

## Electric Field Distributions in an Ionized Gas\*

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A method for improving systematically the Holtsmark distribution is described. It is based on a cluster-type expansion and takes into account increasing orders of correlation. In this paper, it is applied to the calculation of the distribution of the high-frequency component of the electric field in an ionized gas in thermal equilibrium.

### I. INTRODUCTION

THE knowledge of the probability distribution function for the electric field in an ionized gas is a prerequisite to the solution of a number of problems, in particular that of the calculation of the broadening of spectral lines emitted by atoms in the gas. This problem was first attacked a long time ago by Holtsmark,<sup>1</sup> who solved it by neglecting the correlations between the various charged particles producing the field. Since then, various attempts have been made to include these correlations.<sup>2-6</sup> In the present paper, we shall propose a method for including them which is somewhat similar to the cluster expansion method of statistical mechanics.<sup>7</sup> The idea is to set up the calculation in such a way as to yield exactly the Holtsmark result in the high-temperature limit when correlations become unimportant. The corrections to this appear as the result of taking into account increasingly complicated types of correlation, the first correction being provided by two-body correlations, the next one by three-body correlations, etc. Obviously, such an approach will be valid only if the result does not deviate too far from the original Holtsmark distribution. This restricts it to the high-temperature, low-density limit, i.e., to the case of an ionized gas in the usual sense as opposed to that of a very hot solid for which some of the previous theories were also intended.<sup>2</sup> On the other hand, when used in its region of validity, it constitutes the first few terms in a series expansion in powers of a small parameter and can, therefore, be trusted completely. Although we shall write the complete series expansion in a formal way, we shall eventually calculate only the correction due to two-body correlations. For the correlation function we shall use the Debye-Hückel<sup>8</sup>

result. The conditions under which it is correct are well known,<sup>9</sup> and are the same as those that make our power expansion meaningful. Stated in physical terms, they say that the volume of the cloud which surrounds each charged particle and shields it should be large compared to the volume per particle.

We distinguish two parts in the electric field, which we call the high-frequency and the low-frequency components. The high-frequency component is that part of the electric field whose time variation is governed by the motion of the electrons, while the time variation of the low-frequency component is governed by the motion of the ions. It is obviously desirable to separate the two parts and calculate their distributions separately since they involve such radically different frequencies. But it would be incorrect to believe that the high-frequency part is just the sum of the fields of all the electrons and the low-frequency part the same for the ions. Indeed, the low-frequency part contains a contribution from the electrons since each ion is surrounded by a cloud of electrons which moves at the same speed it does and follows it everywhere. The correct way to calculate the distribution of the low-frequency component is to assume first that the ions are fixed in space and to take the time average of the electric field at the point where the distribution is desired. It is this time average which constitutes the low-frequency component, and its distribution can then be obtained by letting the ions themselves be distributed in space in the appropriate manner. In other words, the low-frequency component consists of the sum of the shielded ionic fields, the shielding being caused by the electronic cloud surrounding each ion, and the average being taken over times long compared to typical electronic relaxation times, but short compared to ionic times. For the shielded ionic field, it is appropriate again to use the Debye-Hückel result<sup>8</sup> since this is a long-time average. We emphasize that this shielding does not include any contribution from the other ions. The Debye-Hückel theory does not apply there, and this effect will be properly taken into account when we introduce the ion pair correlation function. Having thus defined the low-frequency component, we see that the average of the remainder, the high-frequency

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<sup>1</sup> J. Holtsmark, *Ann. Physik* **58**, 577 (1919).

<sup>2</sup> A. A. Broyles, *Phys. Rev.* **100**, 1181 (1955); *Z. Physik* **151**, 187 (1958).

<sup>3</sup> F. N. Edmonds, *Astrophys. J.* **123**, 95 (1956).

<sup>4</sup> G. Ecker, *Z. Physik* **148**, 593 (1957); G. Ecker and K. G. Müller, *Z. Physik* **153**, 317 (1958).

<sup>5</sup> H. Hoffmann and O. Theimer, *Astrophys. J.* **127**, 477 (1958).

<sup>6</sup> M. Lewis and H. Margenau, *Phys. Rev.* **109**, 842 (1958).

<sup>7</sup> J. E. Mayer and M. G. Mayer, *Statistical Mechanics* (John Wiley & Sons, Inc., New York, 1940), Chap. 13.

<sup>8</sup> P. Debye and E. Hückel, *Physik. Z.* **24**, 185 (1923).

