terms of the physical polarization parameters can be obtained only when the transformation matrix V is known.

5. CONCLUDING REMARKS

The explicit considerations of the previous sections thus show that it is indeed possible to extend the usual methods of polarization parameters to the study of the dynamics of higher spin ensembles. In the present paper we have deliberately restricted ourselves to the case of spin 1, since the algebraic methods developed here become quite tedious for higher spins, though no difficulty of principle remains. Some economy is achieved by the use of Clebsch-Gordan coefficients. But the most powerful approach to the problem of constructing invariants is in terms of group characters and the general case will be so treated elsewhere. Needless to say, such a discussion of the polarization dynamics is of application in scattering problems also.

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Interference Effects in Leptonic Decays*

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It is proven that in any leptonic decay experiment in which the lepton masses and charges may be neglected, and in which no pseudoscalar correlations are measured, all $V\cdot A$ interference terms will be antisymmetric under exchange of the two leptons, while the pure V and A terms will be symmetric. If the experiment measures a pseudoscalar correlation, these conclusions are reversed. Even if the lepton masses cannot be ignored (e.g., for $\Lambda^0 \rightarrow \mu^- + \bar{\nu} + p$, or low-energy β decay) it is still true that no $V\cdot A$ interference may appear when scalars are measured, and only $V\cdot A$ interference may contribute when pseudoscalars are measured, providing that the lepton spins and momenta are not directly observed. Thus experiments can be devised that involve no interference effects, or only interference effects. This theorem holds independently of the strangeness change, spin change, energy transfer, or of any particular assumptions about the form of the V and A currents. It proves most useful when it is difficult or tedious to calculate transition rates directly. Applications are discussed, including possible tests of the Feynman–Gell-Mann theory in nonunique forbidden β decay, of the nature of the leptonic Λ^0 and K^0 decay interaction, and of the charge symmetry properties of weak interactions.

I. INTRODUCTION

IT is generally supposed that leptonic decay processes proceed through vector and axial vector interaction forms, so that the rate for any given process will, in general, involve positive terms from V alone and A alone, plus an interference between V and A. It is the purpose of this note to point out that the interference terms are always different qualitatively in their dependence on lepton variables from the "pure" terms, and that experiments can always be devised to measure the pure terms only, with interference terms vanishing, or to measure the interference terms only. Our results are independent of any multipole expansion, and prove most useful when such expansions cannot easily be used to calculate the transition probability directly.

II. THE INTERFERENCE THEOREM

Let us consider a process $\alpha \rightarrow \beta + l + \bar{\nu}$, where α and β are states of the strongly interacting particles, and $l=\nu$, or e^-, or μ^-. We make no assumption about the spin or strangeness change in the transition $\alpha \rightarrow \beta$. We will consistently neglect the charge of l, but may include other electromagnetic effects. The various correlations that may be measured among the momenta and polarizations of α, β, l, $\bar{\nu}$, may be closed as either scalar (if the experiment is arranged to be space-inversion invariant) or pseudoscalar (e.g., a $\boldsymbol{\sigma}\cdot\mathbf{p}$ correlation). The total decay rate is of course a scalar.

The interaction Hamiltonian for such decays is given by

$$\begin{aligned}\mathcal{H} &= J_\lambda^{(V)}\bar{\psi}_l\gamma^\lambda\psi_\nu + J_\lambda^{(V')}\bar{\psi}_l\gamma^\lambda\gamma_5\psi_\nu + J_\lambda^{(A)}\bar{\psi}_l(i\gamma_5\gamma^\lambda)\psi_\nu \\ &\quad + J_\lambda^{(A')}\bar{\psi}_l(i\gamma_5\gamma^\lambda)\gamma_5\psi_\nu + \text{H.c.} \\ &= (J_\lambda^{(V)} - iJ_\lambda^{(A')})\bar{\psi}_l\gamma^\lambda\psi_\nu \\ &\quad + (J_\lambda^{(A)} + iJ_\lambda^{(V')})\bar{\psi}_l i\gamma_5\gamma^\lambda\psi_\nu + \text{H.c.} \quad (1)\end{aligned}$$

