairs (Wheeler-Lamb) (b/atom)	Total cross section (b/atom)	$\left(\frac{\text{Total exp.}\sigma}{\text{Total theor.}\sigma}\right)$
Al 13	0.0147	1.2383	0.0029	0.1114	1.3615	1.001±0.013
Ti 22	0.025	3.421	0.025	0.180	3.601	1.023±0.014
Cu 29	0.033	5.832	0.078	0.230	6.017	0.986±0.014
Mo 42	0.047	11.905	0.349	0.321	11.924	1.021±0.013
Sn 50	0.067	16.626	0.667	0.375	16.401	0.997±0.008
Ta 73	0.08	34.38	2.69	0.52	32.29	0.992±0.019
Pb 82	0.09	43.04	4.10	0.58	39.61	0.993±0.010
U 92	0.10	53.54	6.14	0.64	48.14	0.985±0.011

done in two steps. First a semiquantitative search was made; this gave upper limits on a large number of elements. Then if there were some impurities present that could affect the cross section by more than 0.1%, a quantitative analysis was performed on these alone. The correction described above was based on these major impurities. However, the small amounts of other elements present cause a small uncertainty in the cross section of about 0.1%.[6]

6. In calculating C, the number of counts/sweep, it was assumed that the efficiency of the real coincidence and accidental coincidence channels was identical. Their efficiencies were measured to be the same to within 10%. The pair spectrometer counting rate was kept low enough so that the accidental rate was $\leq 10\%$ of the real rate. Under these conditions, assuming that the efficiencies are equal causes an uncertainty of a few tenths of a percent.

The above six sources of error are independent and the total error is the root mean square of the above numbers, which is 1.1% for lithium and less for the other elements.

IV. THEORETICAL CALCULATIONS

Theoretical cross sections have been calculated as follows: Compton scattering was calculated using the Klein-Nishina formula. Nuclear-field pair production was calculated using the theory of Davies, Bethe, and Maximon.[1] In this theory the cross section is obtained by calculating the Born approximation result and subtracting a Coulomb correction from it. The Born approximation result is found by numerical integration of the Bethe-Heitler differential cross section[7,8] using the appropriate screening functions. The screening functions are based on the Thomas-Fermi statistical model of the atom except for hydrogen where exact wave functions have been used. Both the Thomas-Fermi and hydrogen screening functions are given in the article by Wheeler and Lamb.[9] There is an empirical energy-dependent Coulomb correction which is very small at high energies. It has been calculated using constants based on low-energy experiments as given by Grodstein.[10]

Electron-field pair production was calculated using the formula derived by Wheeler and Lamb.[9] This is based on the Born approximation and except for hydrogen the screening functions have been calculated from the Thomas-Fermi model.

The electron field pair calculation of Joseph and Rohrlich[11] predicts a total cross section which differs from that of Wheeler and Lamb by a constant amount independent of screening and energy. Wheeler and Lamb neglect certain Feynman diagrams, and the difference between the two calculations is due to Joseph and Rohrlich's estimate of the contribution of these diagrams. These terms are exchange diagrams and diagrams in which the atomic electron first absorbs the incident photon and then re-emits it, the re-emitted

---

[6] The chemical analysis of the absorbers was done by the New England Spectrochemical Laboratories, Ipswich, Massachusetts, and the National Spectrographic Laboratories, Cleveland, Ohio. The analysis of the uranium absorbers was furnished by the Atomic Energy Commission, Oakridge, Tennessee. The beryllium absorber was obtained on loan by Professor B. D. McDaniel through the courtesy of the Division of Research of the Atomic Energy Commission.

[7] H. A. Bethe and W. Heitler, Proc. Roy. Soc. London **A146**, 83 (1934).

[8] H. A. Bethe, Proc. Cambridge Phil. Soc. **30**, 524 (1934).

[9] J. A. Wheeler and W. E. Lamb, Phys. Rev. **55**, 858 (1939) and Phys. Rev. **101**, 1836 (1956).

[10] G. W. Grodstein, National Bureau of Standards Circular No. 583 (1957).

[11] J. Joseph and F. Rohrlich, Revs. Modern Phys. **30**, 354 (1958).

