Production of Σ^+ Hyperons by 990-Mev Positive Pions in Liquid Hydrogen*

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The production and subsequent decay of six Σ^+ hyperons was observed in the central region of a hydrogen bubble chamber exposed to π^+ mesons of kinetic energy 990±30 Mev at the Cosmotron. The corresponding total production cross section, corrected for scanning inefficiencies, is $0.16_{-0.06}^{+0.08}$ mb. This value, combined with weighted averages of Σ^0 and Σ^- production cross sections in hydrogen obtained by Brookhaven, Columbia, and Berkeley groups, is used to compute the three triangular inequalities imposed by charge independence. The inequality

$$[\sigma(\Sigma^+)]^{\frac{1}{2}}+[\sigma(\Sigma^-)]^{\frac{1}{2}}-[2\sigma(\Sigma^0)]^{\frac{1}{2}}=0.12_{-0.10}^{+0.11}(\text{mb})^{\frac{1}{2}}\geqq 0$$

is the only one that might not be satisfied. The near-zero value of the left-hand side indicates the difficulty to be encountered in demonstrating a failure of charge independence at this energy.

I. INTRODUCTION

FOLLOWING the development of the "strangeness hypothesis,"[1,2] it was thought that total isotopic spin might be conserved along with strangeness in associated production processes. Recently, however, Sakurai[3] and the Michigan bubble chamber group[4] have called attention to the fact that the differential cross sections for Σ^+ hyperon production in propane by 1.1-Bev pions did not satisfy one of the triangular inequalities imposed by charge independence.[5] The inequality considered was (in obvious notation)

$$\left[\frac{d\sigma(\Sigma^+)}{d\Omega}\right]^{\frac{1}{2}}+\left[\frac{d\sigma(\Sigma^-)}{d\Omega}\right]^{\frac{1}{2}}-\left[2\frac{d\sigma(\Sigma^0)}{d\Omega}\right]^{\frac{1}{2}}\geqq 0.$$

The Michigan data, obtained in a propane bubble chamber, indicated that the neutral Σ's were produced preferentially backwards in the center-of-mass system. The charged Σ's, however, tended toward a vanishing cross section for the same angles. Since all the total cross sections were of comparable size, the left side of the inequality assumed a negative value for the backward direction.

The most serious uncertainty in the Michigan data, as the authors themselves noted, is that in a propane chamber without magnetic field it may be difficult to distinguish Σ^0 events produced in hydrogen collisions from Λ^0 events produced in carbon collisions. To a lesser degree the presence of carbon in the chamber may also have influenced the observed cross sections for the charged Σ's.

Since it was not clear to what extent the propane cross sections might be expected to differ from those of simple π-p collisions, it seemed desirable to repeat the Michigan experiments in liquid hydrogen. At present, at least three groups have collected data on Σ^0 and Σ^- production in hydrogen at 960 Mev.[6–8] We therefore selected an energy as close to this one as we could for an exploratory Σ^+ production experiment in a hydrogen chamber.

II. APPARATUS AND PROCEDURE

A. Beam Arrangement

The π^+ beam for this experiment was obtained from a one inch wide polyethylene target placed in the 3-Bev external proton beam of the Cosmotron (Fig. 1). Pions were collected by a strong-focusing magnet at an angle of 7° to the incident beam and were doubly momentum

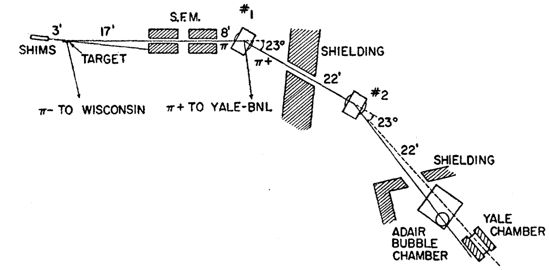

FIG. 1. Floor plan of beam arrangement used in this experiment. The same external beam was shared by four bubble chambers.

