Heavy Nuclei and α Particles between 7 and 100 Bev/Nucleon. II. Fragmentations and Meson Production*

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Three hundred and seventeen heavy-nuclei and 175 α-particle interactions of more than 7 Bev/nucleon have been found in a systematic way and analyzed. We obtained average values and fluctuations of the individual values for the energy of heavy nuclei as measured by the opening angle of their fragments by comparing with the energy found by scattering measurements. At an average energy of 10 Bev/nucleon and 40 Bev/nucleon the average number of charged mesons produced by α-particle interactions with emulsion nuclei was found to be 4.6 and 8.2, respectively. About one half of the nucleons of the α particle participate in the collision. The meson multiplicities per participating nucleon agree for proton and α-particle collisions at 10 Bev/nucleon.

We measured the angular distribution of the shower particles emitted from the interactions. We calculated the energy of the interactions from the angular distribution by using the median angle formula or equivalent formulas, based on the "meson spectrum independent" approximation. These methods overestimate the true primary energy on the average by a factor of 2 (for interactions with less than 5 heavily ionizing prongs). This is due to the relatively large proportion of slow mesons in the c.m. system. The distribution of primary energy as obtained by this method around the true value is shown for two groups having average energies of 10 Bev/nucleon and 40 Bev/nucleon. Equivalent results are given for the heavy-nuclei interactions.

The average number of mesons produced in collisions between heavy nuclei and emulsion nuclei at an average energy of 20 Bev/nucleon is given. It increases with the charge of the incident nucleus in agreement with a crude geometrical model.

1. INTRODUCTION

During the past years a considerable amount of work has been done on the nuclear interactions of the heavy primaries of the cosmic radiation.[1–11] Relatively little, however, is known at energies above 10 Bev/nucleon. At these energies multiple production of mesons becomes an important phenomenon. We have, therefore, carried out a study of heavy-nuclei and α-particle interactions in a stack of nuclear emulsions exposed near the geomagnetic equator. It covers an energy range between 7 Bev/nucleon and about 100 Bev/nucleon. A description of the emulsion stack, the flight, and the experimental procedure used for this work was given in the preceding paper, hereafter denoted by I. The investigations of this paper are based on 99 interactions of nuclei of the H-group ($Z\geq 10$), 218 nuclei of the M-group ($6\leq Z\leq 9$), and 175 α-particle interactions found in a systematic way. We shall first discuss methods of energy measurements on these particles. The second part deals with meson production proper, particularly its dependence on energy and atomic number, and the angular distribution of the mesons.

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2. ENERGY MEASUREMENTS ON FRAGMENTATIONS

An important advantage of this investigation is the possibility of measuring the energies of the particles up to more than 100 Bev/nucleon. We used three independent methods of measuring the energy: (a) measurement of the multiple scattering; (b) measurement of the angles between the fragments of heavy-nuclei fragmentations; (c) measurement of the median angle of meson showers produced by heavy nuclei or α-particles from the fragmentations.

Direct measurement of multiple scattering on our tracks (total energy>8 Bev/nucleon) is impossible in most cases because of spurious scattering. Therefore, track-to-track (relative) scattering measurements have to be applied. This is possible for all cases where the heavy nucleus breaks up into two or more α particles or heavier fragments. Kaplon, Peters, *et al.*[1] have pointed out that one can treat a fragmentation process as an evaporation process in the center-of-mass system of the incoming heavy nucleus. Therefore, in the laboratory system we assume that all the fragments must have the same energy per nucleon as the incident nucleus; thus, a measurement of their relative multiple scattering yields the correct value of the energy of the primary nucleus. Actually, the finite evaporation energy of the fragments in the center-of-mass system introduces a small spread of the energy in the laboratory system around the energy of the primary nucleus. The influence of this effect on energy measurements above 7 Bev/nucleon was calculated and can be neglected since it would introduce a correction of about 1%.

Even if one carries out track-to-track scattering measurements, the influence of spurious scattering and of emulsion distortions cannot be disregarded completely. Therefore, we limited our measurements to tracks having a separation of not more than 40 μ in any direction. The noise was eliminated by making two sets of measurements on each track with a displacement of 100 μ. A scattering constant $k=32$ Mev deg/$(100\ \mu)^{\frac{1}{2}}$ was used.[12]

As has been pointed out,[8] it is not possible to make measurements on all tracks with this method if the cutoff energy of the heavy nuclei is not fairly high. At low energies the angles between the fragments can be rather large and only a small track length is available for measurement before the separation between tracks becomes too large. This would produce a bias favoring the selection of high-energy events. In our case, however, this difficulty does not exist because at our cutoff energy of 7 Bev/nucleon, the angles between fragments are much smaller on the average than those observed for heavy nuclei entering at high latitudes. It was possible to make scattering measurements in the case of 96% of all multiple fragmentations. Part of the remaining 4% which could not be measured were due to fragments interacting in the emulsion or leaving the stack after a short distance. This, of course, does not depend on the energy of the event. Therefore, in our analysis there is no bias favoring events with small opening angles between the fragments.

The second method of energy measurement makes use of the angular distribution of the fragments. For energies above 7 Bev/nucleon and for α-particle fragments the basic relation as given first by Kaplon, Peters, *et al.*[1] can be written in the form

$$\langle\theta_\alpha{}^2\rangle^{\frac{1}{2}}=K/E, \tag{1}$$

where

$$K=(\langle T_\alpha\rangle M/3)^{\frac{1}{2}}. \tag{2}$$

Here, E is the total energy per nucleon of the incident heavy nucleus; M, the proton mass; $\langle T_\alpha\rangle$, the average kinetic energy of evaporation of the α particles in the rest system of the incident heavy nucleus; and $\langle\theta_\alpha{}^2\rangle^{\frac{1}{2}}$, the root-mean-square angle between the α particles and the line of flight of the primary nucleus.

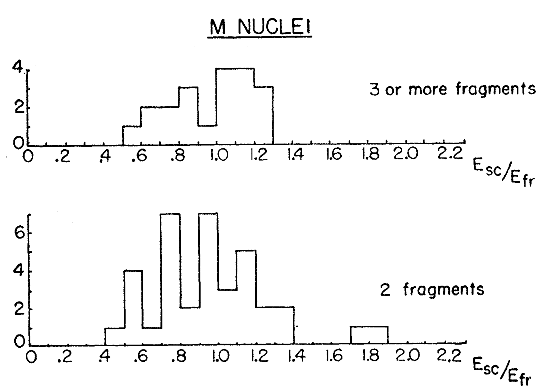

FIG. 1. Comparison between energy E_{sc} obtained by scattering measurements and energy E_{fr} obtained from the opening angle of fragments (M nuclei). Ordinate: number of events.

[12] C. Fichtel and M. W. Friedlander (to be published).