TABLE IV. List of total absorption experiments above 40 Mev.

Experimenters	Source of photons	Detector	Energy (Mev)	Absorbers	Number of measurements
Wyckoff and Koch[a]	Betatron, U.S. Bur. of Standards	Sodium iodide spectrometer	60	C, Al	2
Lawson[b]	Betatron, General Electric	Pair spectrometer	88	Be, Al, Cu, Sn, Pb, U	6
Moffatt, Thresher, Weeks, and Wilson[c] Moffatt and Weeks[d]	Synchrotron, Oxford, England	Biased liquid scintillation counter	42.5, 68.5, 94.0	H H (H_2 and CH_2-C), C, Cu, Ag, Pb, U	9
DeWire, Ashkin, and Beach[e]	Synchrotron, Cornell	Pair spectrometer	280	Be, Al, Cu, Sn, Pb, U	6
Anderson, Kenney, and McDonald[f] Anderson, Kenney, McDonald, and Post[g]	Synchrotron, Berkeley	Pair spectrometer	319	H, C, Be, Pb	4
Cooper[h]	Synchrotron, Cal. Tech.	Pair spectrometer	375–390	C, Al, Cu, Sn, Pb	20
			400–410	C, Al, Cu, Sn, Pb	
			425–440	C, Al, Cu, Sn, Pb	
			450–465	C, Al, Cu, Sn, Pb	
This experiment	Synchrotron, Cornell	Pair spectrometer	412	Cu	
			700	Cu	14
			985–1005	H, Li, Be, C, Al, Ti, Cu, Mo, Sn, Ta, Pb, U	

[a] See reference 17.
[b] See reference 18.
[c] See reference 19.
[d] See reference 20.
[e] See reference 21.
[f] See reference 22.
[g] See reference 23.
[h] See reference 24.

photon making a pair; however, these contributions should become relatively unimportant as the photon energy increases. At 1 Bev it is expected that the Wheeler-Lamb formula will give accurate predictions; at 100 Mev the neglect of these other diagrams might become significant. Suh and Bethe have concluded that the Wheeler-Lamb calculation is accurate at high energy. They discuss these points in detail in their article.[12]

Photonuclear effects are small and can be neglected for $Z > 13$. Photodisintegration cross sections are taken as A times the cross section for photodisintegration of the deuteron. This is taken from Wilson's phenomenological theory[13] which fits the experimental data up to 400 Mev. Above this energy it is small and is neglected. At the high energies photoproduction of mesons is the dominant nonelectronic process. Meson photoproduction cross sections in hydrogen are obtained from Bethe and de Hoffmann.[14] At high energies where double, triple, and K-meson production also contribute the total meson photoproduction cross section is estimated using Wilson's isobaric state model.[15] The total meson photoproduction cross section in other elements is taken as $A^{\frac{2}{3}}$ times the cross section in hydrogen. In beryllium, e.g., photodisintegration contributes ~0.2% of the total cross section at 100 Mev and ~0.1% at 300 Mev. Photomesons contribute ~0.7% at 300 Mev and ~0.2% at 1 Bev.

V. EXPERIMENTAL RESULTS

It is interesting to take the results in low-Z absorbers and obtain an "experimental" electron-field pair production cross section by subtracting the theoretical Compton, nuclear-field pair, and photonuclear cross sections from the experimental total cross section. The results shown in Table II are compared with the theoretical cross section of Wheeler and Lamb. In addition the ratio $Z\sigma_{\text{E.F.}}/\sigma_{\text{N.F.}}$ is calculated using the "experimental" E.F. cross section and the theoretical N.F. cross section. For hydrogen, the Wheeler-Lamb formulas predict 1.09 for this ratio at 1 Bev; for the other elements (using the Thomas-Fermi model) the Wheeler-Lamb calculation predicts ratios around 1.18 and the Joseph-Rohrlich calculation predicts ratios around 0.95 and 0.89 in hydrogen. Further evidence on the production of E.F. pairs has been obtained in a cloud chamber experiment by Hart, Cocconi, Cocconi, and Sellen, described in an accompanying paper.[16]

The theoretical calculations at 1 Bev for the heavier elements are tabulated in Table III. The ratios of the experimental to theoretical total cross sections are also shown.