<sup>9</sup> L. Landau and E. Lifshitz, *Statistical Physics* (Pergamon Press Limited, London, 1958), Sec. 74.

component, over times large compared to typical electronic times always vanishes. It is, therefore, plausible to assume that the distribution of the high-frequency component is the same as that of a gas of electrons with uniform neutralizing background, and we shall do so.

The method which we shall develop applies equally well to both components of the electric field. Since the calculations are simpler for the high-frequency part, only the results of that case will be reported here. The low-frequency part of the distribution will be published later. In Sec. II we derive the Fourier transform of the field distribution by a cluster expansion method, and in Sec. III we apply it to the calculation of the probability distribution of the high-frequency component at a neutral point.

## II. THE CLUSTER EXPANSION

We consider  $N$  particles with coordinates  $\mathbf{x}_1, \dots, \mathbf{x}_N$ , enclosed in volume  $\mathcal{V}$ , and we call  $n$  the density,  $n=N/\mathcal{V}$ . We call  $\mathbf{E}_1$  the electric field produced at a given point by the particle located at  $\mathbf{x}_1$ . The total electric field we call  $\mathbf{E}$ ,

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \dots + \mathbf{E}_N. \quad (1)$$

We are looking for the probability distribution  $W(\mathbf{E})$  of the total field, but it is simpler to calculate its Fourier transform,

$$F(\mathbf{k}) = \int \exp(i\mathbf{k} \cdot \mathbf{E}) W(\mathbf{E}) d^3E. \quad (2)$$

We do this by changing over to the variables  $\mathbf{x}_1, \dots, \mathbf{x}_N$ , instead of  $\mathbf{E}$ . We call  $P(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N)$  the probability of occurrence of a given configuration of the  $N$  particles. Therefore,

$$F(\mathbf{k}) = \int \exp[i\mathbf{k} \cdot (\mathbf{E}_1 + \dots + \mathbf{E}_N)] \times P(\mathbf{x}_1, \dots, \mathbf{x}_N) d^3x_1 \dots d^3x_N. \quad (3)$$

At this point we use a standard trick and replace each exponential,  $\exp(i\mathbf{k} \cdot \mathbf{E}_i)$ , by the sum of two terms

$$\exp(i\mathbf{k} \cdot \mathbf{E}_i) = 1 + [\exp(i\mathbf{k} \cdot \mathbf{E}_i) - 1]. \quad (4)$$

In order to abbreviate notations, we write this in the form

$$\exp(i\mathbf{k} \cdot \mathbf{E}_i) = 1 + \varphi_i, \quad (5)$$

and expand the product of  $N$  such factors. The result is to replace the exponential in (3) by the following series:

$$1 + \sum_1 \varphi_i + \sum_2 \varphi_i \varphi_j + \dots, \quad (6)$$

where  $\sum_1$  means the sum over all particles,  $\sum_2$  the sum over all pairs, etc. When the  $M$ th term of this series is substituted in (3), we can integrate immediately over all coordinates but those of the  $M$  particles to which the  $\varphi$ 's refer. This means replacing  $P(\mathbf{x}_1, \dots, \mathbf{x}_N)$  by

$P_M(\mathbf{x}_i, \dots, \mathbf{x}_s)$ , the probability function that the  $M$  particles  $i, \dots, s$ , will be located at  $\mathbf{x}_i, \dots, \mathbf{x}_s$ . The result is

$$F(\mathbf{k}) = \sum F_M(\mathbf{k}), \quad (7)$$

with

$$F_M(\mathbf{k}) = \sum_M \int \varphi_i \dots \varphi_s P_M(\mathbf{x}_i, \dots, \mathbf{x}_s) d^3x_i \dots d^3x_s. \quad (8)$$

The sum in the last equation is over all possible combinations of  $M$  particles among the  $N$ .

We now introduce explicitly the idea that correlations will have a small effect. This is expressed by writing the function  $P_M$  in the form of a series containing increasing orders of correlation. As a first approximation, we can say that the  $M$  particles are completely independent and therefore  $P_M$  will be the product of  $M$  one-body probability functions. The latter can be taken equal to  $\mathcal{V}^{-1}$  when we compute the field distribution at a neutral point, but not if we require it at a charged point. As a correction to our first approximation, we may introduce in  $P_M$  a two-body function which will take care of those cases where two particles approach each other closely and are correlated. If two different pairs of particles come close together in two different parts of the gas, we shall just take the product of two such corrections. But if three particles should all come close together, we would need a further correction since the sum of the pair corrections is probably not accurate there. Therefore, we introduce a three-body correction and it will also occur when several distant groups of three particles come in close contact. Next we need a four-body correction, etc. All this is expressed in the following equation<sup>10</sup>:

