estimate of possible systematic effects) for the decay energy of tritium and the Jones[17] half-life of 12.262 ±0.004 years, the *ft* value[18] of H^3 is 1122±20 sec. Popov *et al.*[13] report a longer lifetime 12.58±0.18 years and give an average (which includes the value of Jenks *et al.*[14] as well as that of Jones[17]) of 12.43±0.04 years. This average along with 18.6±0.1 kev gives $ft=1137\pm20$ sec.

Some recent comments on the problem of the ratio of Gamow-Teller to Fermi coupling constants were made by Kistner and Rustad[19] at the Gatlinburg Conference on Weak Interactions. Using $E_0(H^3)=18.6$ kev from the mass difference work, and magnetic moment data to correct $|\int\boldsymbol{\sigma}|^2$, they pointed out that H^3, O^{15}, and F^{17} are consistent with $C_{GT}^2/C_F^2=1.16\pm0.05$ while the neutron data (along with the average of $0\rightarrow0$ transitions) point to a higher value (1.4) for this ratio. However, if the uncorrected single-particle value for $|\int\boldsymbol{\sigma}|^2$ is used for H^3, then tritium and the neutron are consistent with $C_{GT}^2/C_F^2\simeq1.45$.

[17] W. M. Jones, Phys. Rev. **100**, 124 (1955).
[18] E. Feenberg and G. Trigg, Revs. Modern Phys. 22, 399 (1950), Eqs. (A6) and (A7) for *f*.
[19] O. C. Kistner and B. M. Rustad, Bull. Am. Phys. Soc. Ser. II, **4**, 79 (1958); and private communication of work to be published.

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Magnetic Moments of Strongly Deformed Odd-Odd Nuclei*

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The strong-coupling magnetic moment relation for odd-odd nuclei is evaluated with the use of Nilsson wave functions for finite values of core distortion. The theoretical predictions are compared with all of the data available on odd-odd nuclei in the strong-coupling region. There is no evidence for appreciable error having been introduced by the assumption of no interaction between the odd proton and odd neutron.

I. INTRODUCTION

THE magnetic moments of odd-odd nuclei have been calculated using a variety of approaches. The simplest model to meet with some success is the *j-j* coupling scheme wherein one assumes that the magnetic moment arises from an odd proton and odd neutron, each in its appropriate shell-model state.[1] An improvement in agreement of theory with experiment[2] was obtained when Schwartz[3] suggested that, still using the *j-j* coupling scheme, the *g*-factors for the odd proton and neutron states could be obtained empirically from nearby odd-even nuclei where the shell-model states being occupied are presumably the same as in the odd-odd nucleus under consideration. Caine[4] has shown that this semiempirical approach is successful because it takes into account a large part of the effects resulting from configuration mixing.

The work described above is based on the assumption that the particles are in a spherically symmetrical potential well. Calculations of the magnetic moments of distorted core, odd-odd nuclei were performed first by Bohr and Mottelson.[5] The odd proton and odd neutron were assumed to be coupled strongly to an ellipsoidal core, but both nucleons were considered to be in pure *j*-states. The effects of *j*-mixing arising from the noncentral potential were considered qualitatively. Recently, Gallagher and Moszkowski[6] have calculated moments of odd-odd nuclei using the "asymptotic wave functions" obtained by Nilsson.[7] These are wave functions for particles so tightly coupled to the distorted core that the effects of spin-orbit coupling are insignificant. Thus these calculations represent the opposite extreme from those based on the spherical-core shell model.

Nilsson, however, has solved exactly a Hamiltonian which includes both the spin-orbit and particle-core couplings, so that for each value of the core distortion parameter one is given a level order and corresponding

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[1] E. Feenberg, Phys. Rev. **76**, 1275 (1949).
[2] R. J. Blin-Stoyle, *Theories of Nuclear Moments* (Oxford University Press, London, 1957), p. 68.
[3] H. M. Schwartz, Phys. Rev. **89**, 1293 (1953).
[4] C. A. Caine, Proc. Phys. Soc. (London) **A64**, 999 (1956).
[5] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **27**, No. 16 (1953).
[6] C. J. Gallagher, Jr., and S. A. Moszkowski, Phys. Rev. **111**, 1282 (1958).
[7] S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **29**, No. 16 (1955).

wave functions. In the case of no core distortion the energy levels and wave functions reduce to those of the shell model, while for very large core distortions one obtains the asymptotic eigenvalues and eigenfunctions. In the present article we shall use these intermediate-coupling results of Nilsson to calculate the magnetic moments for all of the nuclei in or very near the strong-coupling region ($150<A<190$) with experimentally determined magnetic moments.