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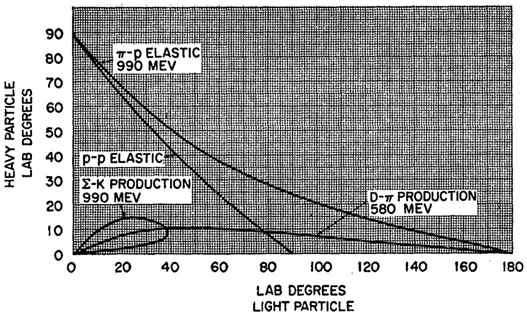

FIG. 2. Kinematics for observed coplanar reactions at 1.12-Bev/c incident momentum. Corresponding kinetic energies are shown.
Note: The caption for D-π Production should read 530 Mev.

analyzed before being focused at the bubble chamber. In contrast to previous 7° π^+ beams used at the Cosmotron,[9,10] our arrangement did not use tight collimation and thus allowed several experiments to operate simultaneously.

The hydrogen bubble chamber was constructed by Adair and Leipuner and has been discussed in a recent article.[6] Its inside dimensions were 6 in. along the beam×3 in. high×2 in. deep. No magnetic field was used.

Photography of the chamber employed dark field illumination and a stereo angle of approximately 15°. Measurements were made on the reprojected images by using drafting machines to determine distances and angles with respect to fiducial marks on the front chamber window. The data were processed with a digital computer, which reconstructed spatial coordinates, distances, and angles, and which also provided parameters for estimating coplanarity and the effect of measuring errors.

B. Data Analysis

Because of a large proton contamination in the π^+ beam, four coplanar processes were observed in the chamber:

$$p+p \rightarrow p+p, \tag{1}$$

$$p+p \rightarrow d+\pi^+, \tag{2}$$

$$\pi^+ + p \rightarrow \pi^+ + p, \tag{3}$$

$$\pi^+ + p \rightarrow \Sigma^+ + K^+. \tag{4}$$

In a sample of 1600 events each of these was unambiguously identified by the 1.12-Bev/c kinematics shown in Fig. 2, except for points of obvious overlap. Events in the vicinity of these points were separated statistically by drawing upon the available p-p data in this energy region.[11]

The p-p experiments have determined that the noncoplanar inelastic p-p events are 18% as abundant as p-p elastic events. Furthermore, an accurate correction can be made for the systematic scanning loss of small-angle scatterings. This information allowed the inelastic events to be assigned in the proper proportion to p-p or π^+-p collisions. Thus the total number of π^+-p events in the measured sample was established by direct classification of elastic events and a statistical division of the inelastic events.[12]

The ratio of protons to pions in the beam obtained with this procedure was 3.0±0.4. It differs considerably from the more favorable ones obtained in other experiments with this beam arrangement,[9,10] and must be attributed mainly to the absence of a tightly collimating beam channel. The correctness of this ratio is supported by the fact that it can be used to determine an absolute π^+-p elastic cross section in the forward direction that agrees well with the dispersion relations.

III. $\Sigma^+ - K^+$ PRODUCTION CROSS SECTION

Approximately 30 000 pictures of acceptable quality were scanned in both stereo views for all kinds of interactions. In a central volume of hydrogen, accessible through the thin "beam window" 2 in. high by $1\frac{1}{2}$ in. wide, the scanners located 5500 interactions including six definite production events in which the Σ^+ was observed to decay. An additional Σ^+ event was found outside the central region.

Data on the six production events are listed in Table I. The moderation times and observed decay times give a Σ^+ mean life of $(0.7_{-0.4}^{+0.6})\times 10^{-10}$ second, in agreement with the average value of $(0.83_{-0.05}^{+0.06})\times 10^{-10}$ second given by Barkas and Rosenfeld.[13] It was

TABLE I. Data on Σ^+-K^+ production and decay. θ_Σ is the center-of-mass production angle for the Σ^+. The laboratory angle between the Σ^+ and the charged decay product is small enough to be consistent with either $\Sigma^+ \rightarrow p+\pi^0$ or $\Sigma^+ \rightarrow n+\pi^+$ in every case but the first one listed. The last event in the table did not occur in the central volume of the chamber and was not used for cross-section calculations.

Event number	Cos θ_Σ c.m. system	Laboratory decay angle	Σ^+ mean life (seconds)	Moderation time (seconds)
9057–3 291	−0.01	79.0°	13.0×10^{-11}	41.2×10^{-11}
9057–2 83	0.34	16.1°	6.9×10^{-11}	25.5×10^{-11}
9047–17 329	−0.22	18.2°	5.0×10^{-11}	48.9×10^{-11}
9057–5 301	0.82	13.1°	2.5×10^{-11}	26.2×10^{-11}
9057–2 395	0.22	13.8°	4.3×10^{-11}	42.1×10^{-11}
9047–8 156	−0.54	11.4°	4.6×10^{-11}	39.4×10^{-11}
9127–2 122	−0.09	15.9°	6.1×10^{-11}	11.4×10^{-11}

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not possible to distinguish the Σ^+ decay modes with complete certainty.