$$\begin{aligned} \mathcal{V}^M P_M(\mathbf{x}_i, \dots, \mathbf{x}_s) = & \prod g_1(\mathbf{x}_i) + \sum_2 g_2(\mathbf{x}_j, \mathbf{x}_k) \prod g_1(\mathbf{x}_i) \\ & + \sum_{22} g_2(\mathbf{x}_j, \mathbf{x}_k) g_2(\mathbf{x}_l, \mathbf{x}_m) \prod g_1(\mathbf{x}_i) + \sum_{222} \dots + \dots \\ & + \sum_3 g_3(\mathbf{x}_j, \mathbf{x}_k, \mathbf{x}_l) \prod g_1(\mathbf{x}_i) \\ & + \sum_{33} g_3(\mathbf{x}_j, \mathbf{x}_k, \mathbf{x}_l) g_3(\mathbf{x}_m, \mathbf{x}_n, \mathbf{x}_p) \prod g_1(\mathbf{x}_i) + \sum_{333} \dots \\ & + \sum_{32} g_3(\mathbf{x}_j, \mathbf{x}_k, \mathbf{x}_l) g_2(\mathbf{x}_m, \mathbf{x}_n) \prod g_1(\mathbf{x}_i) + \dots \\ & + \sum_4 \dots + \dots. \end{aligned} \quad (9)$$

We have called the one-body probability function  $\mathcal{V}^{-1}g_1(\mathbf{x})$ . The function  $g_2$  is the two-body correction, etc. By extracting the factor  $\mathcal{V}^M$  as we did, we have defined the  $g$  functions in a way which is independent of  $\mathcal{V}$  for large  $\mathcal{V}$ . Here  $\sum_2$  stands for the sum over all pairs among the  $M$  particles, while  $\sum_{22}$  is the sum over all distinct pairs of pairs. Similarly,  $\sum_{32}$  is the sum over all possible combinations of one triplet and one pair involving five distinct particles out of the  $M$ , etc. In each term, the product  $\prod g_1(\mathbf{x}_i)$  extends over all remaining particles among the  $M$ , i.e., all those which are not already involved in pairs, triplets, etc. Each term in (9) may be represented by a cluster diagram in the customary manner.<sup>7</sup>

<sup>10</sup> The  $g$  functions appear occasionally in the literature. See, for instance, J. E. Mayer and E. Montroll, J. Chem. Phys. **9**, 4 (1941); or E. E. Salpeter, Ann. phys. **5**, 217 (1958).

We must now substitute expression (9) back into (8) and sum over all  $M$ 's. We assert that, in the limit where  $N$  and  $\mathcal{V}$  become very large, the density  $n$  remaining constant, the result for  $F(\mathbf{k})$  is the same as the following:

$$F(\mathbf{k}) = G_1(\mathbf{k})G_2(\mathbf{k})G_3(\mathbf{k})\cdots, \quad (10)$$

with

$$G_P(\mathbf{k}) = 1 + \mathcal{V}^{-P} \sum_P \int \varphi_i \cdots \varphi_s g_P(\mathbf{x}_i, \cdots, \mathbf{x}_s) d^3x_i \cdots d^3x_s \\ + \mathcal{V}^{-2P} \sum_{PP} \int \varphi_i \cdots \varphi_v g_P(\mathbf{x}_i, \cdots, \mathbf{x}_s) g_P(\mathbf{x}_t, \cdots, \mathbf{x}_v) \\ \times d^3x_i \cdots d^3x_v + \mathcal{V}^{-3P} \sum_{PPP} \cdots + \cdots \quad (11)$$

Here,  $\sum_P$  runs over all possible combinations of  $P$  particles out of the  $N$ ,  $\sum_{PP}$  over all possible distinct combinations of two clusters containing  $P$  particles each, all particles being different, and so on. The difference between (7) and (10) is only that in every term of (7), as expanded using (8) and (9), all particles must be different, while in (10), as we have defined it, it could happen that one of the particles involved in  $G_P$ , say, would be the same as a particle involved in  $G_Q$ . Thus (10) contains some terms which are not included in (7). But the reader will easily convince himself that the number of such terms is smaller than the number of terms appearing in both expressions by a factor of order  $N$ , and therefore this is a negligible correction when  $N$  tends to infinity.