By this method we determined the energy of all our nuclei undergoing interactions with more than one fragment. The procedure of finding $\langle\theta_\alpha{}^2\rangle^{\frac{1}{2}}$ was the same as that described by Kaplon, Peters, *et al.*[1]

In order to evaluate E from Eq. (1), the value of K must be known. Since we have measured the energy of all our fragmentations by track-to-track scattering as well as by the opening-angle method, K can be determined directly. The energy values found by scattering and opening-angle measurements shall be denoted by E_{sc} and E_{fr}, respectively. The error of E_{sc} is 30% on the average. Neglecting this as compared to the relatively large fluctuations of E_{fr} around the true energy, K can then easily be found by assuming E_{sc} to be the true energy. Another method of finding K is by means of Eq. (2). In order to be able to compare the results of both methods, we calculated the average value of K from our measurements according to

$$\bar{K}=\langle K^2\rangle^{\frac{1}{2}}=\langle\langle\theta_\alpha{}^2\rangle E_{sc}{}^2\rangle^{\frac{1}{2}} \tag{3}$$

because [see Eq. (2)] the experimentally known quantity, $\langle T_\alpha\rangle$, is proportional to K^2. It has been pointed out previously[7,8] that K might be a function of both the

TABLE I. Values of K for different groups of fragmentations.

Charge group	Number of fragments	K	Av. K	K calculated from Eq. (2)
$M(6\leq Z\leq 9)$	2	0.054	0.052	0.056
	>2	0.046		
$H(Z\geq 10)$	2	0.051	0.055	0.066
	>2	0.060		

charge of the incoming nucleus and the number of fragments. We have, therefore, calculated K separately for 4 groups of events, namely events initiated by M and H nuclei and events with only two and with more than two fragments, respectively. The results are shown in Table I. In Table I the values of K have to be compared with those calculated from Eq. (2) and with the results of other authors. Kaplon, Peters, *et al.*[1] used a value of $K=0.056$ for nuclei of the H group. This corresponds to a value of $\langle T_\alpha\rangle=10$ Mev derived from results of Perkins[13] on the energies of evaporation α particles in cosmic-ray stars. Kaplon's[1] direct measurements on the fragmentations were in agreement with this value and it also agrees well with our results as given in Table I. It is, however, expected that the evaporation energy of α particles depends on the charge of the incident nucleus. The Turin group[14] has shown this by measuring directly the average energy of α particles evaporating from collisions between heavy primaries and emulsion nuclei. $\langle T_\alpha\rangle$ was found to depend on the charge of the target nucleus and is 10.0 Mev for the C, N, O group and 21.6 Mev for the Ag, Br group. In order to compare these

[13] D. H. Perkins, Phil. Mag. 41, 138 (1950).
[14] Garelli, Quassiati, and Vigone, Nuovo cimento 8, 731 (1958).

values with our results, we took the value of 10 Mev for our M nuclei; for our H nuclei we assumed an average energy of the α particles of 14 Mev. We arrived at this value of 14 Mev by interpolating between the two energy values quoted for the C, N, O, and the Ag, Br nuclei ($\bar{Z}\approx 41$), using the average charge $\bar{Z}\approx 14$ for the H group. Our actual factor K is thus smaller than the one given by the Turin group for the H nuclei. The comparison between our experimental values of K and those calculated from the evaporation energy and from Eq. (2) is given in Table I. Our experimental values seem to be slightly smaller than the values derived from Eq. (2). There are two possible reasons for this small discrepancy of about 10%. It is possible that the scattering measurements were slightly affected by distortions and spurious scattering. Second, we notice in the table that K seems to depend on the number of fragments emitted. A more detailed study of the evaporation process would be necessary in order to obtain a more accurate comparison

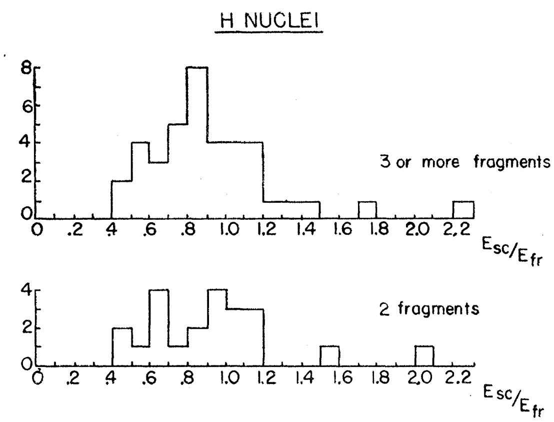

FIG. 2. Comparison between energy E_{sc} obtained by scattering measurements and energy E_{fr} obtained from the opening angle of fragments (H nuclei). Ordinate: number of events.

with our values. Within an accuracy of about 10% there is, however, a satisfactory agreement between the various measurements of K. If one disregards the dependence of K on the number of fragments, all results are within the indicated limits of error compatible with the following relations:

$$E=(0.06\pm 0.006)/\langle\theta_\alpha^2\rangle^{\frac{1}{2}} \text{ for } H \text{ nuclei,} \tag{4}$$

$$E=(0.055\pm 0.006)/\langle\theta_\alpha^2\rangle^{\frac{1}{2}} \text{ for } M \text{ nuclei.} \tag{5}$$

No dependence of K on the energy was found.

Equation (1) being true only in the limiting case of very many α particles emitted, it is expected to give rise to large fluctuations for an individual event having only a few fragments. This is shown in Figs. 1, 2, and 3 where we have plotted the distribution of E_{sc}/E_{fr} for the four groups mentioned above and combined for all our events. The distributions are asymmetric, as is to be expected from the mechanism of the evaporation

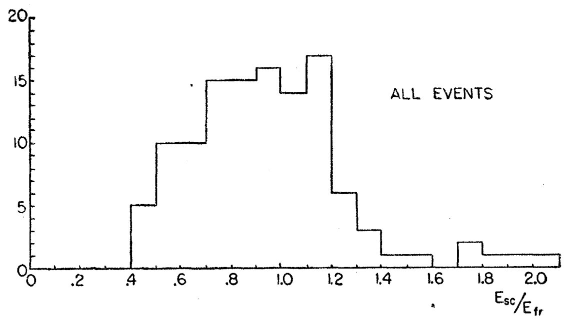

FIG. 3. Comparison between E_{sc} and E_{fr} for all events combined ($M+H$ nuclei).

process and as was already shown by Kaplon, Peters, *et al.*[1] They have also pointed out that no values of E_{fr} smaller than about two times the true energy should occur. Our experimental distribution agrees with this.

There have been reported somewhat higher values of K than given here by the Bristol[7] and Turin[8] groups. They were, however, derived in a more indirect way by averaging the angles of all the fragments without measuring the energy of each individual event and assuming a certain primary energy spectrum. In such a case it seems that great care must be taken when averaging over the distribution of various quantities, as has also been emphasized by the Turin group.[14] This point will be discussed further in our paper (Part III), on the energy spectrum. Apart from this, there is a qualitative agreement between our results and those of the Bristol group regarding the dependence of K on the charge of the incoming nucleus and on the number of fragments. This indicates that for more accurate work, the dependence of Eqs. (1) and (2) on the number of fragments should be taken into account.

The third independent method which measures the energy of heavy nuclei uses the angular distributions of the shower particles produced in high-energy collisions. This will be discussed in Secs. 4 and 7.

3. ANGULAR DISTRIBUTION OF SHOWER PARTICLES

As already described in I, we found 175 interactions of α particles originating from heavy nuclei fragmentations by following the α particles along the track. In 157 cases the energy of the α particle could be measured by the methods described in Sec. 2 because it was emitted together with other fragments. These 157 interactions were divided into two groups of approximately equal statistical weight according to their energy. The average energy of the groups was 10 Bev/nucleon and 40 Bev/nucleon, respectively. The general characteristics of the stars and the values of the cross sections were given in I.