The operators $J_\lambda^{(V,A)}$ and $J_\lambda^{(V',A')}$ are "currents" involving strongly interacting fields. They are defined to be vectors $(J_\lambda^{(V)}, J_\lambda^{(V')})$ and axial vectors $(J_\lambda^{(A)}, J_\lambda^{(A')})$ but we will not need to make any assumptions about their form. Our theorem has two parts:

(A) Suppose that the mass of l may be neglected, so

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that the energy transfer must be $\gg m_l$. If a scalar correlation is measured, then the interference terms between vector and axial vector currents will be antisymmetric under interchange of l and neutrino, while the "pure" terms will be symmetric. Thus, it we measure any quantity symmetric under $l \leftrightarrow \bar{\nu}$ [such as the total transition rate, or *average* $l-\bar{\nu}$ angular correlation, or $\langle (E_l - E_\nu)^2 \rangle$] there will appear *no* interference between V and A or V' and A', while if we measure antisymmetric quantities (such as $\langle E_l - E_\nu \rangle$) we get contributions *only* from the interference terms. On the other hand, if a pseudoscalar correlation is measured, then it is the vector-axial vector interference terms that are symmetric functions of lepton spins and momenta, and the "pure" terms that are antisymmetric. As a trivial corollary, if we sum over all lepton polarizations and momenta, scalar (pseudoscalar) correlations will involve no (only) vector-axial vector interference, since only terms symmetric under $l \leftrightarrow \bar{\nu}$ will contribute.

(B) If we do not neglect m_l (we cannot if l is a μ meson, or if the energy transfer is small and l is an electron), then these rules don't apply. *However, in experiments in which no lepton spins or momenta are directly observed, the above corollary is still correct;* scalar correlations involve no vector-axial vector interference, while pseudoscalar correlations involve interference terms only. Part (B) is applicable in a measurement of the total decay rate, or of any correlations between the spins and momenta of α and β.

Proof

(A) If we neglect the mass and charge of l, the free Hamiltonian is invariant under the canonical transformation

$$\psi_l \to \psi_\nu^c, \quad \psi_\nu \to \psi_l^c. \tag{2}$$

Under this transformation, we have

$$\begin{aligned} \bar{\psi}_l \gamma_\lambda \psi_\nu &\to -\bar{\psi}_l \gamma_\lambda \psi_\nu, \\ \bar{\psi}_l \gamma_5 \gamma_\lambda \psi_\nu &\to \bar{\psi}_l \gamma_5 \gamma_\lambda \psi_\nu. \end{aligned} \tag{3}$$

Thus we can make the entire Hamiltonian invariant by extending (2) formally, to include

$$\begin{aligned} J_\lambda^{(V)} &\to -J_\lambda^{(V)}, \quad J_\lambda^{(V')} \to J_\lambda^{(V')}, \\ J_\lambda^{(A)} &\to J_\lambda^{(A)}, \quad\;\;\; J_\lambda^{(A')} \to -J_\lambda^{(A')}. \end{aligned} \tag{4}$$

Therefore the pure terms (V^2, V'^2, A^2, A'^2) and the $V \cdot A'$, $V' \cdot A$ interference must be symmetric under $l \leftrightarrow \bar{\nu}$, while the $V \cdot V'$, $A \cdot A'$, and $V' \cdot A'$ terms must be antisymmetric. If a scalar is measured, the $V \cdot V'$, $V \cdot A'$, $A \cdot V'$, $A' \cdot A$ terms drop out; if a pseudoscalar correlation is measured these are the only terms that don't drop out.