If we let  $N$  become infinite, we can write  $G_P(\mathbf{k})$  in closed form if we realize that in every sum all terms are equal and that we only have to count the number of terms. The result is

$$G_P(\mathbf{k}) = \exp[(n^P/P!)h_P(\mathbf{k})], \quad (12)$$

with

$$h_P(\mathbf{k}) = \int \varphi_1 \cdots \varphi_P g_P(\mathbf{x}_1, \cdots, \mathbf{x}_P) d^3x_1 \cdots d^3x_P, \quad (13)$$

and therefore we obtain the Fourier transform of the field distribution function in the form

$$F(\mathbf{k}) = \exp \left[ \sum_{P=1}^{\infty} (n^P/P!) h_P(\mathbf{k}) \right]. \quad (14)$$

As is customary in other cluster expansion problems, the result is the exponential of a power series, each term of which involves bigger and bigger clusters. By taking the first term only, one obtains the Holtmark distribution in the case of the field at a neutral point. In our calculations we shall include the second term, substituting for the function  $g_2$  the result of the Debye-Hückel theory. The field distribution itself is obtained by inverting (2),

$$W(\mathbf{E}) = (2\pi)^{-3} \int \exp(-i\mathbf{k} \cdot \mathbf{E}) F(\mathbf{k}) d^3k. \quad (15)$$

The question, how fast does the series in (14) converge, might be raised at this point. It is possible to show that, for given  $n$  and  $\mathbf{k}$ , successive terms contain increasing inverse powers of the temperature. Thus, the series becomes increasingly better as we approach the high-temperature limit, as we have stated earlier. In this respect, the present problem differs radically from the imperfect gas problem,<sup>7</sup> for which the cluster method, when applied to Coulomb interactions, gives divergent answers. The reason for the difference is that the electric field, for which we want the probability distribution, depends mainly on the positions of the particles in the immediate vicinity of the point where we are calculating it, and any long-range interaction of these particles with others elsewhere in the gas is not very important. By using the Debye-Hückel correlation function, we have already taken care of all relevant long-range aspects of the problem. It is also clear that, although both theories use the "cluster trick," they put it to different purposes; in (4) it is applied to  $\exp(i\mathbf{k} \cdot \mathbf{E}_i)$ , while in the imperfect gas problem it is used for the Boltzmann factor itself.

### III. DISTRIBUTION OF THE HIGH-FREQUENCY COMPONENT AT A NEUTRAL POINT

Our model consists of a gas of electrons of density  $n$  with uniform neutralizing background. We choose the origin of coordinates to be the point at which we want the field distribution. The proper field to use is the unshielded Coulomb field,

$$\mathbf{E}_i = e\mathbf{x}_i/r_i^3. \quad (16)$$

Since the origin is neutral,  $g_1$  is unity. The integral (13) is straightforward for  $h_1$  and yields

$$h_1(\mathbf{k}) = -(4/15)(2\pi e k)^3, \quad (17)$$

which is the Holtmark result.

According to the Debye-Hückel theory,<sup>8</sup> the pair correlation function is

$$P_2(\mathbf{x}_1, \mathbf{x}_2) = \mathcal{V}^{-2} \exp(-e^2\Phi_{12}/\kappa T), \quad (18)$$

with

$$\Phi_{12} = |\mathbf{x}_1 - \mathbf{x}_2|^{-1} \exp(-|\mathbf{x}_1 - \mathbf{x}_2|/\lambda), \quad (19)$$

and

$$\lambda = (\kappa T/4\pi n e^2)^{1/2}. \quad (20)$$

The corresponding expression for  $g_2$  follows from (9),

$$g_2(\mathbf{x}_1, \mathbf{x}_2) = \exp(-e^2\Phi_{12}/\kappa T) - 1. \quad (21)$$

The fact that  $h_2(\mathbf{k})$  is a small correction to  $h_1(\mathbf{k})$  for hot ionized gases whose densities are not too large is a consequence of the smallness of  $g_2$  for all configurations except those where electrons 1 and 2 approach each other very closely. Similar reasoning leads one to expect that three or more electrons have even less chance of being close enough together and that the pair correction gives most of the correlation effect. In accordance with

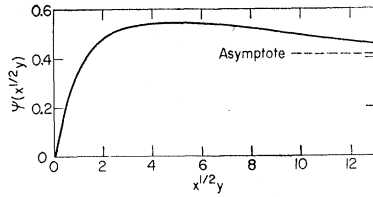


FIG. 1. The function  $\psi(x^{1/2}y)$  in terms of which the correction due to pair correlations is evaluated.