The angular distribution of the shower particles was measured for all events. Moreover, for each individual event we have plotted $\log[f/(1-f)]$ *vs* $\log\tan\theta$, where

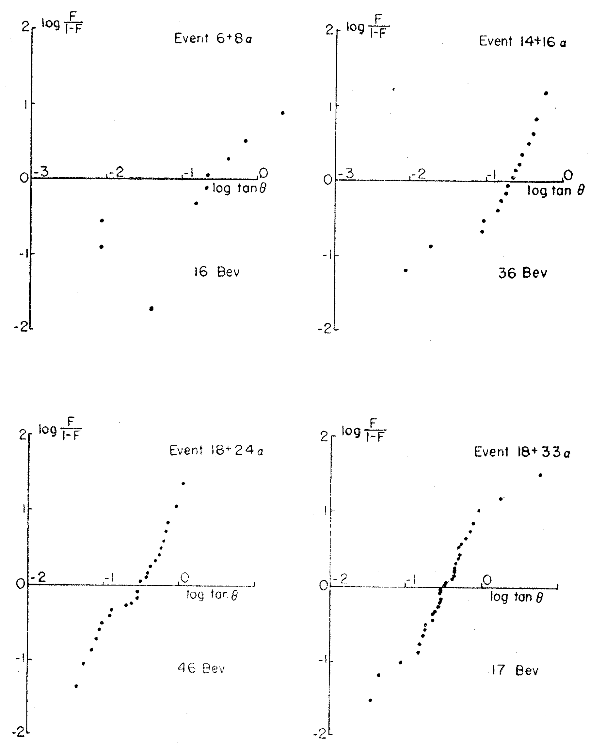

FIG. 4. Angular distribution of some α-particle interactions. The values given in Bev are the primary energies per nucleon.

f is the fraction of all particles having an angle $<\theta$ in the laboratory system. This method, first introduced by Duller and Walker,[15] has recently been used in connection with models of multiple meson production.[16–19] Assuming that the interactions can be analyzed in terms of one or more independent nucleon-nucleon collisions and neglecting the influence of secondary interactions in the same nucleus, it has been shown[18,19] that diagrams of this sort can give some information about the angular distribution of the shower particles in the center-of-mass (c.m.) system. An isotropic distribution in the c.m. system, for example, gives a straight line of slope 2 in this diagram. In Figs. 4 and 5 we show a few characteristic examples, including also 4 events without heavy prongs, which possibly may satisfy the assumptions mentioned above. However, the occurrence of secondary interactions can be seen in the diagrams in some cases, for example, by the presence of an isolated group of particles at large angles. In most of our events the points are grouped approximately along a straight line, the slope of which is about 2 or slightly less. This indicates that in the center-of-mass system the angular distribution of the mesons deviates only a little from isotropy, slightly favoring the forward and backward direction (The average energy of our α-particle events is 20 Bev/nucleon.) Apart from these remarks which depend on the assumptions about the collision process as mentioned above, there is a distinct and well separated group of particles to be seen at small angles in most of the events. These particles can also be seen in the ordinary angular distribution for all of our events combined as shown in Fig. 6. Here a sharp peak occurs at angles $<2°$. The Bombay group,[20] in a similar investigation of α-particle interactions, have shown that these particles are probably protons originally belonging to the α particle which continue without appreciable deflection and without strongly participating in the collisions. We have determined the number of these particles by various methods —for example, from Figs. 4, 5, and 6, by counting the number of tracks in the separated groups mentioned and applying a correction for meson background. This correction turns out to be small. We find that the average number of protons continuing without appreciable deflection after the collision of the α particle is $n_p=0.93$. This figure agrees well with the one quoted by Appa Rao *et al.*[20] which is $n_p=0.82$. They, however, have left 6 events of the sort $0+2\alpha$ out of the analysis. They considered these to be Coulomb interactions. In view of the fact that the cross section for this process is not well known, we preferred not to make any allowance for

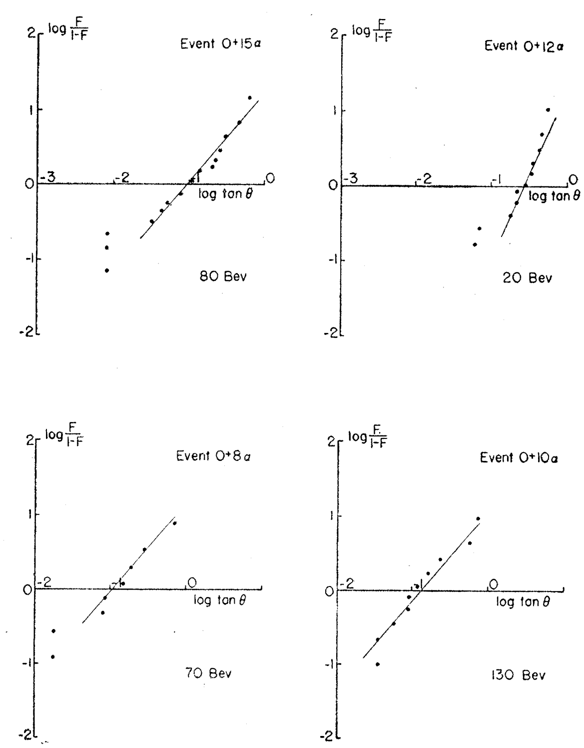

FIG. 5. Angular distribution of some α-particle interactions without heavy prongs.

[15] N. M. Duller and W. D. Walker, Phys. Rev. **93**, 215 (1954).
[16] S. Takagi, Progr. Theoret. Phys. (Kyoto) **7**, 123 (1952).
[17] W. L. Kraushaar and L. J. Marks, Phys. Rev. **93**, 326 (1954).
[18] G. Cocconi, Phys. Rev. **111**, 1699 (1958).
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[20] Appa Rao, Daniel, and Neelakantan, Proc. Indian Acad. **43A**, 181 (1956).

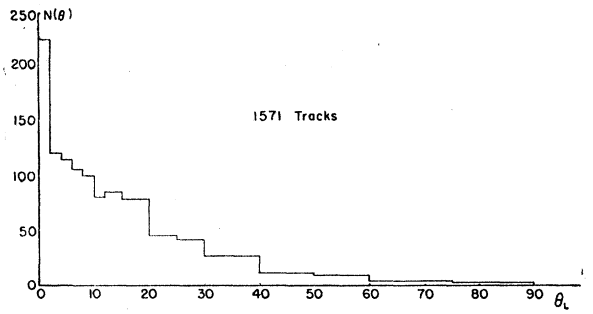

FIG. 6. Angular distribution of all shower particles from α-particle interactions in the laboratory system. $N(\theta)$ is the number of tracks with angle θ_L in a 2° interval. 26 tracks with $\theta_L>90°$ are not plotted.

Coulomb interactions. As can easily be seen, this has a very small effect on our conclusions. Adding the $0+2\alpha$ events, the Bombay value[20] would be $n_p=0.88$. This is in good agreement with ours. Knowing the number of protons we can estimate the number of nucleons which on the average take part in a collision of a high-energy α particle. From charge symmetry it appears reasonable to assume that the average number of protons and neutrons not participating in a collision is equal. The total number of nucleons not taking part in a collision is, therefore, $2n_p$. Another contribution to this number comes from events in which the α particle as a whole unit continues after the collision. The most reasonable assumption for this kind of event is the stripping of a neutron from the α particle. We have 9.7% events of this kind. Assuming that in most cases the particle going on is a He^3 nucleus, the average number of nucleons not taking part in a collision is $2n_p+0.097\times 3=2.15$. The average number of nucleons participating in the collision between an α particle and a nucleus of the emulsion is, therefore, 1.85. Treating the Bombay data[20] in the same way, we get an average of 2.1 nucleons which is, again, in good agreement.