(B) For a transition $\alpha \to \beta$ (where we may measure the momenta and polarizations of α, β, but must sum over the momenta and polarization of l and $\bar{\nu}$) the decay rate will contain terms quadratic in $J_\lambda^{(V)} - iJ_\lambda^{(A')}$ and in $J_\lambda^{(A)} + iJ_\lambda^{(V')}$, plus an interference term given by

$$\begin{aligned} \omega_{\alpha\to\beta}^{(\text{int.})} = 4\pi \int \frac{d^3p_l d^3p_\nu}{E_l E_\nu} \delta^4(p_l + p_\nu + p_\beta - p_\alpha) \epsilon^{\lambda\eta\rho\sigma} \\ \times (p_l)_\rho (p_\nu)_\sigma \operatorname{Re}\{\langle \beta | J_\lambda^{(V)} - iJ_\lambda^{(A')} | \alpha \rangle \\ \times \langle \beta | J_\eta^{(A)} + iJ_\eta^{(V')} | \alpha \rangle^* \}, \end{aligned} \tag{5}$$

where

$$p_x = [\mathbf{p}_x, E_x = (m_x^2 + |\mathbf{p}_x|^2)^{\frac{1}{2}}].$$

Now let us make a nonlinear change of variables, given by

$$\begin{aligned} p_l' &= (1+\zeta) p_\nu + \zeta p_l, \\ p_\nu' &= (1-\zeta) p_l - \zeta p_\nu, \\ \zeta &= -m_l^2 / (p_l + p_\nu)^2. \end{aligned} \tag{6}$$

[In the limit $m_l = 0$, (6) is the same as (2).] This transformation leaves l and ν on their respective mass shells, i.e., $p_l'^2 = -m_l^2$, $p_\nu'^2 = 0$, and has the properties

$$p_l' + p_\nu' = p_l + p_\nu, \tag{7}$$

$$\epsilon^{\lambda\eta\rho\sigma} (p_l')_\rho (p_\nu')_\sigma = -\epsilon^{\lambda\eta\rho\sigma} (p_l)_\rho (p_\nu)_\sigma, \tag{8}$$

$$\frac{d^3p_l' d^3p_\nu'}{E_l' E_\nu'} = \frac{d^3p_l d^3p_\nu}{E_l E_\nu}. \tag{9}$$

Therefore, making the substitution $p \to p'$ in (5), we get the same integral, with a minus sign from (8); thus $\omega_{\alpha\to\beta}^{(\text{int.})} = 0$. Therefore we can have no interference between $J_\lambda^{(V)} - iJ_\lambda^{(A')}$ and $J_\eta^{(A)} + iJ_\eta^{(V')}$ in such an experiment. If a scalar correlation is measured, the only terms that can enter are V^2, V'^2, A^2, A'^2, while for a pseudoscalar correlation the only terms are $V \cdot A'$, $V' \cdot A$.

As an example, let us consider the second forbidden corrections to allowed β decay.[1] One of the V-type matrix elements $\int \boldsymbol{\alpha} \times \mathbf{R}$, may interfere with the main A-type matrix element, $\int \boldsymbol{\sigma}$. This interference makes a contribution to the electron energy spectrum proportional to

$$\begin{aligned} \omega^{(V \cdot A)}(E_e) dE_e \\ \sim p_e E_e (W_0 - E_e)^2 \left(E_e - \frac{W_0}{2} - \frac{m_e^2}{2E_e} \right) dE_e, \end{aligned} \tag{10}$$

where $E_e = (p_e^2 + m_e^2)^{\frac{1}{2}}$. (We neglect Coulomb corrections.) Part (A) is obviously correct here, since if we neglect m_e we have $p_e = E_e$, and

$$\omega^{(V \cdot A)}(E_e) dE_e \sim E_e^2 (W_0 - E_e)^2 (E_e - W_0/2) dE_e, \tag{11}$$

and this is an antisymmetric function of E_e and $W_0 - E_e = E_\nu$. Part (B) is also correct; we may show by direct calculation that even if $m_e \neq 0$,

$$\int_{m_e}^{W_0} \omega^{(V \cdot A)}(E_e) dE_e = 0. \tag{12}$$

[1] See, e.g., M. Gell-Mann, Phys. Rev. **111**, 362 (1958).

This is not at all obvious upon inspection of (10). Similar examples may be found in published formulas for first forbidden transitions.[2]

It is easy to extend this theorem to a general S, V, T, A, P mixture; the results are given in Table I. (Part B holds even if $m_\nu \neq 0$.)