the spirit of the Debye-Hückel theory, we shall linearize (21) and use instead

$$g_2(\mathbf{x}_1, \mathbf{x}_2) = -e^2\Phi_{12}/\kappa T, \quad (22)$$

hence

$$h_2(\mathbf{k}) = -\left(\frac{e^2}{\kappa T}\right) \iint \varphi_1 \varphi_2 \Phi_{12} d^3x_1 d^3x_2. \quad (23)$$

This last integral seems at first sight rather difficult to evaluate. But some insight into the behavior of the integrand is obtained if one notices that a fairly good approximation for  $\varphi_i$  is  $iek \cdot \mathbf{x}_i/r_i^3$  for small  $\mathbf{k}$  or large  $r_i$ , and minus unity for large  $\mathbf{k}$  or small  $r_i$ . This indicates that, if we expand  $\varphi_i$  in spherical harmonics, the first two terms, i.e., the isotropic and the dipole term, will be most important. Therefore, we expand every factor in the integrand of (23) in spherical harmonics as follows (taking the vector  $\mathbf{k}$  along the  $z$  direction):

$$\varphi_i = \sum_l i^l [4\pi(2l+1)]^{1/2} [j_l(ek/r_i) - \delta_{l0}] Y_{l0}(\theta_i, \omega_i), \quad (24)$$

$$\begin{aligned} \Phi_{12} &= \sum_l [(2l+1)/4\pi]^{1/2} f_l(r_1, r_2, \lambda) Y_{l0}(\theta_{12}, \omega_{12}) \\ &= \sum_{lm} f_l(r_1, r_2, \lambda) Y_{lm}^*(\theta_1, \omega_1) Y_{lm}(\theta_2, \omega_2). \end{aligned} \quad (25)$$

The last line is obtained from the previous one by using the addition theorem for spherical harmonics.<sup>11</sup> The functions  $j_l$  are spherical Bessel functions.<sup>11</sup> The functions  $f_l$  can be obtained in closed form and are fairly simple for the lower  $l$  values. The angular integrations can now be performed with the help of the orthogonality relations for spherical harmonics, and one is left with a series over  $l$ , each term consisting of a double integral over  $r_1$  and  $r_2$ . It turns out that the main contribution comes from the  $l=1$  term in conformity with our expectations. The series is alternating and convergent and is well approximated by its first three terms. The sign of  $h_2(\mathbf{k})$  is opposite to that of  $h_1(\mathbf{k})$ , but its absolute value is always smaller.

For the numerical calculations it is convenient to define a unit length  $r_0$  by

$$(4/15)(2\pi)^{3/2} r_0^3 n = 1. \quad (27)$$

This is numerically almost equal to  $r_s$  defined by  $(4\pi/3)r_s^3 n = 1$ , but more convenient. We also define a unit of field strength

$$E_0 = e/r_0^2, \quad (28)$$

and we call  $\beta = E/E_0$  and  $x = kE_0$ . Since  $W(\mathbf{E})$  does not

depend on angles, the probability distribution  $H(\beta)$  for the scalar quantity  $\beta$  is

$$\begin{aligned} H(\beta) &= 4\pi\beta^2 W(\beta) \\ &= 2\beta\pi^{-1} \int_0^\infty \sin(\beta x) F(x) x dx. \end{aligned} \quad (29)$$

According to (17), the first term of the series in (14) is

$$nh_1(\mathbf{k}) = -x^3, \quad (30)$$

while the second term can be written

$$\frac{1}{2}n^2 h_2(\mathbf{k}) = x^3 \psi(x, y), \quad (31)$$

with

$$y = r_0/\lambda. \quad (32)$$

It turns out that  $\psi$  depends only on  $x^{1/2}y$  and this function is plotted in Fig. 1. Our approximation for  $F(x)$  is, therefore,

$$F(x) = \exp\{-x^3[1 - \psi(x^{1/2}y)]\}. \quad (33)$$

We expect it to be good when the two-body correction

TABLE I. Distribution function  $H(\beta)$  of the high-frequency component at a neutral point for several values of  $r_0/\lambda$ . (The Holtzmark distribution corresponds to  $r_0/\lambda=0$ .) For values of  $r_0/\lambda$  not tabulated, one may use linear interpolation for most values of  $\beta$ . The accuracy of the numerical integration is such that the last figure quoted is uncertain.