The fragmentation of an α particle into one or more protons is especially striking in many events of small multiplicity. Knowing the α-particle energy and the deflection of the protons, one is able to calculate their transverse momentum. We determined the transverse momentum for 28 cases having low multiplicity and two closely collimated minimum tracks continuing on, which we considered to be protons. The distribution of the transverse momentum P_t is plotted in Fig. 7. There seems to be a distinct group of particles with transverse momentum $P_t<400$ Mev/c in addition to a tail extending to very high momenta $P_t>2000$ Mev/c. This tail is most probably due to mesons, the energy of which is much smaller than that of the protons and which, therefore, give much too high values for the transverse momentum if treated as protons. Assuming the group with $P_t<400$ Mev/c to be mostly protons, their average transverse momentum is 200 ± 30 Mev/c. If the angular distribution of the protons in the rest system of the α particle is isotropic, the average momentum is 250 ± 50 Mev/c. This corresponds to a kinetic energy $E_t=33\pm 10$ Mev. If the proton distribution is anisotropic, we get a lower limit of 200 ± 30 Mev/c for the average momentum and 21 ± 7 Mev for the kinetic energy.

If one tries to use the angle of fragmentation of the protons for an energy determination of the α particles in the same way as in the case of heavier fragments, the width of the distribution together with the tail at high momenta will introduce considerable uncertainties and fluctuations in the energy determination. As a consequence, the energy determination will be quite unreliable.

Of course, part of the tracks will be due to one of the heavier H isotopes (D or T nuclei). In this case P_t means the transverse momentum/nucleon.

4. THE "MEDIAN ANGLE" METHOD

As was already mentioned in Sec. 2, a widely used method to determine the energy of high-energy particles is to deduce it from the angular distribution of the mesons produced in the nuclear collisions.

The method depends on the following assumptions:

(1) The collision can be described as one or more independent collisions between the individual nucleons of the colliding nuclei.

(2) Secondary interactions of the nucleons and mesons within the target nucleus can be neglected.

(3) In the center-of-mass system of the collision, the velocity of all particles is the same, namely, equal to the velocity of the c.m. system.

The primary energy of the colliding particle can be estimated in various ways:

(a) Median angle method:

$$\gamma_c=\cot\theta_{\frac{1}{2}}, \tag{6}$$

where $\theta_{\frac{1}{2}}$ is the angle enclosing half of all the shower particles in the laboratory system, γ_c is the energy of the colliding particle in the c.m. system in units of its rest energy.

(b) The method given by Castagnoli *et al.*[21]:

$$\log\gamma_c=\langle\log\cot\theta\rangle, \tag{7}$$

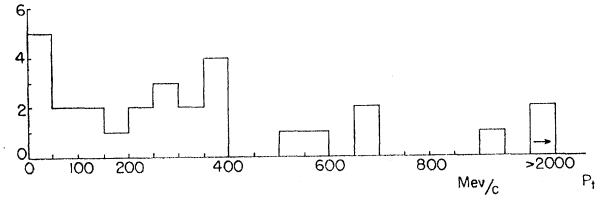

FIG. 7. Distribution of tranvserse momentum P_t/nucleon for protons, deuterons, and tritons from α-particle fragmentations. Ordinate: number of events per 50 Mev/c.

[21] Castagnoli, Cortini, Franzinetti, Manfredini, and Moreno, Nuovo cimento **10**, 1539 (1953).

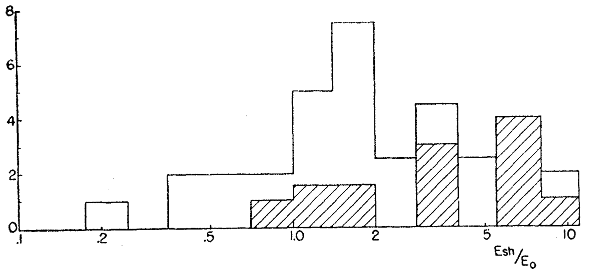

FIG. 8. Comparison of the true primary energy of α-particle interactions E_0 with the energy E_{sh} obtained from the angular distribution of the shower particles. (Average primary energy 10 Bev/nucleon.) Events with $n_H \leq 4$ are crosshatched. Ordinate: number of events.

where θ is the angle of a shower particle in the laboratory system.

(c) The graphical method given first by Duller and Walker[15] consisting in plotting $\log[f/(1-f)]$ *vs* $\log\tan\theta$ and finding the intercept for $f=0.5$. (The meaning of f was explained in Sec. 3.) The primary energy per nucleon E in the laboratory system is then found according to

$$E = Mc^2(2\gamma_c^2 - 1), \tag{8}$$

where M is the nucleon rest mass.

It is well known that assumptions (2) and (3) made above are generally not satisfied. Nevertheless methods (a), (b), and (c) have been widely used because in most cases they are the only ones available at very high energies. In our case, however, we know within about 25% the energies of our α particles by track-to-track scattering and by the fragmentation angles, as discussed in Sec. 2. Hence we were able to check the validity of methods (a), (b), and (c) directly. To every single α-particle interaction we have applied each of the three methods (a), (b), and (c) directly. In all cases the results agreed reasonably with one another. The best value of the energy was found by taking the average of methods (b) and (c), which is better than using method (a) alone because (b) and (c) make use of the actual values of all angles of the shower particles. There is an additional difficulty due to the fact that only mesons, among the shower particles, should be considered in these methods.

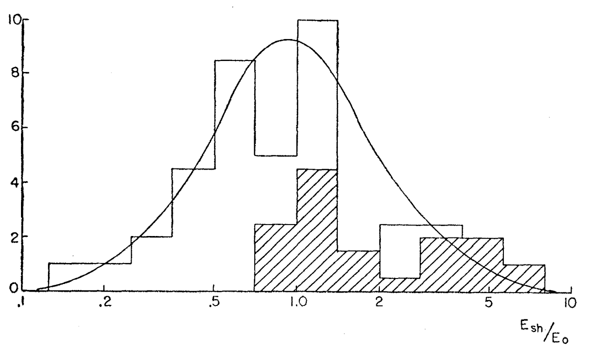

FIG. 9. Same as Fig. 8. Average primary energy 40 Bev/nucleon.

Because the average multiplicity of our showers is not too high, the nucleons among the shower particles do influence the results appreciably, in particular since there is a group of nucleons present having very small angles and since methods (b) and (c) give large statistical weight to these tracks. As was pointed out before, this group of nucleons (0.93 per star on the average) can be recognized and hence left off. This will decrease the primary energy of the α particle as determined by method (b) and (c) on the average by a factor of 1.65. We have left off one more track in each event, namely the one having the next smallest angle apart from the group of noninteracting nucleons. This results in leaving off roughly two tracks per interaction having the smallest angles which can be considered to correspond to the two original protons in the α particle which are expected to continue after the collision with a relatively small deflection only, due to their high initial energy. This will decrease the energy of the stars on the average by a factor of 2.1 compared to the procedure in which one is using all shower particles. The energy value of each α particle thus obtained by methods (b) and (c) was compared with the energy actually measured by scattering, or the opening angle of the fragments. In Figs. 8 and 9 we have plotted the distribution of the ratio E_{sh}/E_0; E_{sh} is the energy determined from the angular distribution of the mesons according to methods (b) and (c) and E_0 is the actual energy of the event measured from scattering and the angles of the fragments. Again we divided the showers into the two energy groups of average energies of 10 Bev/nucleon (Fig. 8) and 40 Bev/nucleon (Fig. 9), respectively. We found the method to give extremely big fluctuations for showers having 7 or fewer shower particles. We have, therefore, restricted ourselves to showers with $n_s \geq 8$ shower particles (Figs. 8 and 9, based on 77 events altogether). Even for these events very large fluctuations occur. One sees that the energy as determined from the angular distribution of the mesons can be overestimated, as well as underestimated by a factor up to 10 in the measured energy range. About 50–60% of all events agree with the true energy within a factor of 2. That these large fluctuations are not due to some undetected systematic error in the measurement of the true energy can be seen from the fact that we have observed the same fluctuations in events where several α particles coming from the same fragmentation interacted.