III. APPLICATION—THE FEYNMAN–GELL-MANN THEORY

Let us consider a "nonunique" forbidden β decay, in which either $\Delta\Pi = (-)^{\Delta J}$ (i.e., $|\Delta J| = 1$ *yes*, $|\Delta J| = 2$ *no*, etc.), or $\Delta J = 0$, *yes* (not $0 \to 0$). We assume the 2-component neutrino theory, so that $J_\lambda^{(V)} = J_\lambda^{(V')}$, $J_\lambda^{(A)} = J_\lambda^{(A')}$. The dominant matrix elements will be, from the vector interaction,[3]

$$\mathfrak{M}_L{}^M(\boldsymbol{\alpha}) = \left\langle \beta \left| \int d^3\mathbf{R}\, \mathbf{J}^{(V)}(\mathbf{R}) \cdot \mathbf{Y}_{L,L-1}{}^M(\mathbf{R}) R^{L-1} \right| \alpha \right\rangle, \quad (13)$$

$$\mathfrak{M}_L{}^M(1) = \left\langle \beta \left| \int d^3\mathbf{R}\, J_0{}^{(V)}(\mathbf{R}) Y_L{}^M(\mathbf{R}) R^L \right| \alpha \right\rangle, \quad (14)$$

and from the axial vector interaction

$$\mathfrak{M}_{L,L}{}^M(\boldsymbol{\sigma}) = \left\langle \beta \left| \int d^3\mathbf{R}\, \mathbf{J}^{(A)}(\mathbf{R}) \cdot \mathbf{Y}_{L,L}{}^M(\mathbf{R}) R^L \right| \alpha \right\rangle, \quad (15)$$

$$\mathfrak{M}_{L+1,L}{}^M(\boldsymbol{\sigma}) = \left\langle \beta \left| \int d^3\mathbf{R}\, \mathbf{J}^{(A)}(\mathbf{R}) \cdot \mathbf{Y}_{L+1,L}{}^M(\mathbf{R}) R^L \right| \alpha \right\rangle, \quad (16)$$

where the order of forbiddenness is $L = |\Delta J|$. (If $\Delta J = 0$, take $L = 1$; there are then additional A-type matrix elements.) For example, if $L = 1$ these are respectively proportional to $\int \boldsymbol{\alpha}$, $\int 1$, $\int \boldsymbol{\sigma} \times \mathbf{R}$, and $\int (\sigma_i R_j + \sigma_j R_i - \frac{2}{3}\delta_{ij}\boldsymbol{\sigma}\cdot\mathbf{R})$. The last term, (16), does not appear if $J_\alpha + J_\beta < L + 1$.

TABLE I. Summary of the interference theorem in the general case: Here S, V, T, A, P and S', V', T', A', P' refer to parity-conserving and parity-nonconserving currents respectively. Cross terms marked "S" or "A" or "0" are respectively symmetric or antisymmetric under lepton exchange, or zero, providing lepton masses may be neglected. Even if lepton masses are not neglected, cross terms marked "A" cannot contribute if no lepton properties are observed *directly*. If only scalars are measured, there can be no interference between primed and unprimed coupling; if only pseudoscalars are measured, such terms are the only ones present.

	S, P'	V, A'	T, T'	A, V'	P, S'
P, S'	S	0	A	0	S
A, V'	0	A	0	S	
T, T'	A	0	S		
V, A'	0	S			
S, P'	S				

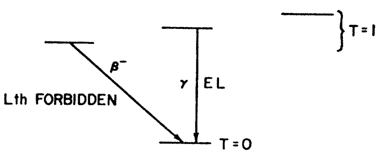

FIG. 1. Proposal for a test of the Feynman–Gell-Mann theory in forbidden β decay. A strict lower limit (19) may be placed on the ratio of the β- and γ-decay rates.

