exceed a factor 2 to 3 on either side, if one restricts one-self to interactions with more than 6 to 8 charged mesons.

The distribution and the average values of the number of charged mesons produced by high-energy α particles are given. Meson production by α particles rises proportional to $E^{0.40\pm0.08}$ between 10 Bev/nucleon and 40 Bev/nucleon. At 40 Bev/nucleon, on the average 8.2 charged mesons are produced per collision. In about 10% of all cases the α particle continues on after the collision.

The average number of mesons produced by heavy nuclei increases first proportional to $A^{\frac{2}{3}}$ and more slowly for $A>16$. The average number of charged mesons produced per collision is 16.1 for incident M nuclei, 17.2 for H nuclei, and 23.2 for VH nuclei at an average energy of 20 Bev/nucleon.

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Energy Spectrum of the Heavy Nuclei in the Cosmic Radiation between 7- and 100-Bev/Nucleon*

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The energy spectrum of the heavy nuclei of the cosmic radiation was determined between 7 Bev/nucleon and 100 Bev/nucleon. The distribution for the $M(6\leq Z\leq 9)$ and $H(Z\geq 10)$ charge groups agree with one another within the limits of error. Combining both charge groups, the integral spectrum is of the form $N(>E)\sim E^{-1.6\pm0.15}$ (E=total energy/nucleon). Flux values for nuclei of the M, H, and VH $(Z\geq 20)$ charge groups at the geomagnetic equator are given. Comparing these results with flux values obtained at high latitudes, it is concluded that a power spectrum of the form $E^{-1.6}$ fits all three charge groups within the limits of error between 2.5 Bev/nucleon and 7 Bev/nucleon. From the observation of α-particle showers of very high energy we conclude that under certain assumptions the integral spectrum of α particles can be represented by $N(>E)\sim E^{n}$ with $n=-1.58_{-0.18}^{+0.21}$ for energies ≤ 1500 Bev/nucleon.

INTRODUCTION

TEN years ago the energy spectrum of the cosmic radiation was known only at the high-energy end above 10^{13} ev from measurements of extensive air showers, where a power law of the form $N(>E)=CE^{-\gamma}$ could be well established. The value of the exponent was found to be $\gamma=1.75$. As soon as rocket flights permitted measurements of the total intensity above the top of the atmosphere, in 1947, one could show by varying the geomagnetic latitude that for singly charged particles a similar power law holds, the exponent being close to 1.[1–4] Due to the albedo effect of the earth the total intensities of singly charged particles, measured in those experiments, were not the intensities of the primary proton component. Attempts have been made to derive the proton spectrum by estimating and subtracting the albedo effect.[5] After the discovery of heavy nuclei in the primary cosmic radiation the meaning of total primary intensities as a function of latitude got more complicated. Kaplon, Peters, *et al.*[6] deduced an energy spectrum of the heavy-nuclei component not only by varying the geomagnetic latitude, but by observing and evaluating fragmentations of heavy nuclei in nuclear emulsions. Their method has been discussed in a previous paper (II).[7] They could prove that the number of heavy nuclei $(Z\geq 10)$ between 3 and 30 Bev/nucleon is well represented by a power-law spectrum with an exponent $\gamma=1.35\pm0.15$. Since 1952 a number of measurements of the heavy-nuclei flux have been carried out at different latitudes near the top of the atmosphere by using the method of nuclear emulsion and by greatly improving the techniques of charge determination.[6,8–19]

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In this way one can deduce energy spectra of heavy nuclei up to 7 Bev/nucleon. In summarizing their own data and those of other workers, the Bristol group came to the conclusion that between 1.8 and 3.0 Bev/nucleon the exponent is 1.5 ± 0.18.[20] Recently Singer re-evaluated all available data on the energy spectrum of heavy nuclei.[21] He tried to deduce spectra for individual charge groups, such as M(C, N, O, F) and $H(Z\geq 10)$, and put forward the hypothesis of a possible increase of γ with Z. Such a behavior of the heavy-nuclei component, if confirmed by better statistics and extended to higher energies, would have far-reaching consequences upon our basic ideas of the origin of the cosmic radiation.