In order to look further into the nature of these fluctuations, we selected those events having four or fewer black or gray prongs. These events can be regarded to be less contaminated by secondary interactions inside the same nucleus than the rest of the events. They are shown crosshatched in Figs. 8 and 9. As expected, the tail at the low energies E_{sh} is almost entirely due to events in which secondary interactions play an important role. The diagrams also show that by restricting ourselves to events with fewer than 5 black or gray prongs, assumption (2) made at the beginning of this analysis seems much better satisfied. Also, the fluctuations are reduced by about a factor of 4. There are no events in which the energy was underestimated by more than a factor of 1.5.

However, especially for this class of events, the energy can be overestimated greatly (up to a factor 8 to 10). In addition, the energy of these events, on the average, is also overestimated. Thus to give a better estimate of the energy, Eq. (7) has to be modified. As we selected events with 8 or more shower particles, there is the possibility that for the events in the low-energy group (Fig. 8) there is a systematic effect due to this selection because the average multiplicity in this group is smaller than 8. We shall restrict ourselves, therefore, to the high-energy group (Fig. 9) which has a higher average multiplicity. Equation (7) is now to be modified in such a way that it gives the correct result for the energy if it is applied to a great number of minimum tracks originating in events having all the same primary energy. This implies

$$\langle\log(E_{\mathrm{sh}}/E_0)\rangle=0. \tag{9}$$

Using the hatched distribution, Fig. 9, one obtains, therefore, the following modification of Eq. (7):

$$\log(1.4\gamma_c)=\langle\log\,\mathrm{cotan}\theta\rangle. \tag{10}$$

This means that energies are, on the average, overestimated by a factor of about 2 by using Eq. (7). Because the results of all three methods (a), (b), and (c) are in good agreement with each other this means that, in general, the energy of most events will be overestimated by using any of the three methods involving the angular distribution of the shower particles and selecting events which are little influenced by secondary interactions. By using the modified Eq. (10), the energy is overestimated in only 6 out of 14 cases. Eighty percent of all events agree with the energy values given by Eq. (10) within a factor of 2.2 on either side. The energy of events which are strongly affected by secondary interactions will generally be underestimated. Considering the average of all events, this effect seems about to compensate the modification introduced by Eq. (10) as shown in Fig. 9. This, of course, increases the width of the distribution greatly, as does every sort of cascade mechanism.

The fact that the energies by the normally used methods (a), (b), and (c) come out too high must be caused by a violation of assumption (1) or (3) made above. Most probably, assumption (1) can be ruled out. That the nuclei within the α particle do not greatly influence one another during the collision process can, for example, be seen by the fact that half of the nucleons continue unaffected after the collision. In particular, we have some collisions where the α particle as a He isotope continues after producing a shower, the shower having been produced most probably by the stripping of a single neutron. Also, in these cases, the energy is mostly overestimated by using methods involving the angular distribution. According to our present knowledge, it seems quite difficult to consider a model which would give an overestimate in energy which could occur in the case of a coherent action of all the nucleons of the α particle or the heavy nucleus. In particular, it has been suggested[22] that a high-energy collision might be looked at as the collision of the incident nucleus with a whole column of nuclei in the target nucleus. This would have the effect of giving primary energies which are too low according to Eqs. (6) through (8). Clearly this is in contradiction with our measurements, at least for the events having less than 5 black prongs. We are, therefore, left with assumption (3) to explain the discrepancy. This assumption is certainly violated for protons and other heavy particles among the shower particles. As was explained above, we made an effort at least to leave off some of the protons in our analysis. We thus do not think that the discrepancy is due to the remaining protons among the shower particles, since the average multiplicity of our showers was quite high (>8). Therefore, most of the particles must be π mesons. We have, therefore, to assume that the velocity of the mesons in the center-of-mass system is not always equal to the velocity of the center-of-mass system itself. This explanation has also been put forward previously in connection with similar observations[23,24] for proton interactions at lower energy. This work will be discussed at the end of this section.

Several direct measurements on the energy distribution of π mesons in the center-of-mass systems for meson showers at primary energies between 30 and 30 000 Bev have been carried out in recent years.[25–34] They all show that an appreciable part of the mesons have quite low energies in the center-of-mass system which is, for example, compatible with Heisenberg's theory[35] of multiple meson production. The influence of the energy spectrum of the mesons in the center-of-mass system on the angular distribution in the laboratory system was

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TABLE II. Interactions of protons and α particles with energies >6 Bev/nucleon.[a]

Protons		α particles					
$E=6.2$ Bev Bombay[b]	$E=9$ Bev Dubna[c]	$\langle E\rangle=10$ Bev Bombay[b]	$\langle E\rangle=10$ Bev This work	$\langle E\rangle=40$ Bev This work	$\langle E\rangle=20$ Bev This work	$\langle E\rangle=3000$ Bev Chicago[d] Bristol[e]	
99	120	170	81	76	175	14	No. of events
6.4	7.8	7.7	7.5	7.2	7.2	7.5	$\langle n_H\rangle$
2.6	3.2	8.0	7.3	10.8	9.0	$\lesssim 60$	$\langle n_s\rangle$
1.6	2.2	5.3	4.6	8.2	6.3	$\lesssim 60$	$\langle n_m\rangle$
1.6	2.2	2.5	2.5	4.4	3.4	$\lesssim 30$	$\langle n_m\rangle$/nucleon
...	0.138	0.16	0.16	0.145	0.15	3/14	q_0
1.9[f]	2.6	4.0	5.6	6.5	6.0	...	$\langle n_s\rangle_0$
...	3.4	...	4.7	5.9	5.3	...	$\langle n_s\rangle_L$
...	...	0.076	0.09	0.12	0.097	1/14	$q_{\alpha\alpha}$
...	...	1.1	1.1	6.2	3.8	...	$\langle n_s\rangle_{\alpha\alpha}$

[a] E: energy/nucleon; $\langle n_H\rangle$: number of black and gray prongs; $\langle n_s\rangle$: number of shower particles; $\langle n_m\rangle$: number of charged mesons produced; $\langle n_s\rangle_0$: number of shower particles from stars with $n_H=0$; $\langle n_s\rangle_L$: number of shower particles from stars with $n_H\leq 4$; $\langle n_s\rangle_{\alpha\alpha}$: number of shower particles from stars with α particle continuing; q_0: fraction of stars with $n_H=0$; $q_{\alpha\alpha}$: fraction of stars with α particle continuing.
[b] See reference 20.
[c] See reference 24.
[d] Chicago group (unpublished).
[e] Brisbout, Dahanayake, Engler, Fujimoto, and Perkins, Phil. Mag. 1, 605 (1956).
[f] Kalbach, Lord, and Tsao (to be published), mesons only.