The theory of the vector interaction suggested by Feynman and Gell-Mann[4] states that the vector current is divergenceless, i.e., $\nabla \cdot \mathbf{J}^{(V)} = \dot{J}_0{}^{(V)}$, and equal, up to an isospin rotation and a constant C_V/e, to the isovector part of the electric current. If $J_\lambda{}^{(V)}$ is divergenceless, a relation is implied between $\mathfrak{M}_L{}^M(\boldsymbol{\alpha})$ and $\mathfrak{M}_L{}^M(1)$, namely

$$\mathfrak{M}_L{}^M(\boldsymbol{\alpha}) = \frac{iW_0}{[L(2L+1)]^{\frac{1}{2}}} \mathfrak{M}_L{}^M(1), \quad (17)$$

where $W_0 = m_\alpha - m_\beta$. If, furthermore, $J_\lambda{}^{(V)}$ is related to the electric current, $\mathfrak{M}_L{}^M(1)$ is analogously related to the electric 2^L-pole moment for the analogous γ transition. The γ transition is parity favored, and therefore pure electric, so that the magnitude of the EL moment, and thus of $\mathfrak{M}_L{}^M(1)$ and $\mathfrak{M}_L{}^M(\boldsymbol{\alpha})$, is given by the decay rate for the γ transition. For example, if the nuclear level scheme is as shown in Fig. 1, then the γ-decay rate is given by

$$T(EL) = \frac{8\pi\omega^{2L+1}}{(2L+1)!!^2} \frac{L+1}{L} \frac{1}{137|\sqrt{2}C_V|^2} \frac{1}{2J_\alpha+1} \times \sum_{M, M_\alpha, M_\beta} |\mathfrak{M}_L{}^M(1)|^2, \quad (18)$$

where ω is the photon energy.

So far, we have said nothing about the A-type matrix elements (15) and (16). However, by part (B) of our theorem, the total β-decay rate is given by a positive term involving $\mathfrak{M}_L{}^M(1)$ and $\mathfrak{M}_L{}^M(\boldsymbol{\alpha})$, plus a positive term involving $\mathfrak{M}_{L,L}{}^M(\boldsymbol{\sigma})$ and $\mathfrak{M}_{L+1,L}{}^M(\boldsymbol{\sigma})$, *with no cross term.* Thus if we measure the γ-decay rate $T(EL)$, and use (18) and (17), we obtain a strict lower bound on the β-decay rate $T(\beta)$, independent of any nuclear model. Neglecting Coulomb effects, we obtain for the decay scheme of Fig. 1,

$$T(\beta)/T(EL) \geq \frac{137}{48\omega^{2L+1}} \frac{\ln 2}{(ft)_0 {}^{14}} \int_1^{W_0{}^2} (W_0{}^2 - X)^{L-\frac{1}{2}} \times \left(W_0{}^2 - \frac{LX}{L+1} \right) \frac{(X-1)^2}{X^2} (2X+1) dX. \quad (19)$$

[2] See, e.g., M. Morita and R. S. Morita, Phys. Rev. **109**, 2048 (1958).

[3] For the definition of the vector spherical harmonics $\mathbf{Y}_{L,l}{}^M(\mathbf{R})$, see J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physcis* (John Wiley and Sons, New York, 1952), Appendix A.

[4] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

Nuclei for which this prediction might be tested include Cl^{38}, K^{42} ($\Delta J=0$ *yes*), and Na^{24} and Cl^{36} ($|\Delta J|=2$ *no*). The greatest experimental difficulty comes in measuring the γ lifetime. (It may be possible in the case $|\Delta J|=0$ or 1, *yes*, to measure $T(E1)$ by an (n,γ) or (p,γ) resonance width measurement. It should be fairly easy to pick out the $T=1$ resonance, since all $T=0\rightarrow T=0$ $E1$ widths are smaller by several orders of magnitude than a $T=1\rightarrow T=0$ $E1$ width.)