Recently counter techniques have been greatly improved by making use of a combination of Čerenkov and scintillation counters.[22] This made it possible to obtain precise measurements of α-particle fluxes near the top of the atmosphere at different geomagnetic latitudes. By summarizing his own measurements between $\lambda=41°$ N (Texas) and $\lambda=0°$ (Guam), MacDonald[23] got an exponent of 1.5 for α particles. With the same detector he also tried to measure the primary proton spectrum. After corrections for the albedo effect he came to the conclusion that his proton data are in agreement with a power law spectrum having the same exponent of $\gamma=1.5$. Together with recent data on α particles and heavy nuclei obtained by Fowler and Waddington[20] and by Waddington[19] for the same energy range, this would indicate that protons, α particles, and heavy nuclei $(Z\geq 6)$ have a spectrum with the same value of γ rather than one in which γ increases with Z. Until now little has been known concerning the accurate energy spectrum of the primary light nuclei (Li, Be, B) which, of course, would be of considerable interest for the theory.

Reviewing this situation it seemed desirable to obtain more precise flux values for the different charge groups of heavy nuclei near the geomagnetic equator by increasing the statistics, and to extend the measurements of Kaplon, Peters, *et al.*[6] to still higher energies.

FLUX OF HEAVY NUCLEI AT $\lambda=0°$ (GUAM)

The details of the emulsion experiment of this paper are the same as those described in a previous paper (I).[24]

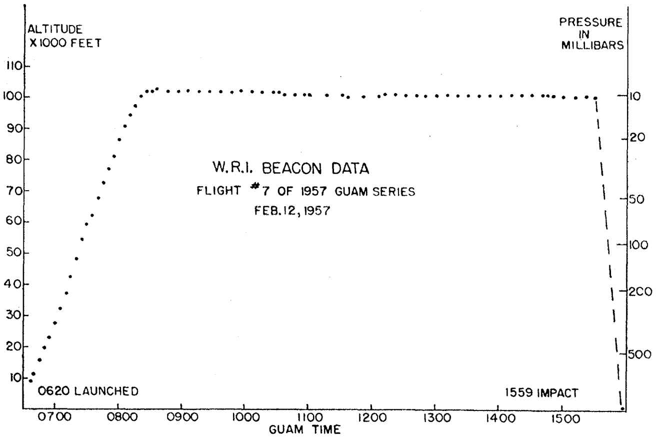

FIG. 1. Flight curve of stack.

The flight curve of the balloon is shown in Fig. 1. For the calculation of flux values we used only those tracks which entered the stack through the top surface having a projected zenith angle $\leq 45°$ in the plane of the emulsion and a dip angle $\leq 11°$ (≥ 3 mm per plate). Thirty-nine tracks of H nuclei and 98 tracks of M nuclei were accepted under these criteria. The "effective" thickness of the matter above our scanning line was 13.7 g/cm^2 taking into account the residual atmosphere of 10 g/cm^2, the material of the gondola, and the solid angle accepted under the above-mentioned criteria. In paper I it was shown that the interaction mean free paths and the fragmentation probabilities do not depend on the energy of the heavy nuclei in the energy range under consideration in this paper. Therefore, in evaluating the diffusion equations[25,26] for the heavy nuclei based on our data and results obtained by other groups discussed in I we used the following sets of constants:

Interaction mean free paths (in air):

M nuclei 27 g/cm^2,

H nuclei 20 g/cm^2,

VH nuclei 16 g/cm^2.

Fragmentation probabilities P_{ik} (in air):

$$P_{M,M}=0.18,$$
$$P_{H,M}=0.35,$$
$$P_{H,H}=0.31,$$
$$P_{VH,VH}=0.13.$$

The fragmentation probabilities are calculated by averaging over the results, obtained by various authors, for l events (see I) in emulsion. Our set of P_{ik} values is nearly the same as the one reported by the Bristol group.[27] The differences are well within statistical errors,

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which is also true for the corresponding fragmentation probabilities obtained by Noon and Kaplon.[13,26] (The discrepancies for P_{HL} and P_{ML} do not have to be discussed here.)

The solution of the diffusion equation gives the following flux values for the top of the atmosphere near the geomagnetic equator:

M nuclei ($6\leq Z\leq 9$): $I_M{}^0=(0.89\pm0.15)$ particles/m^2 sec sterad,

H nuclei ($Z\geq 10$): $I_H{}^0=(0.33\pm0.07)$ particles/m^2 sec sterad,

VH nuclei ($Z\geq 20$): $I_{VH}{}^0=(0.10\pm0.03)$ particles/m^2 sec sterad,

These results are corrected for the time of ascent of the balloon.