estimated by von Lindern,[30] Symanzik,[36] and by Castagnoli *et al.*[21] Castagnoli *et al.* found that, according to Heisenberg's theory Eq. (7) should be modified to

$$\log(C\gamma_c)=\langle\log\cot\!\operatorname{an}\theta\rangle. \tag{11}$$

This means that the "energy-spectrum independent" method, Eq. (7), would overestimate the primary energy in the laboratory system by a factor of about C^2 as compared to Eq. (11). According to Castagnoli *et al.*, C is roughly energy-independent and equal to 1.4 and 2.0 for a cutoff energy of $1.5m_\pi c^2$ and $1.1m_\pi c^2$ of the π-meson spectrum at its lower end. This compares well with the factor $C=1.4$ found by us experimentally. Symanzik has given a result for the limiting case of high energies and isotropic angular distribution in the c.m. system, as applied to the half-angle method. In terms of Eq. (11) it would give $C=1.3$. We have also calculated the value of C by using the actually measured energy spectra and angular distributions of some events[26,28–31] having comparable energies to our α-particle jets. This yielded the value of $C=1.4$. All these values are in reasonable agreement in view of the fact that the correction C depends in a sensitive way on the shape of the energy spectrum at the low-energy end and that this is not sufficiently well known. Using recent measurements of transverse momentum of shower particles and Landau's[37] theory, Gramenizkij *et al.*[38] have also arrived at the conclusion that the methods (a), (b), and (c) [in particular Castagnoli's method, Eq. (6)] should overestimate the true primary energy by a factor 2–2.5 (corresponding to $C=1.4$ to 1.6). The value of $C=1.4$ found by us experimentally is in good agreement with all these theoretical predictions.

There is also other experimental work which is in qualitative agreement with our observations. Hänni[5,6] and Zdanov *et al.*[39] found big fluctuations in the energy determination when doing work similar to that described here. Statistically well-founded results have, in particular, been obtained by Haber-Schaim[23] and by a Russian group[24] at lower energies. Studying stars produced by 6.2-Bev protons at the Bevatron, Haber-Schaim found an energy of 11.7 ± 2.2 Bev for proton interactions with fewer than 8 black prongs by using the half-angle method. A very similar result was obtained by Beliakov *et al.*[24] working with the Dubna accelerator at 8.7 Bev. Applying the half-angle method to possible p-p and p-n interactions, they arrived at an average energy of 16 Bev. Thus in both cases the energy was overestimated by roughly a factor of 2. Although this agrees very well with our results, both groups of experiments cannot be strictly compared. Our energy was considerably higher on the average and the influence of the nucleons should be smaller for our analysis. Also, the method of averaging over the angular distribution is somewhat different.

5. THE AVERAGE MESON MULTIPLICITY

Results on the average number of particles produced in the α-particle interactions are given in Table II. We have listed our results for the two energy groups of average energy 10 Bev/nucleon and 40 Bev/nucleon and for all our events together. For comparison we have given work done by other groups on α particles and protons at comparable energies. There is good agreement between our results at 10 Bev/nucleon and those of the Bombay[20] group. Certain remarks should be made regarding Table II. The number of black tracks $\langle n_H\rangle$ is a good measure for the nuclear excitation given to the nucleus. It is observed to be quite independent of energy and is the same for protons and α particles for energies ≥ 10 Bev/nucleon. The distribution of the numbers of

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shower particles for the α particle group having an average energy of 40 Bev/nucleon is shown in Fig. 10. The number of events having a certain number of shower particles has approximately an exponential distribution. Figure 10 shows that there are large fluctuations of the multiplicity and, in particular, that the multiplicity can be very low.

The number of shower particles given for the 14 events of $\langle E\rangle \approx 3000$ Bev/nucleon can be regarded as an upper limit only, since these events were found by scanning for cascades and this introduces a bias for finding events of high multiplicity. In order to find the number of charged mesons produced ($\langle n_m\rangle$), we have to subtract the number of protons among the shower particles. For the α-particle interactions we first subtracted 1.8 tracks to account for the two protons originally belonging to the incident α particle and correcting for the 10% of all events in which the α particle reappears. Moreover, knowing that 1.8 nucleons interact, on the average, roughly the same number is hit directly in the target nucleus. We have thus subtracted another 0.9 fast knock-on protons. For the proton interactions we subtracted one track since the effect of charge exchange of the incoming proton is roughly expected to compensate for the production of fast knock-on protons. We believe that at least for the α-particle interactions this procedure will not lead to serious errors in the determination of $\langle n_m\rangle$ because the average multiplicity is quite high compared to the correction for protons. The next column in Table II shows the number of charged mesons produced in the collision of one nucleon. It is obtained from $\langle n_m\rangle$ by dividing by the number of interacting nucleons, as was discussed in Sec. 3. The average meson production by collision of one nucleon with emulsion nuclei as a function of energy is shown in Fig. 11. The 9-Bev proton point agrees well with the 10-Bev α-particle points. This again indicates that the nucleons of the α particles do not influence one another greatly in the collision process. Our results indicate that the average charged meson multiplicity for 25-Bev proton collisions will be about 3.5. It rises proportional to $E^{0.40\pm0.08}$ between 10 Bev and 40 Bev and the exponent probably decreases at higher energies. The fraction of events having no heavy prongs and which, therefore, could possibly be interactions with hydrogen also seems to be independent of energy. The value at 3000 Bev/nucleon is consistent with this conclusion, although the statistical accuracy is rather poor. From the cross sections in emulsion, about 7% of all interactions are expected to occur on hydrogen; the rest are due to glancing collisions with heavier nuclei. For events of this kind, a smaller number of the nucleons of the incident α particle is expected to hit the target nucleus. This is reflected in the smaller number of shower particles emitted as compared to all events. The number of shower particles is also smaller for events having fewer than 5 black or gray tracks ($\langle n_s\rangle_L$) and is, within the statistical errors, equal to the number of shower particles in the stars without any heavy prongs. This shows that the influence of secondary interactions and the average number of nucleons hitting is about the same in both classes of events. In about 10% of all cases the α particle seems to continue after the collision. These events are probably due to stripping of a neutron from the α particle. The average number of shower particles produced in these events is in agreement with the number calculated for events in which one nucleon is colliding ($\langle n_m\rangle$/nucleon); however, the number of events observed so far is rather small and no definite conclusion can be drawn.

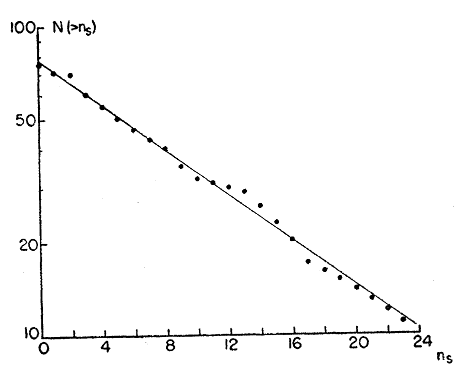

FIG. 10 Integral distribution of the number of shower particles n_s originating from α-particle interactions (average energy 40 Bev/nucleon). $N(>n_s)$ is the number of interactions having $\geq n_s$ shower particles.