$\beta$	$r_0/\lambda=0^a$	$r_0/\lambda=0.2$	$r_0/\lambda=0.4$	$r_0/\lambda=0.6$	$r_0/\lambda=0.8$
0.2	0.01667	0.02189	0.02853	0.03646	0.04521
0.4	0.06308	0.08158	0.10433	0.13062	0.15877
0.6	0.12960	0.16348	0.20298	0.24611	0.29010
0.8	0.20327	0.24818	0.29670	0.34564	0.39219
1.0	0.27132	0.31873	0.36499	0.40686	0.44293
1.2	0.32402	0.36487	0.39943	0.42591	0.44514
1.4	0.35620	0.38390	0.40218	0.41144	0.41437
1.6	0.36726	0.37906	0.38138	0.37630	0.36731
1.8	0.36004	0.35674	0.34655	0.33195	0.31610
2.0	0.33694	0.32393	0.30578	0.28627	0.26765
2.2	0.30684	0.28661	0.26464	0.24372	0.22500
2.4	0.27275	0.24910	0.22641	0.20623	0.18887
2.6	0.23822	0.21404	0.19255	0.17427	0.15890
2.8	0.20557	0.18274	0.16347	0.14752	0.13427
3.0	0.17606	0.15562	0.13894	0.12533	0.11409
3.25	0.14437	0.12741	0.11389	0.10294	0.09389
3.50	0.11837	0.10476	0.09401	0.08529	0.07804
3.75	0.09741	0.08672	0.07825	0.07132	0.06550
4.00	0.08067	0.07235	0.06569	0.06017	0.05549
4.25	0.06733	0.06087	0.05562	0.05121	0.04743
4.50	0.05667	0.05164	0.04749	0.04394	0.04086
4.75	0.04811	0.04418	0.04087	0.03800	0.03547
5.00	0.04118	0.03809	0.03543	0.03309	0.03100
5.25	0.03553	0.03308	0.03093	0.02901	0.02727
5.50	0.03089	0.02893	0.02718	0.02559	0.02413
5.75	0.02704	0.02546	0.02402	0.02269	0.02146
6.00	0.02382	0.02254	0.02135	0.02023	0.01919
6.5	0.01901	0.01795	0.01711	0.01631	0.01555
7.0	0.01525	0.01456	0.01396	0.01337	0.01280
7.5	0.01249	0.01200	0.01156	0.01112	0.01068
8.0	0.01038	0.01003	0.00970	0.00936	0.00902
8.5	0.00874	0.00849	0.00824	0.00797	0.00771
9.0	0.00745	0.00726	0.00706	0.00685	0.00664
9.5	0.00641	0.00627	0.00611	0.00594	0.00577
10.0	0.00556	0.00545	0.00533	0.00519	0.00505

<sup>11</sup> J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952), p. 784.

<sup>a</sup> The Holtzmark distribution, except for  $\beta$  from 2.0 through 6.0, was taken from S. Chandrasekhar, *Revs. Modern Phys.* **15**, 73 (1943).

is small compared to the original Holtsmark term, for those values of  $x$  which do not make  $F(x)$  very small. Figure 1 shows this to be the case for values of  $r_0/\lambda$  ranging from 0 to approximately unity. This is also the range of  $r_0/\lambda$  for which we can expect the Debye-Hückel theory to give a reasonable approximation to the pair correlation function.

The final Fourier transformation (29) was performed by numerical integration. Table I lists the field distribution for several values of  $r_0/\lambda$  including the Holtsmark case,  $r_0/\lambda=0$ . Temperatures and densities in most laboratory experiments and many astrophysical problems fall in the range of  $r_0/\lambda$  for which the curves were calculated. The main feature of the results is the shift to smaller fields of the peak of the distribution as

$r_0/\lambda$  increases. For large values of  $\beta$  not included in Table I,  $H(\beta)$  is well represented by the following asymptotic expansion which consists of the asymptotic expansion for the Holtsmark distribution,  $H_0$ , minus a correction for the pair correlations,

$$H(\beta) \approx H_0 - (r_0/\lambda)\beta^{-7/2}[0.5453(r_0/\lambda) + 11.78\beta^{-1} + 114.6\beta^{-5/2} + \dots], \quad (34)$$

where

$$H_0 \approx 1.496\beta^{-5/2} + 7.639\beta^{-4} + 21.60\beta^{-11/2} + \dots \quad (35)$$

In another paper we shall apply this method to several other field distribution problems, in particular that of the distribution of the low-frequency component, and compare our results with those of other workers.