The errors quoted in these results are not statistical errors only but include also estimated errors resulting from scanning efficiency, interaction mean free paths, fragmentation probabilities, flight duration, and a small altitude change of the balloon.

For the flux of the VH nuclei the acceptance criterion must be changed in order to get statistically significant values.

In Table I our flux values at 0° geomagnetic latitude are compared with those obtained by Waddington[19] and Webber[17] at the same geomagnetic latitude but in a different flight. All three values agree within the limits of errors. The flux values obtained by Danielson and others[16] using oriented stacks of nuclear emulsions and observing the zenith angle dependence of heavy nuclei in a balloon flight at $\lambda=10°$N (Galapagos Islands) are lower both for M and H nuclei. Singer collected all available data on flux values for heavy nuclei published until 1957. It seems to us very difficult to deduce a

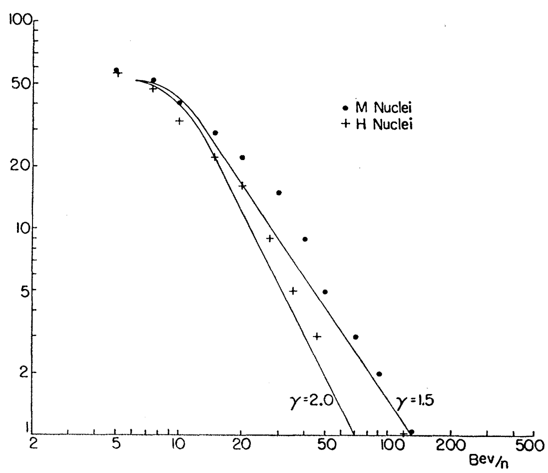

FIG. 2. Integral energy spectrum of heavy nuclei determined from scattering measurements. The full lines are calculated for an assumed power-law spectrum with an exponent 2.0 and 1.5. Ordinate: number of events. Abscissa: energy in Bev/nucleon.

TABLE I. Flux values at the top of the atmosphere for heavy nuclei near the geomagnetic equator (Guam) (particles/m^2 sec sterad).

	Atmospheric depth	M ($6\leq Z\leq 9$)	H ($Z\geq 10$)	VH ($Z\geq 20$)
This work	10 g/cm^2	0.89±0.15	0.33±0.07	0.10±0.03
Waddington[a]	17	0.95±0.11	0.35±0.08	
Webber[b]	17	1.02±0.26	0.52±0.17	
Average		0.93±0.08	0.36±0.06	

[a] See reference 19.
[b] See reference 17.

power law spectrum in this way because the meaning of the errors quoted by various investigators is sometimes not obvious. Furthermore, one has to consider that balloon techniques as well as methods for charge determination improved considerably during the last ten years. Therefore, we used only the equatorial flux values mentioned above and very recent data obtained under similar conditions in Texas ($\lambda=41°$N)[28,29] and northern Italy ($\lambda=46°$N)[20,30] which are of comparable statistical accuracy. The cutoff energy for both locations seems to be about 1.5 Bev/nucleon.[17,31] All these data are in good agreement with the assumption that within the present limits of measurements a power law $N(>E)=CE^{-\gamma}$ with the same exponent $\gamma=1.6\pm0.15$ holds for both charge groups between 1.5 and 7 Bev/nucleon. For C we obtain, from the average flux values quoted in Table I,

$$C_{\mathrm{CNOF}}=26.0\pm2.2 \text{ particles/m}^2 \text{ sterad sec},$$

$$C_{Z\geq 10}=11.9\pm2.0 \text{ particles/m}^2 \text{ sterad sec}.$$

Even the very heavy nuclei (VH) seem to have a spectrum with the same value of γ. Our results at the equator (cutoff energy 7 Bev/nucleon) yield a ratio

$$N_{VH}/N_H=0.30\pm0.07,$$

to be compared with results obtainable from data for northern Italy (cutoff energy 1.5 Bev/nucleon) by the Turin[32] group, which give $N_{VH}/N_H=0.29\pm0.06$.

ENERGY SPECTRUM OF HEAVY NUCLEI BETWEEN 7 Bev NUCLEON AND 100 Bev/NUCLEON

Heavy nuclei used for this work were first located and then followed the same way as described in I. As explained in II, the energy of the particles was determined by three methods; (a) from track-to-track scattering measurements between α particles or heavier fragments originating from fragmentations of heavy nuclei, (b) from the opening angle of fragments, (c) from the angular distribution of the shower particles produced in high-energy interactions. The experimental procedures were described in II. In particular, it was shown that

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all three methods yield consistent results. Fluctuations around the true value of energy as determined by method (c), however, were shown to be very large. Therefore, we mainly used methods (a) and (b) for determining the energy spectrum. Our results were grouped and evaluated depending on the method [(a), (b), or (c)] used and depending on the charge of the various nuclei (*M* or *H*).