II. STRONG-COUPLING MAGNETIC MOMENT RELATION FOR ODD-ODD NUCLEI

In order to use the Nilsson wave functions most directly for odd-odd nuclei, we shall assume that the two extra-core particles interact much more strongly with the nuclear core than with each other. Should the moments calculated on this assumption differ from the experimental values by more than the corresponding calculations for odd-even nuclei, we might conclude that the odd-proton and odd-neutron wave functions are strongly correlated.

To investigate this possibility we have used the Nilsson "intermediate" wave functions along with the Nilsson-Mottelson energy level diagrams[7,8] to calculate the magnetic moments of several odd-even nuclei in the strong-coupling region. We have obtained results similar to those of Gottfried[9] who has calculated odd-even nuclear moments using energy levels and wave functions which he obtained by solving a Hamiltonian with a different particle-core potential from that employed by Nilsson. By "similar" we mean that the theoretical predictions for the odd-even nuclei result in disagreements with experiment which, as in Gottfried's calculations, average about one fourth of a nuclear magneton for odd-neutron nuclei and about one half a nuclear magneton for odd-proton nuclei. Thus, we expect discrepancies of at least this size in the odd-odd moment calculations, since there has been no refinement of the theory in the consideration of the even-A nuclei, even though a possibly significant new interaction (odd-proton—odd-neutron) has arisen.

To obtain the strong-coupling value of the magnetic moment, μ, for odd-odd nuclei, we write

$$\mu=\langle\boldsymbol{\mu}\cdot\mathbf{I}\rangle/(I+1), \quad (M_I=I) \tag{1}$$

where $\boldsymbol{\mu}$ is the magnetic moment operator and I is the nuclear spin. The magnetic moment operator in Eq. (1) is given by

$$\boldsymbol{\mu}=g_{sp}\mathbf{s}_p+g_{lp}\mathbf{l}_p+g_{sn}\mathbf{s}_n+g_{ln}\mathbf{l}_n+g_R\mathbf{R}, \tag{2}$$

where $\mathbf{s}$, $\mathbf{l}$, and $\mathbf{R}$ represent intrinsic spin, orbital, and core rotational angular momenta, respectively; the subscripts p and n refer to proton and neutron; and the g's represent the appropriate g factors for the various angular momenta.

[8] B. R. Mottelson and S. G. Nilsson, Phys. Rev. **99**, 1615 (1955).
[9] K. Gottfried, Phys. Rev. **103**, 1017 (1956).

The total angular momentum of the nucleus is given by

$$\mathbf{I}=\mathbf{s}_p+\mathbf{s}_n+\mathbf{l}_p+\mathbf{l}_n+\mathbf{R}. \tag{3}$$

In obtaining an expression for μ we see from inspection of Eqs. (1), (2), and (3) that we must obtain expectation values of $\mathbf{l}_p\cdot\mathbf{I}$, $\mathbf{l}_n\cdot\mathbf{I}$, $\mathbf{s}_p\cdot\mathbf{I}$, and $\mathbf{s}_n\cdot\mathbf{I}$. These quantities can be obtained from pages 27 and 28 of reference 7. These expressions derived by Nilsson have complicating terms for the case $\Omega=K=\frac{1}{2}$,[10] but since we are considering here only even-A nuclei, K can never equal $\frac{1}{2}$ and these additional terms will not arise even though the Ω values of the individual neutron or proton states may equal $\frac{1}{2}$. Combining Eqs. (1), (2), and (3) and assuming that $I=\Omega=K$ for the ground state, we obtain

$$\mu=[I/(I+1)][g_{sp}\langle s_{p3}\rangle+g_{sn}\langle s_{n3}\rangle+g_l\langle l_{p3}\rangle+g_R]. \tag{4}$$

The subscripts, 3, refer to the components of angular momenta along the core axis of symmetry. If there were no spin-orbit coupling, then the spin and orbital angular momenta components along the symmetry axis would be constants of motion and the expectation values in Eq. (4) would be replaced by the appropriate eigenvalues, Λ and Σ, of the Nilsson asymptotic wave functions. In this case Eq. (4) reduces to the relation used by Gallagher and Moszkowski for odd-odd nuclei.