To get an over-all view of the experimental and theoretical situation at high energies, the results of 7

---

[12] K. S. Suh and H. A. Bethe, this issue [Phys. Rev. **115**, 672 (1959)]. Discussions with Professor Bethe and Mr. Suh have been very helpful to me. I am grateful to them for showing me their theoretical calculations prior to publication.

[13] R. R. Wilson, Phys. Rev. **104**, 218 (1956).

[14] H. A. Bethe and F. de Hoffmann, *Mesons and Fields* (Row, Peterson and Company, Evanston, Illinois, 1955), Vol. 2.

[15] R. R. Wilson, Phys. Rev. **110**, 1212 (1958).

[16] Hart, Cocconi, Cocconi, and Sellen, preceding paper [Phys. Rev. **115**, 678 (1959)].

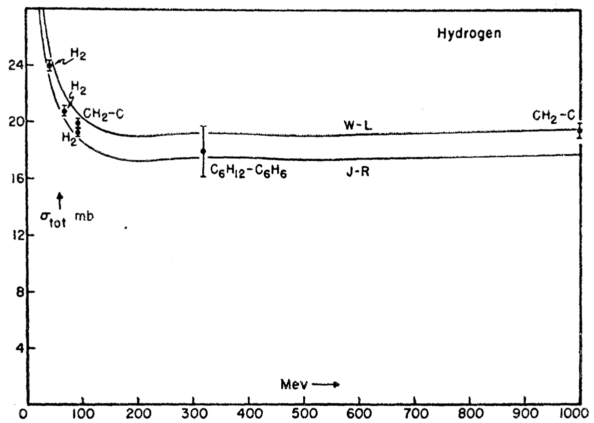


FIG. 3. Total absorption cross section in hydrogen plotted as a function of energy. The two theoretical curves shown were obtained using the two different calculations of electron-field pair production discussed in the text. The experimental results of various authors are shown and the absorbers used to obtain the hydrogen cross sections are noted. These results have not been corrected for the effect of molecular binding.

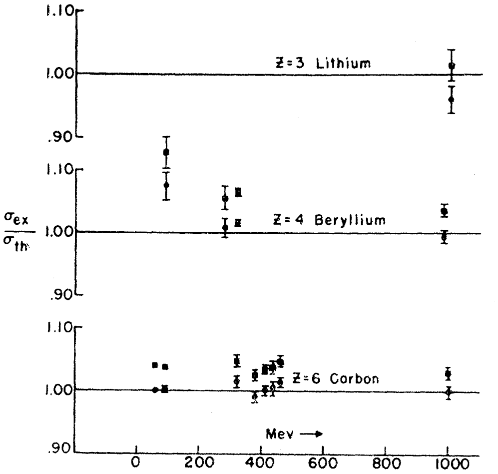


FIG. 4. Ratios of the experimental to theoretical total cross section plotted as a function of energy. These ratios have been computed using both theories of electron field pair production. The different experiments in which these data were obtained are listed in Table IV.

accurate attenuation experiments above 40 Mev[17–24] have been collected together and compared with theory. The experiments are listed in Table IV. There are 61 experimental results altogether. Results quoted by the authors in cm^2/g have been converted to barns and probable errors have been converted to standard errors. The theoretical calculations in the various papers are not used. Instead independent calculations were made following the procedure outlined above. The results are shown in Figs. 3, 4, 5, and 6. Figure 3 is a plot of total cross section in hydrogen as a function of energy. The various measurements are shown. In Figs. 4, 5, and 6 are plotted the ratio of experimental to theoretical cross section. Figure 4 shows the ratios for the light elements computed using both E.F. theories. Figures 5 and 6 contain the experiment/theory ratios for medium and heavy elements.