As was explained in part II, method (a) or (b) can be used only if the heavy nucleus breaks up into two or more fragments of charge ≥ 2. A total of 540 heavy nuclei were located and followed; 339 of these interacted in the stack. From the 339 interactions, 115 fragmentations yielded two or more fragments. The energy spectrum, as determined by using methods (a) and (b) on these 115 events, will then represent the true spectrum only if the following two conditions are satisfied:

(1) The interaction mean free path does not depend on energy.

(2) The probability of an interaction to be a fragmentation yielding two or more fragments does not depend on energy.

In I we have compared our results obtained at a cutoff energy of 7 Bev/nucleon with the results of other investigators at higher latitudes (cutoff energy about 1.5 Bev/nucleon). The average energy is different by a factor of about 3. As was pointed out in I, we found good agreement within experimental errors for both the mean free paths and for the fragmentation probabilities when comparing our results with those at lower energies. We conclude, therefore, that these quantities are independent of energy within the statistical accuracy of this experiment. Hence conditions (1) and (2) are satisfied.

The energy spectra for the *M* and *H* nuclei as determined by scattering measurements and by the opening angle of the fragments are given in Fig. 2 and Fig. 3. Due to the finite resolution of individual energy measurements, the energy spectrum as determined from our events will not represent the true energy spectrum, but will deviate from it. The deviation increases as the average error in the individual energy measurements increases. This has to be taken into account for determining the true shape of the spectrum. We have calculated the spectrum to be expected under different assumptions regarding the shape of the true spectrum and depending on the energy resolution of the measurements. For the true integral energy spectrum we chose $N(>E)=CE^{-\gamma}$ (E= total energy/nucleon) with a sharp geomagnetic cutoff at a total energy of 8 Bev/nucleon. Two values of γ were tried: $\gamma=1.5$ and $\gamma=2$. This spectrum was corrected by the energy resolution of the measurements. For the scattering measurements we used a normal distribution with an rms error of 30%. The distribution for the opening angle measurements was taken from the experimentally measured distributions which were shown in II (Figs. 1, 2, and 3). The same procedure was used for determining values by the angular distribution of the shower particles. We used the curve drawn to fit all the events in Fig. 9 of II. As was described in II, this distribution holds for α-particle interactions found in an unbiased way. We have furthermore shown in II that energy measurements by using the angular distribution of shower particles give the same results for α particles and for heavy nuclei. Because the heavy primary interactions which were used for the energy spectrum were also selected in an unbiased way, we can assume that the energy spread is the same as the one found for the α-particle interactions. In each figure we have given the two spectra calculated for $\gamma=1.5$ and for $\gamma=2$ for comparison. These curves bend at the lower end of the spectrum; for high energies they give asymptotically the expected slope. In this region they should be parallel to the true spectrum, but somewhat displaced towards higher energies. The shift increases as the width of the energy resolution increases. One method of finding the true value of γ from the experimentally measured points consists, therefore, in disregarding the low-energy events and measuring the slope from the high-energy events only.

It is seen, that for both methods (Figs. 2 and 3) the points for the *M* nuclei lie mostly above the points for the *H* nuclei. This probably comes from the higher precision of energy measurements possible for the *H* nuclei, because they frequently break up into more than two fragments, which increases the number of independent measurements which can be performed and thus the statistical accuracy. As demonstrated above, a better energy resolution would shift the distribution towards lower energies. In Figs. 2 and 3 the calculated curves bend somewhat more sharply than the experimental curves at the low-energy end. This we attribute to the fact that the theoretical curves were calculated for a sharp cutoff at 8 Bev/nucleon (total energy). However, particles having smaller energies can reach the stack, for example, if they come from the west under higher