The number of Σ^+ hyperons observed in the central volume was less than the number actually produced because of three effects. (1) Events in which the Σ^+ proceeds in nearly the same direction as the incident pion may be missed by the scanners. This loss can be estimated through a study of the d-π production [reaction (2)] which is similar kinematically and whose differential cross section is well known. (2) Σ's that decay too near the production vertex (<0.1 in.) cannot be positively identified. (3) Those Σ's which travel out of the chamber before decaying also cannot be positively identified. These three losses amount to 4%, 7%, and 14%, respectively. Thus the 6 Σ^+-K^+ productions observed imply $8.0_{-2.8}^{+3.7}$ expected events. Possible errors in the corrections are of little consequence compared to the statistical uncertainty.

The number of all other events found had to be increased by 6.7% to correct for scanning losses. From the resulting total number of expected events, the number of π^+-p interactions, 1175±129, was obtained using the proton-to-pion ratio determined above and the known[11] p-p and[9] π^+-p total cross sections (31.7 mb and 23.5 mb, respectively). Normalized to the total π^+-p cross section, our corrected Σ^+ production cross section is $0.16_{-0.06}^{+0.08}$ mb. The true cross section lies within the indicated limits with a $\frac{2}{3}$ probability.

IV. DISCUSSION

The angular distribution of only six Σ^+-K^+ production events is not very meaningful. Since the triangular inequalities apply at every angle, however, they must also apply to the total cross sections, and can be used in this way with a smaller amount of data. In order to compute the inequalities we make use of weighted averages for the Σ^0 and Σ^- production cross sections in liquid hydrogen obtained by other groups.[6–8]

$$\sigma(\Sigma^-)=0.21\pm0.03 \text{ mb (960 Mev)},$$
$$\sigma(\Sigma^0)=0.26\pm0.04 \text{ mb (960 Mev)},$$
$$\sigma(Z^+)=0.16_{-0.06}^{+0.08}, \text{ mb (990 Mev)}.$$

None of these are at the same center-of-mass momentum of the Σ. Therefore, before substitution in the inequalities the Σ^- and Σ^+ cross sections have been corrected to the center-of-mass momentum of the Σ^0 cross section by arbitrarily assuming a linear momentum dependence. Any error introduced in this way must be quite small compared to experimental uncertainties.

Two of the inequalities imposed by charge independence are definitely satisfied. These are

$$[\sigma(\Sigma^+)]^{\frac{1}{2}}+[2\sigma(\Sigma^0)]^{\frac{1}{2}}-[\sigma(\Sigma^-)]^{\frac{1}{2}} =0.61_{-0.10}^{+0.11} \text{ mb}^{\frac{1}{2}}\geqq 0, \quad (1)$$

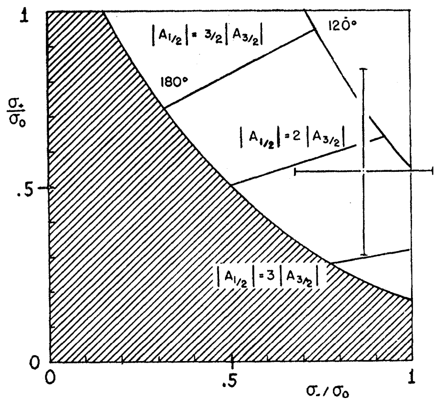

FIG. 3. $\sigma(\Sigma^+)/\sigma(\Sigma^0)$ *vs* $\sigma(\Sigma^-)/\sigma(\Sigma^0)$. For the simple case where the $T=\frac{1}{2}$ and the $T=\frac{3}{2}$ production amplitudes are related by $A_{\frac{1}{2}}=Ne^{i\varphi}A_{\frac{3}{2}}$, conservation of isotopic spin in production excludes points from the shaded region. The relative magnitude, N, and phase angle, φ, associated with cross-section ratios are mapped in the unshaded area.

$$[\sigma(\Sigma^-)]^{\frac{1}{2}}+[2\sigma(\Sigma^0)]^{\frac{1}{2}}-[\sigma(\Sigma^+)]^{\frac{1}{2}} =0.83_{-0.10}^{+0.11} \text{ mb}^{\frac{1}{2}}\geqq 0. \quad (2)$$