VI. ENERGY SHARING EXPERIMENT

The Bethe-Heitler theory predicts that the differential pair cross section, $d\sigma/df$, is symmetrical about $f=0.5$; f is the fraction of photon energy given to the positron. Small departures from symmetry can be caused by the breakdown of the Born approximation and by the Pauli exclusion principle in the electron-field case. The departures from symmetry will be significant only at low energy and very close to $f=0$ and $f=1$. The symmetry of the curve has been observed at lower energies.[25–29]

This symmetry was experimentally confirmed above 600 Mev by a short experiment using 3 counters placed in the pair spectrometer as shown in Fig. 7. One counter, X, has been added, and A and B have been moved slightly from the symmetrical position used in the total

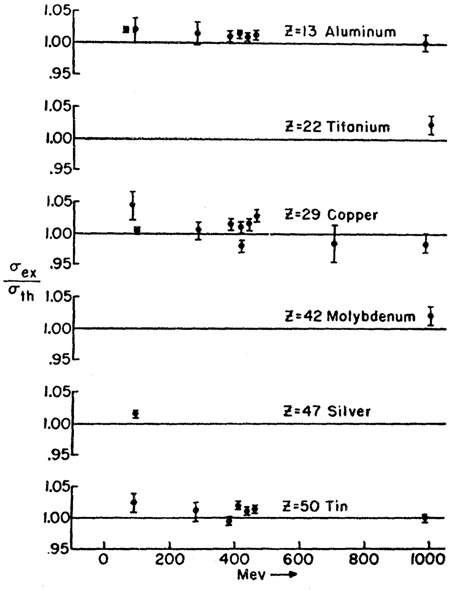


FIG. 5. Ratios of the experimental to theoretical cross section for medium-Z elements plotted as a function of energy.

---

[17] J. M. Wyckoff and H. W. Koch, Bull. Am. Phys. Soc. Ser. II, 3, 174 (1958).

[18] J. L. Lawson, Phys. Rev. **75**, 433 (1949).

[19] Moffatt, Thresher, Weeks, Wilson, Proc. Roy. Soc. London **A244**, 245 (1958).

[20] J. Moffatt and G. C. Weeks, Proc. Phys. Soc. London **73**, 114 (1959).

[21] DeWire, Ashkin, and Beach, Phys. Rev. **83**, 505 (1951).

[22] Anderson, Kenney, and McDonald, Phys. Rev. **102**, 1626 (1956).

[23] Anderson, Kenney, McDonald, and Post, Phys. Rev. **102**, 1632 (1956).

[24] D. H. Cooper, thesis, California Institute of Technology, 1955 (unpublished).

[25] W. B. Dayton, thesis, Cornell University, 1951 (unpublished).

[26] E. R. Gaertner and M. L. Yeater, Phys. Rev. **78**, 621 (1950).

[27] C. R. Emigh, Phys. Rev. **86**, 1028 (1952).

[28] J. W. DeWire and L. A. Beach, Phys. Rev. **83**, 476 (1951).

[29] Powell, Hartsough, and Hill, Phys. Rev. **81**, 213 (1951).

Table V. Results of energy-sharing experiment.

Photon energy (Mev)	Fractional positron energy (field normal)	Ratio (field normal/field reversed)		Ratio (Be result/Cu result)
		Be radiator	Cu radiator	
968	$f=0.44$	0.957 ± 0.025	0.971 ± 0.044	0.986 ± 0.053
662	$f=0.18$	0.997 ± 0.032	0.958 ± 0.059	1.041 ± 0.070

absorption measurements. The counters detect electrons of the following mean energies: A, 544 Mev; B, 424 Mev; X, 118 Mev. Therefore A and B look at pairs with $f=0.44$ and A and X look at pairs with $f=0.18$. By reversing the magnetic field in the pair spectrometer the counters then looked at pairs with $f=0.56$ and $f=0.82$. Bias curves, variation of coincidence counting rate with discriminator setting, were taken for the counters with both signs of magnetic field. In all cases a wide plateau was found at about the same discriminator settings indicating that the photomultiplier gain was unchanged by reversing the field. Setting the discriminator in the middle of this plateau insured that all the pairs were counted. Beryllium and copper radiators, producing different relative amounts of electron and nuclear field pairs, were used. Table V gives the results of this experiment. The ratio of the results for the two radiators, Be and Cu, will be independent of any change in counter gain with magnetic field. This ratio is consistent with 1, thus implying no variation in the symmetry of the curve with Z. The errors shown are statistical.