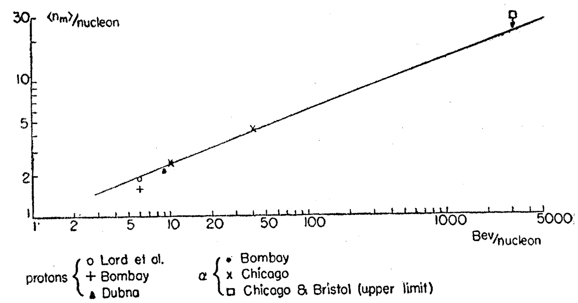

FIG. 11. Average number of charged mesons $\langle n_m\rangle$/nucleon produced in nuclear interactions in emulsion as a function of primary energy per nucleon.

6. MESON PRODUCTION BY HEAVY NUCLEI ($Z\geq 6$)

Average Multiplicities

Interactions made by the primary heavy nuclei and by their heavy fragments were investigated the same way as the α-particle interactions. Table III shows the results of our analysis based on 317 interactions of the heavy nuclei of the cosmic radiation with emulsion nuclei. No interactions of secondary heavy fragments were included in Table III.

The number of heavy prongs is seen to decrease as the charge of the incident nucleus increases. This might be explained by the fact that the percentage of glancing collisions and collisions with light nuclei of the emulsion

TABLE III. Interactions of Heavy nuclei (average energy 20 Bev/nucleon).[a]

Charge group	Number of events	$\langle Z\rangle$	$\langle n_H\rangle$	$\langle n_s\rangle$	$\langle n_m\rangle$	n_i
M: $6\leq Z\leq 9$	218	6.9	9.1	20.6	16.1	4.9
H: $10\leq Z$	99	15	8.4	25.0	17.2	5.2
VH[b]: $20\leq Z$	20	23	6.2	33.5	23.2	7.0

[a] $\langle n_H\rangle$: average number of gray and black tracks; $\langle n_s\rangle$: average number of shower particles; $\langle n_m\rangle$: average number of charged mesons; n_i: average number of nucleons participating in the collision.
[b] Also included in H group.

increases with increasing charge of the incident nucleus, and that the number of black tracks is smaller in this type of collisions. We cannot, however, exclude the possibility that the effect is due to statistical fluctuations. The distribution of the number of black tracks and a more detailed discussion of them was given in I. The number of protons among the shower particles was determined by the customary procedure of applying conservation of charge of the incident nucleus to the interactions: the number of incident charges minus the number of outgoing charges contained in fast fragments with $Z\geq 2$ was considered equal to the number of protons. Subtracting the number of protons from n_s gives the number of charged mesons produced, n_m. This value of n_m is probably an upper limit to the number of mesons. The contribution of fast knock-on protons was neglected. Due to the high multiplicity at our average primary energy of 20 Bev/nucleon, this contribution is not too important. Assuming a crude model for the collisions between heavy nuclei, one might expect that the number of nucleons participating in the collision is roughly proportional to the impact area, hence to $A^{\frac{2}{3}}$, where A is the number of nucleons of the incident nucleus. Assuming, further, that the collision can be regarded as a combination of individual nucleon-nucleon collisions, which is compatible with all our findings, the number of mesons produced should be approximately proportional to the number of nucleon pairs involved and hence also to $A^{\frac{2}{3}}$. The number of mesons produced should level off if the charge of the incident nucleus becomes greater than the charge of C, N, O nuclei of the emulsion, because at least for collisions with these

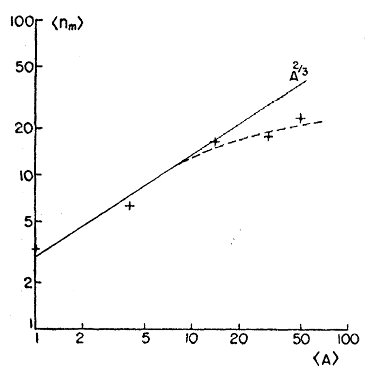

FIG. 12. Average number of charged mesons $\langle n_m\rangle$ produced in collisions of nuclei of mass number $\langle A\rangle$ with emulsion nuclei. The average energy is 20 Bev/nucleon.

nuclei, the maximum number of nucleons involved is then limited by the A of the target nucleus. In Fig. 12 we plotted the average number of charged mesons produced as a function of the atomic number A. The average energy of the events is 20 Bev/nucleon. The values for $A\geq 4$ were taken from Tables II and III; the proton point from Fig. 11. The predictions of our crude model are seen to be well fulfilled. One can then calculate the average number of nucleons n_i of the incident nucleus which participate in the collision. Assuming that meson production is independent of the number of simultaneously interacting nucleon pairs, we have $n_i=\langle n_m\rangle/\langle n_m\rangle_p$, where $\langle n_m\rangle_p$ is the average number of mesons produced by a single nucleon. The value of n_i for the different charge groups is included in Table III.

The energy dependence of meson production by heavy nuclei was studied by dividing our events into two classes of about equal statistical weight according to their energy in the same way as was done for α particles. The energy of the fragmentations was determined as shown in Sec. 2. In addition, the energy of a statistically unbiased sample of interactions having only one or no heavy fragment was measured by means of the angular distribution of the shower particles. The results are shown in Table IV. Here we have given the ratio of q the number of mesons produced at an average energy of 40 Bev/nucleon and 10 Bev/nucleon, respectively. For comparison, the value for α particles as found from Table II is also given. The value of q is somewhat lower for the heavy nuclei than for the α particles. Both values are, however, statistically compatible with one another. Another possibility might be that with increasing energy the number of nucleons participating in the collision decreases for increasing charge of the heavy nuclei. An indication of this effect has been found by the Turin[8] group.

7. ANGULAR DISTRIBUTION OF SHOWER PARTICLES

Our knowledge about interactions between heavy nuclei and emulsion nuclei is so far compatible with the assumption that they can be analyzed in terms of collisions between the individual nucleons. In this case one can use the angular distribution of the shower particles to determine the energy of the incident heavy nucleus by using methods (a), (b), and (c) explained in Sec. 2 [Eqs. (6) through (8)]. In order to check this, we selected 26 interactions of heavy primaries producing 7 or more charged mesons (excluding nucleons). The energy E_0 of these heavy primaries was known from scattering measurements and from the opening angle of the fragments. They were either primary nuclei or secondary heavy fragments. The energy E_{sh} as given by the angular distribution of the shower particles was determined by plotting $\log[f/(1-f)]$ vs $\log\tan\theta$ [see method (c), Sec. 2]. Correction for protons among the shower particles was made in the following way: the number of protons was determined by conservation of charge as explained in Sec. 6. The shower particles having the smallest angles with the direction of the primary were then considered to be these protons and left off. This procedure is analagous to the one applied for α particles. Because most of the nucleons of the heavy nucleus do not participate in the interaction (see Table III), but continue almost undeflected, this procedure of leaving off the above-mentioned tracks with the smallest angles is a necessary one and should not give rise to unreasonably large errors. In Fig. 13 we have plotted the distribution of the ratio E_{sh}/E_0. Comparing it with Fig. 9 one sees that the tendency to overestimate the energy by the angular distribution seems to be still more pronounced for heavy nuclei than for α particles. This is due, however, to the fact that the 26 events of Fig. 13 do not provide a random sample, but were already chosen with the purpose of reducing the influence of secondary interactions. The average energy of the group is 24 Bev/nucleon. Events with 4 and fewer black or gray tracks are again cross-hatched in Fig. 13. The distribution of E_{sh}/E_0 for this class of events is the same as for the corresponding class of α-particle interactions. Therefore, we conclude that the method of estimating the energy from the angular distribution of the shower particles yields similar results in case of heavy-nucleus interactions as in the case of α particles and single nucleons. The method will, in general, also lead to overestimating the true energy if one uses one of the "spectrum-independent" methods [Eqs. (6) through (8)] and has no disturbance by secondary interactions. This result we again attribute to the presence of mesons of low energy in the center-of-mass system. Therefore, Eq. (10) or an equivalent correction should be used when estimating energies of heavy primaries from the angular distribution. This will decrease the energy E_{sh} by a factor of 2 compared to one of the Eqs. (6) through (8). This will bring E_{sh} on the average in agreement with the true energy, at least in the energy region considered here. The fluctuations for individual events are seen to be somewhat larger than in the case of α-particle events (a factor of 4–5 on either side). Thus, the method is not too reliable for energy determinations of individual events. In particular, by using Eqs. (6) through (8) in which the meson spectrum is not taken into account and by failing to eliminate the protons in the inner core of the shower, the energy can be overestimated by a factor of more than 10.