The only inequality which shows any indication that isotopic spin might not be conserved is

$$[\sigma(\Sigma^+)]^{\frac{1}{2}}+[\sigma(\Sigma^-)]^{\frac{1}{2}}-[2\sigma(\Sigma^0)]^{\frac{1}{2}} =0.12_{-0.10}^{+0.11} \text{ mb}^{\frac{1}{2}}\geqq 0, \quad (3)$$

since the left-hand side has a 13% chance of being negative. Although this result is inconclusive, it does indicate the difficulty of obtaining a definitive answer about violation of charge independence. For example, to insure that the left-hand side of (3) is significantly greater than zero, one would prefer an error of ±0.06 or less. This requires relative total cross sections with an accuracy of around 10%. Since there appears to be little evidence for large anisotropy of production at this energy,[7] the angular distributions may not provide a substantially more sensitive test than the total cross sections.

Figure 3 is a plot used by Adair and Leipuner[6] to obtain the relative phase and magnitude of the $T=\frac{3}{2}$ and $T=\frac{1}{2}$ production amplitudes when the total cross sections are due predominantly to a single angular momentum state. In such a case one can write the $T=\frac{1}{2}$ amplitude as a complex constant times the $T=\frac{3}{2}$ amplitude and use this fact to calculate values of $\sigma(\Sigma^+)/\sigma(\Sigma^0)$ and $\sigma(\Sigma^-)/\sigma(\Sigma^0)$ assuming charge independence. Allowed values of these ratios are mapped in the unshaded region of Fig. 3. The point shown uses the same data as the inequality (3).

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Recoil Momentum Distribution in Electron Pair Production*

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Electron pair production by a very energetic photon in the field of a particle of arbitrary mass (in particular in the fields of an electron and a nucleus) is studied following the work of Borsellino. The distribution of recoil momenta q is calculated for q of order of the electron mass and it is shown that the recoil distribution is independent of the mass of the recoil particle if appropriate variables are used. It is also explicitly shown that the mass of the recoil particle does not make any difference in the recoil distribution for very small q (of order $q_{\min}$). The total cross section must therefore be independent of the mass of the recoil particle in the high-energy limit, as previously stated by Borsellino. The Wheeler-Lamb result for pair production in the field of a bound electron is also justified. The results also describe the electromagnetic production of any fermion pair if certain restrictions are satisfied.

I

THE theory of electron pair production by a photon in the field of an electron has been studied in most detail by Votruba[1] and Borsellino,[2,3] by using Dirac's positron theory in Born approximation. Feynman diagrams of the process are given in Fig. 1 and four more diagrams, which are obtained by exchanging the two electrons in the final state, must be added. The processes corresponding to diagrams (*c*), (*d*) and their exchange diagrams are referred to as $\gamma-e$ interactions.[4] Votruba's calculation is complete in that it involves all possible processes. His final expression, however, is so long and complicated that it is difficult to handle; consequently, in order to carry out a general analytic integration, approximations which may introduce errors are required. In particular, Votruba finds that the distribution of recoil momenta q over the region q of order unity[5] is difficult to obtain and, therefore, in evaluating the total cross section, he does not include the contribution from this region correctly.

Borsellino developed his theory for a particle of arbitrary mass M in whose field the electron pair is produced. Consequently he neglected the $\gamma-e$ interaction and exchange terms in his calculation. The errors[6] caused by this procedure are presumably negligible at high photon energies because the probability of large momentum transfer, where the effect of $\gamma-e$ interaction and exchange is important, is negligibly small. Borsellino's calculation should therefore be nearly correct at high incident photon energies, except for the unimportant case when the recoil momentum is of the same order as that of the incident photon.

In this paper, the recoil distribution function for high incident photon energies is obtained from the previous calculation of Borsellino in a simple and tractable form. The recoil distribution function for electron pair production in the field of an electron is

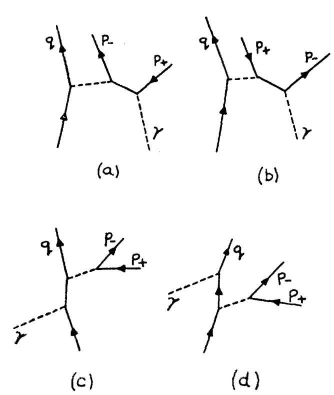

Fig. 1. Feynman diagrams for electron pair production by a photon in the field of an electron.

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