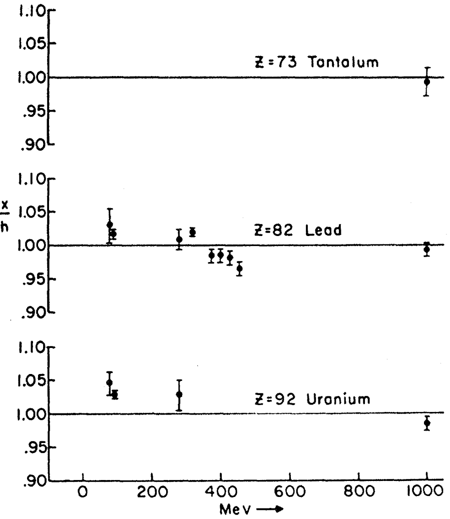


Fig. 6. Ratios of the experimental to theoretical total cross section for high-Z elements plotted as a function of energy.

VII. CONCLUSION

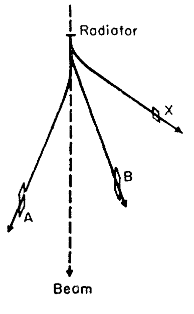
The over-all agreement between experiment and theory is strikingly good over the wide range considered: Z ranging from 1 to 92, energies from 40 Mev to 1 Bev, and total cross sections from 20 millibarns to almost 50 barns. The measurements in medium- and high-Z elements confirm the nuclear-field pair production theory of Davies, Bethe, and Maximon.

The measurements in beryllium and carbon clearly agree with the Wheeler-Lamb prediction and disagree with that of Joseph and Rohrlich. It is assumed in this discussion that the Klein-Nishina formula correctly predicts Compton scattering cross sections. A number of experiments have confirmed this.[18,21,27,30,31]

The measurements in hydrogen and lithium are not conclusive. Nuclear-field pair production in lithium has been calculated using the Thomas-Fermi model which clearly is not a valid procedure. Although hydrogen cross sections were calculated using the correct atomic wave functions, they are complicated by the effects of molecular binding. A calculation of molecular binding in H_2 has been made for high-energy bremsstrahlung (complete screening) by Bernstein and Panofsky.[32] If this correction is applied to the 1-Bev result (measured, however, in polyethylene) it will correct the experimental cross section to a value between the Joseph-Rohrlich and Wheeler-Lamb predictions. The low-energy experiments in hydrogen fall below the Wheeler-Lamb curve. For photon energies ~100 Mev, molecular binding effects are negligible and the measurements indicate that the Wheeler-Lamb calculation is not accurate at low photon energy.

The experiments in general are of sufficient accuracy to warrant more precise theoretical calculations for both nuclear-field and electron-field pairs. More accurate screening functions, based on the Thomas-Fermi model and on exact atomic wave functions in the lighter elements are necessary. The effect of molecular

Fig. 7. Sketch showing the position of the counters used in the energy-sharing experiment. When the magnetic field in the pair spectrometer is "normal," A "sees" electrons. When it is reversed, A "sees" positrons.



---

[30] F. H. Coensgen, University of California Radiation Laboratory Report UCRL-2413, November, 1953 (unpublished).

[31] Kurnosova, Razorionov, and Cherenkov, Zhur. Eksptl. i Theoret. Fiz. U.S.S.R. **30**, 690 (1956) [translation: Soviet Phys. JETP **3**, 546 (1956)].

[32] D. Bernstein and W. K. H. Panofsky, Phys. Rev. **102**, 522 (1956).

binding on screening at various energies is needed in order to evaluate measurements in hydrogen. Conduction electrons in metallic absorbers may also affect the screening. In considering measurements and calculations with accuracy $\lesssim 1\%$ it may be necessary to calculate the contribution of the second order diagrams in the perturbation theory. Until such calculations are made, no significance can be given to the slight deviations of experiment from theory seen in the graphs.