8. CONCLUSIONS

The following results can be derived from the systematic study of 175 interactions of α particles and 317 interactions of heavy nuclei with an energy >7 Bev/nucleon.

The energy of heavy nuclei undergoing fragmentations can be estimated from the opening angle of the fragments by Eqs. (4) and (5), which give average values. The distribution of individual measurements is not symmetric around the average value; the energy is in general not underestimated by more than a factor of 2.

TABLE IV. Ratio q of meson production by heavy nuclei at 40 Bev/nucleon and 10 Bev/nucleon.

Charge group	Number of events	q
M	132	1.52 ± 0.27
H	58	1.12 ± 0.30
Average	190	1.40 ± 0.20
α	327	1.75 ± 0.20

In collisions between high-energy α particles and the nuclei of the photographic emulsion, on the average 1.8 nucleons of the α particle participate strongly. The average transverse momentum of protons which split from an α particle and do not take part in the interaction is 200 Mev/c.

Methods to estimate primary particle energies from the angular distribution of shower particles emitted from high-energy interactions were checked experimentally on 77 interactions of α particles and 26 interactions of heavy nuclei of known high energy. In the inner core of these showers we left off shower particles considered to be protons. Their number was chosen in such a way as to account for all of the incident charges of the primary nucleus (subtracting the charge of outgoing heavier fragments). Big fluctuations of individual energy estimates around the true value were found, which amounted up to a factor of 10 on either side. Restricting ourselves to interactions having ≤ 4 heavily ionizing particles reduces the fluctuations by cutting off events which give too low apparent energies. The energy of the events with ≤ 4 heavily ionizing prongs is on the average overestimated by a factor of 2, if one uses a method not taking into account the energy and angular distribution of the mesons, for example the median-angle formula or the simplest version of Castagnoli's formula. From our measurements and also from theoretical considerations it follows that all primary energies as determined by these methods should be reduced by a factor of 2, for example by using Eq. (10) instead of Eq. (7). In such a case we found that the fluctuations of individual measurements around the average value do in general not

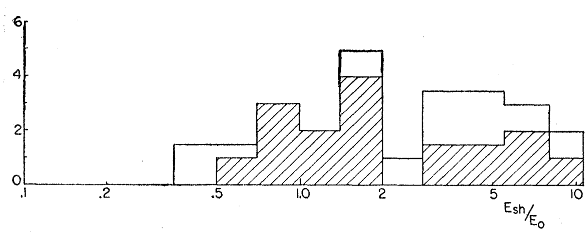

FIG. 13. Comparison of the true primary energy of heavy nuclei interactions E_0 with the energy E_{sh} obtained from the angular distribution of the shower particles. Average primary energy is 24 Bev/nucleon. Events with $n_H \leq 4$ are crosshatched. Ordinate: number of events.

exceed a factor 2 to 3 on either side, if one restricts one-self to interactions with more than 6 to 8 charged mesons.

The distribution and the average values of the number of charged mesons produced by high-energy α particles are given. Meson production by α particles rises proportional to $E^{0.40\pm0.08}$ between 10 Bev/nucleon and 40 Bev/nucleon. At 40 Bev/nucleon, on the average 8.2 charged mesons are produced per collision. In about 10% of all cases the α particle continues on after the collision.

The average number of mesons produced by heavy nuclei increases first proportional to $A^{\frac{2}{3}}$ and more slowly for $A>16$. The average number of charged mesons produced per collision is 16.1 for incident M nuclei, 17.2 for H nuclei, and 23.2 for VH nuclei at an average energy of 20 Bev/nucleon.

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Energy Spectrum of the Heavy Nuclei in the Cosmic Radiation between 7- and 100-Bev/Nucleon*

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The energy spectrum of the heavy nuclei of the cosmic radiation was determined between 7 Bev/nucleon and 100 Bev/nucleon. The distribution for the $M(6\leq Z\leq 9)$ and $H(Z\geq 10)$ charge groups agree with one another within the limits of error. Combining both charge groups, the integral spectrum is of the form $N(>E)\sim E^{-1.6\pm0.15}$ (E = total energy/nucleon). Flux values for nuclei of the M, H, and VH $(Z\geq 20)$ charge groups at the geomagnetic equator are given. Comparing these results with flux values obtained at high latitudes, it is concluded that a power spectrum of the form $E^{-1.6}$ fits all three charge groups within the limits of error between 2.5 Bev/nucleon and 7 Bev/nucleon. From the observation of α-particle showers of very high energy we conclude that under certain assumptions the integral spectrum of α particles can be represented by $N(>E)\sim E^n$ with $n=-1.58_{-0.18}^{+0.21}$ for energies ≤ 1500 Bev/nucleon.

INTRODUCTION

TEN years ago the energy spectrum of the cosmic radiation was known only at the high-energy end above 10^{13} ev from measurements of extensive air showers, where a power law of the form $N(>E)=CE^{-\gamma}$ could be well established. The value of the exponent was found to be $\gamma=1.75$. As soon as rocket flights permitted measurements of the total intensity above the top of the atmosphere, in 1947, one could show by varying the geomagnetic latitude that for singly charged particles a similar power law holds, the exponent being close to 1.[1–4] Due to the albedo effect of the earth the total intensities of singly charged particles, measured in those experiments, were not the intensities of the primary proton component. Attempts have been made to derive the proton spectrum by estimating and subtracting the albedo effect.[5] After the discovery of heavy nuclei in the primary cosmic radiation the meaning of total primary intensities as a function of latitude got more complicated. Kaplon, Peters, *et al.*[6] deduced an energy spectrum of the heavy-nuclei component not only by varying the geomagnetic latitude, but by observing and evaluating fragmentations of heavy nuclei in nuclear emulsions. Their method has been discussed in a previous paper (II).[7] They could prove that the number of heavy nuclei $(Z\geq 10)$ between 3 and 30 Bev/nucleon is well represented by a power-law spectrum with an exponent $\gamma=1.35\pm0.15$. Since 1952 a number of measurements of the heavy-nuclei flux have been carried out at different latitudes near the top of the atmosphere by using the method of nuclear emulsion and by greatly improving the techniques of charge determination.[6,8–19]

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