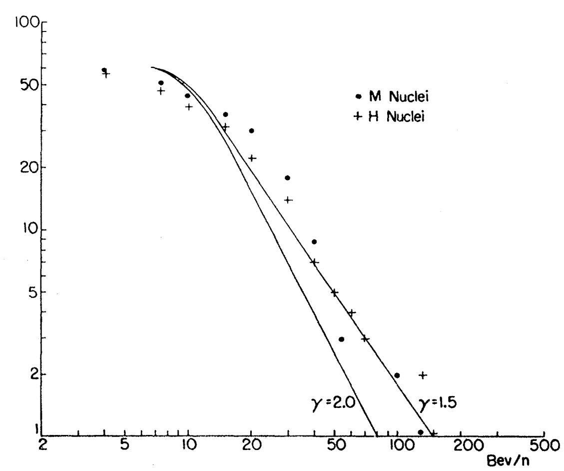

FIG. 3. Integral energy spectrum of heavy nuclei determined from the opening angle of fragments. Ordinate: number of events. Abscissa: energy in Bev/nucleon.

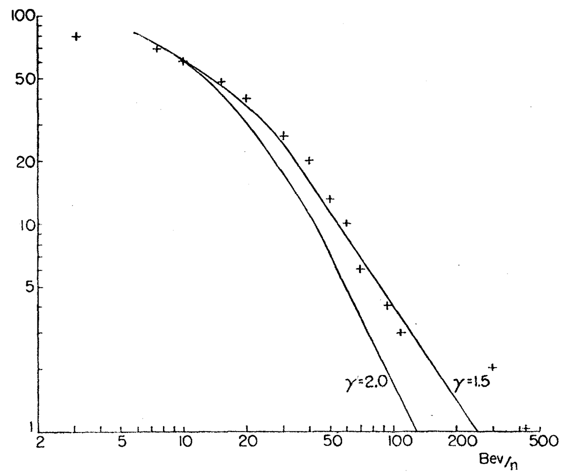

FIG. 4. Integral energy spectrum of heavy nuclei determined from the angular distribution of shower particles. The full lines are calculated values expected for a power law spectrum with exponent 2.0 and 1.5. Ordinate: number of events. Abscissa: energy in Bev/nucleon.

zenith angles. This explains the tail at the low-energy end of the spectrum. Hence our data are compatible with an integral energy spectrum of the form $N(>E)=CE^{-\gamma}$.

In order to find the true value of γ from our measurements, we have used interpolation between the two calculated theoretical curves as well as the slope of our experimental distribution for the high energies only. The values for the exponent found in this way are given in Table II. Within the limits of error the values of γ agree for the different methods and also for the two charge groups. Thus within the indicated limits of error the energy spectrum for both M and H nuclei is the same. Combining both charge groups, we obtain

$$\gamma=1.60\pm0.15$$

between 7 Bev/nucleon and 100 Bev/nucleon. These findings are in agreement with the data shown in Fig. 4, where we have plotted the energy as given by the angular distribution of the shower particles of 79 additional events which have fewer than two fragments. Apart from this they were chosen in a statistically unbiased way. The theoretical curve calculated for $\gamma=1.5$ fits the data, whereas $\gamma=2$ is excluded.

Our results for energies >8 Bev/nucleon can be compared with some measurements published by other authors using methods similar to those used here. The Turin group[30] gave a value $\gamma=1.54$ for both M and H nuclei together, for average energies of about 6.5 Bev/nucleon. This is lower by a factor of 3 compared with our data. Unfortunately, it is not clear from their paper how much their results might be influenced by the finite resolution of their individual energy measurements. Hence our results can perhaps be compared more satisfactorily with the work by Kaplon, Peters, *et al.*[6] For the H-nuclei group they give a value of $\gamma=1.35\pm0.15$ between 3 Bev/nucleon and 20 Bev/nucleon. This is quite compatible with our result.

TABLE II. Exponent γ of the energy spectrum between 8 and 100 Bev/nucleon.

Heavy nuclei charge group	Energy determined by scattering	by opening angle	Average
M	1.52	1.62	1.57 ± 0.20
H	1.79	1.42	1.62 ± 0.20
$M+H$			1.60 ± 0.15

ENERGY SPECTRUM OF α PARTICLES AT AN ENERGY $E\geq1500$ Bev/NUCLEON

For locating events at energies very much exceeding those discussed in section 3 one largely depends, at the present time, on electron cascades originating in extremely high-energy nuclear collisions. From the number of events found in a given amount of material, one may try to get some indication regarding the flux of heavy nuclei at energies >1000 Bev/nucleon. The most important assumptions involved are the following: (1) The collision mean free path does not change with energy. (2) The detection probability is known. (3) The primary energy of the nuclear events is known.