ACKNOWLEDGMENTS

I am grateful to Mr. J. Kirchgessner from whom I inherited the pair spectrometer, and to the operating crews of the Cornell synchrotron whose help was an essential part of this experiment. The late Mr. C. F. Van Amber gave me much assistance in obtaining the high purity absorbers. Above all, I want to thank Professor John DeWire and Professor Albert Silverman, who gave me help, advice, and encouragement throughout the course of the experiment.

Note added in proof.—The final results of Wyckoff and Koch, which Dr. Koch has kindly sent me before publication, seem to indicate a real discrepancy between the experimental and theoretical nuclear-field pair cross sections at energies below 100 Mev. If this discrepancy exists, Figs. 4, 5, and 6 show that it is independent of Z but depends on energy.

PHYSICAL REVIEW VOLUME 115, NUMBER 3 AUGUST 1, 1959

Radiative Muon Capture

JEREMY BERNSTEIN
The Institute for Advanced Study, Princeton, New Jersey

(Received March 6, 1959)

The theory of radiative muon capture is developed. The discussion includes both parity conserving and nonconserving effects. The Gell-Mann weak magnetic term and the induced pseudoscalar are included, along with comparable relativistic effects in the nucleons. The theory is applied to light nuclei and especially to the radiative Godfrey reaction $\mu^- + {}_6C^{12} \rightarrow \nu + \gamma + {}_5B^{12}$. An experiment to detect the induced pseudoscalar directly is proposed.

I. INTRODUCTION

THE theory of radiative K capture has a long history. The first computation was made, at the suggestion of Oppenheimer, by Morrison and Schiff in 1940.[1] They found that the photon spectrum to be expected in allowed electron K capture, neglecting all relativistic and screening effects, is of the form $(1-x)^2 x\,dx$, where x is the photon energy in units of the maximum photon energy. This formula obtains for both Fermi and Gamow-Teller transitions and allows no distinction between them. The first experiments exhibiting this spectrum were done by Bradt *et al.*[2] on the nucleus Fe^{55}. The theory for allowed transitions was refined by Jauch[3] and is most completely given by Glauber and Martin.[4] The latter authors consider relativistic, Coulombic, and screening corrections to the Morrison and Schiff computation and experiments by Lindquist and Wu[4] are in excellent agreement with their elaborate theoretical treatment. The net conclusion of this work is that allowed electron radiative K capture is well described by a four-fermion coupling with photon emission superimposed in the natural way.

The shape of the photon spectrum is, of course, independent of parity conservation or nonconservation in the electron capture event. However, it was realized[5] shortly after the discovery of parity nonconservation, that in a parity-nonconserving interaction the γ's coming from the inner bremsstrahlung of the electron undergoing capture could be circularly polarized, partially or completely. In fact on the two-component theory for V and A the circular polarization is 100% right independent of any details of nuclear matrix elements. Recently Mann *et al.*[6] have measured the circular polarization in K capture in Al^{37} and have obtained close agreement with this prediction of the $V-A$ two-component theory. In general, in a given transition, the degree of circular polarization is a measure of the relative strengths of the covariants involved in the four-fermion coupling. We shall return to this point below when we discuss the "induced" pseudoscalar in radiative muon K capture.

With the advent of intense muon beams[7] one may contemplate the study of radiative muon K capture experimentally. We shall see later that the anticipated

---

[1] P. Morrison and L. I. Schiff, Phys. Rev. **58**, 24 (1940).

[2] H. Bradt *et al.*, Helv. Phys. Acta **19**, 222 (1946).

[3] J. M. Jauch, Oak Ridge National Laboratory Report ORNL-1102, 1957 (unpublished).

[4] R. J. Glauber and P. C. Martin, J. phys. radium **16**, 573 (1955); Glauber, Martin, Lindquist, and Wu, Phys. Rev. **101**, 905 (1956); P. C. Martin and R. J. Glauber, Phys. Rev. **109**, 1307 (1958).

[5] R. E. Cutkosky, Phys. Rev. **107**, 330 (1957).

[6] L. G. Mann *et al.*, Phys. Rev. Letters **1**, 34 (1958).

[7] An intensity of 10^5 muons/cm^2 sec is now obtainable.