If relation (19) is violated in experiment, the Feynman–Gell-Mann theory will be disproven. On the other hand, if it is verified, this will be only weak evidence for their theory. This is not only because we don't known the axial-vector matrix elements, but also because it is not clear whether we would expect relation (18) to hold approximately in any case. Indeed, if we neglect meson effects in β decay and γ decay, it is possible to derive identical formulas for the electric multipole operators[5] and $\mathfrak{M}_L{}^M(1)$. Also, the relation (17) was suggested[6] before there was any suspicion that $J_\lambda{}^{(V)}$ was rigorously divergenceless. Thus our suggested experiment probably falls in the same class, as a test of the Feynman–Gell-Mann theory, with measurements of the ft values of $0^+\rightarrow 0^+$ superallowed transitions, or of the Fermi term in $J\rightarrow J$ *no* transitions between different isospin multiplets.

IV. OTHER APPLICATIONS

(a) The decay process $\Lambda^0\rightarrow p+e^-+\bar{\nu}$ seems anomalously slow. Of course one may try to explain this as an "accident," a damping due to strong interactions. According to part (B), however, this damping must take place *for both V and A interactions separately*, since no $V\cdot A$ interference term appears in the decay rate.

Actually, we have as yet no evidence that both V and A are present; the existence of K_{e3} decay proves that either *V or A* strangeness violating interactions occur, but since K_{e2} decay has not been observed, we cannot be sure that both are present. A clear test would be to measure the average value of (E_e-E_ν) for the Λ^0 decay. Part (A) tells us that $\langle(E_e-E_\nu)^{2n+1}\rangle$ is proportional to the $V\cdot A$ interference, and therefore vanishes unless V and A are both present. If not zero, the magnitude of $\langle E_e-E_\nu\rangle$ would give information on the relative strength of V and A.

(b) If we make the simplest assumptions about the charge-symmetry properties of the strangeness-conserving parts of the currents J_λ, then the transition rates for a process $\alpha\rightarrow\beta+l^-+\bar{\nu}$ and the charge-symmetric process $\tilde{\alpha}\rightarrow\tilde{\beta}+l^++\nu$ (e.g., Σ^+, $\Sigma^-\rightarrow\Lambda^0$, or B^{12}, $N^{12}\rightarrow C^{12}$, or Li^8, $B^8\rightarrow Be^8$) should be equal, except for a change of sign in the $V\cdot A$ interference term. Since there is never any $V\cdot A$ interference in the *total* transition rates, these should be entirely equal. This might serve as one test of the charge-symmetry properties of weak interactions.[7]

(c) The $K_{e3}{}^0$ decay mode proceeds through a V interaction or an A interaction (the difference being one of convention) but not through both. Therefore, by part (A), the differential transition probability (with no pseudoscalar correlations measured) must be totally symmetric under interchange of electron and neutrino spins and momenta. In particular, we must have $\langle E_e-E_\nu\rangle=0$. It would be very surprising if this prediction were not fulfilled; the only reasonable explanation that could then be offered would be the simultaneous presence of S and T, or of P and T, interaction forms.

(d) We can apply our results to μ-meson decay, taking $\alpha=\mu$, $\beta=e$, $l=\nu$, and considering the electromagnetic interaction between μ and e as "strong." In fact this is the only case where the conditions for part (A) are met exactly. The most important experimental parameters[8] are those giving the decay rate $(1/\tau)$ the spectrum shape (ρ) and the $\boldsymbol{\sigma}_\mu\cdot\mathbf{p}_e$ correlation (α,ζ), none of which involve observation of the neutrinos. Since $1/\tau$ and ρ are "scalars," they can involve no $V\cdot A$ interference, while α and ζ, which describe a pseudoscalar correlation, involve *only* $V\cdot A$ interference. This result holds to all orders in the electron charge and mass, and may be checked against the lowest order calculations.

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[5] A. J. F. Siegert, Phys. Rev. **52**, 787 (1937).

[6] M. Yamada, Progr. Theoret. Phys. (Kyoto) **9**, 268 (1958).

[7] S. Weinberg, Phys. Rev. **112**, 1375 (1958). Part (A) of the theorem is mentioned in this reference, but of course it is part (B) that is less obvious, and is needed for application to β decay.

[8] See, e.g., Larsen, Lubkin, and Tausner, Phys. Rev. **107**, 856 (1957).