From the evidence presented in I and from general arguments, assumption (1) is probably reasonably satisfied. It also becomes less critical in a stack the linear dimensions of which are comparable to the nuclear mean free path. Little can be said as yet about assumption (2). Because events of lower energy can more easily be overlooked (small cascade), one should restrict the method to events of very high energies, above $E>10^{12}$ ev/nucleon only. It is possible that even at energies >1000 Bev/nucleon a fraction of the interactions can be overlooked. This will, of course, depend on the stack and the scanning methods used.

The customary method to estimate the energy of these collisions is by means of the angular distribution of the shower particles. This method was evaluated in II. As was pointed out, for individual events, large fluctuations are to be expected. Using only collisions without "heavy" prongs ($N_H=0$), or trying to correct for the effect of secondary interactions in the same nucleus, will in general yield an overestimate of the energy (if one uses the "spectrum-independent" approximation of II). The average energy of a number of interactions to be determined will also be overestimated because of the steepness of the energy spectrum and the width of the energy resolution. This effect has been estimated as described in Sec. 3. Assuming that the width of the energy resolution does not change with energy, it can be deduced from Fig. 4 that the measured energies are shifted towards higher values by a factor of 1.6.

Assuming the energy spectrum to obey a power law, and using the flux value at 1.55 Bev, the Bristol group[20] derived a value of $\gamma=1.49_{-0.20}^{+0.23}$ using 6 α-particle jets observed at $E \geq 800$ Bev/nucleon. Carrying through a similar investigation we limited ourselves to α-particle jets having energies $E \geq 3000$ Bev/nucleon. In view of the discussion given above and in II we, however, think that the true lower limit of the energy is lowered by a factor of about 2 and hence is more like 1500 Bev/nucleon. From the 5 observed events, the geometry of our stack, and the exposure time of the stack we find a flux value of 3.6×10^{-3} α particles/m^2 sterad sec at $E \geq 1500$ Bev/nucleon. Assuming a detection efficiency of 100%, this yields a value of the exponent $\gamma = 1.58_{-0.18}^{+0.21}$. The limits of error include an uncertainty by a factor of 2, of both flux and energy. This value of γ is in satisfactory agreement with the exponent derived for heavy nuclei of energies up to 100 Bev/nucleon and for α particles up to 7 Bev. There is no evidence for a substantial change in γ.

ACKNOWLEDGMENTS

We wish to thank Professor Marcel Schein for the possibility of carrying out this investigation and for his continuous interest and encouragement. We are greatly indebted to D. M. Haskin for his cooperation and to our scanning team for locating the events and for carrying out some of the calculations. We thank the Turin group for sending us some of their results prior to publication.

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Intersecting-Beam Systems with Storage Rings

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The equivalence of fixed- and variable-field particle acceleration systems for the adiabatic damping of synchrotron oscillations is pointed out. These two quite different acceleration methods are therefore able to produce particle beams of equal density for beam stacking purposes.

The transfer mechanism between an accelerator and a storage ring is discussed, and the properties of a fast-rise 3-kilogauss beam-switching magnet are shown. It is concluded that the source interaction rate obtainable in a proton storage ring system would be nearly independent of the focusing properties and repetition rate of the injecting accelerator.

An improved design for intersecting-beam storage rings is described, in which several well-separated interaction regions could be used for simultaneous experiments. Standard types of alternating-gradient magnets would be required, and the over-all weight of synchrotron plus storage rings would be about one-tenth as large as that of comparable beam-stacking accelerators.

I. INTRODUCTION

INTERSECTING-BEAM devices have come to be recognized generally as a necessary next step in accelerator design, mainly as a result of the continuing efforts of the Midwestern Universities Research Association (MURA).[1] Within the last few years a number of designs for achieving high center-of-mass energies have been suggested.[2–5] To obtain adequate interaction rates, it is necessary to accumulate ("stack") many pulses of accelerated particles near the maximum energy containable in the guide field. It has been shown that for proton accelerators one can only perform beam stacking by filling the limited amount of available synchrotron and betatron oscillation phase space.[6–8]

There have been two approaches to intersecting-beam accelerator design. That of the MURA group has been through fixed-field alternating-gradient (FFAG) accelerators[9] which are able to contain, within a single vacuum chamber, all energies from a few Mev up to the stacking energy. Although intersecting-beam FFAG synchrotrons would be large and would require complicated magnetic fields, they would accomplish their purpose within a single magnetic guide field region. Models have proven the design principles to be sound.

We have studied the alternative possibility of trans-

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