In the following we shall make estimates of the core distortion for several nuclei and use the Nilsson wave functions and energy level diagrams corresponding to this value of the distortion to compute the strong-coupling magnetic moment given by Eq. (4). Estimates of the core distortion parameter, δ,[7] are based on the work reported in reference 8. The numerical values of the g factors used in this work are the standard "unquenched" single-particle values

$$g_{sp}=+5.585, \quad g_{sn}=-3.826,$$

together with

$$g_{lp}=1, \quad g_{ln}=0, \quad g_R=0.40.$$

III. NUMERICAL RESULTS

There are comparatively few experimental data available on the odd-odd nuclear magnetic moments in the strong-coupling region. We shall discuss here the six nuclei in this region about which there is some experimental information.

Eu^{152} and Eu^{154}

One of the most significant achievements of the collective model in connection with nuclear magnetic moments is its natural explanation of the large difference

[10] The symbols Ω and K represent respectively the projections of the particle angular momentum and the total angular momentum onto the core axis of symmetry. For an odd-odd nucleus $\Omega=|\Omega_p\pm\Omega_n|$, and we expect for the ground state that $I=\Omega=K$. See, for example, references 5 and 7.

between the magnetic moments of the two spin-$\frac{5}{2}$ isotopes of europium[9] ($\mu_{151}=+3.4$ nm; $\mu_{153}=+1.5$ nm). The observed large difference in the distortion of these nuclei together with the Nilsson or Gottfried energy level scheme indicates that the odd proton is in a different state in these two nuclei. Furthermore, the collective model predicts the occurrence of this sharp change in nuclear distortion in the region of neutron number 88.[8]

Recently Abraham, Kedzie, and Jeffries[11] have measured the spins and magnetic moments of Eu^{152} and Eu^{154} by the paramagnetic resonance method. These nuclei differ from the ones discussed in the preceding paragraph only by the addition of one neutron.

If we assume the core deformation of the heavier isotope, Eu^{154}, to equal that of the relatively non-spherical core of Eu^{153},[8] we see from the Nilsson level scheme that the odd neutron ($N=91$) should have $\Omega_n=\frac{3}{2}$ or $\frac{5}{2}$ depending on which of three close-together energy levels (Nilsson levels 52, 55, and 44) is being filled. The level diagrams indicate that the odd proton should also be in a level with $\Omega_p=\frac{3}{2}$ or $\Omega_p=\frac{5}{2}$. Since we expect that $I=|\Omega_p\pm\Omega n|$[5] for the ground state, Ω_p must equal $\Omega_n=\frac{3}{2}$ in order to be consistent with the observed spin of 3. This means that the proton is not in the same level in Eu^{154} as it is in either of the two spin-$\frac{5}{2}$ isotopes, Eu^{151} and Eu^{153}. Nilsson and Mottelson[8] have pointed out that there is experimental evidence[12] for the existence of this $\Omega_p=\frac{3}{2}$ level very close to the ground state of Eu^{153}, so its occurrence in the ground state of Eu^{154} would not be surprising.

Under the above assumptions, we obtain with the use of the Nilsson wave functions, together with Eq. (4), $\mu_{\text{theor}}=+1.95$ nm which compares favorably with the experimental result of ± 2.1 nm.

Eu^{152} is midway between the two nuclei (Eu^{151} and Eu^{153}) with the previously mentioned large distortion difference, so there is some ambiguity in the core deformation to be assumed. It may be that the addition to Eu^{151} of the neutron in a state whose energy decreases with deformation could tilt the scales in favor of the large distortion increase which is about to occur. We shall, therefore, calculate the magnetic moment for both the large and small distortions.

For the smaller deformation ($\delta=0.16$),[8] the Nilsson levels are such that level 52 (the one assumed in the above Eu^{154} calculation) is again available for occupancy by the odd neutron. If we assume that the proton is still in level 33 as it should be according to the level scheme, we obtain $\mu_{\text{theor}}=+2.33$ nm which agrees relatively well with the observed value of ± 2.0 nm. The prediction of this model is somewhat vague for this case because of the relatively high density of levels in this particular region.

If for Eu^{152} we assume the large deformation associated with Eu^{153} ($\delta=0.30$),[8] we see that a different neutron level is involved (level 57). The resulting theoretical moment value of 2.14 nm is so close to the value corresponding to the smaller distortion that we are unable to gain insight from these calculations concerning the deformation of Eu^{152}.

Lu^{176}

The spin and moments of Lu^{176} have been estimated by Schuler and Gollnow[13] and Klinkenberg[14] on the basis of optical studies of the hyperfine multiplet. Their results are $I\geq 7$ and $\mu=+4.2\pm 0.8$ nm. Estimating the nuclear distortion from the observed large quadrupole moment of around seven barns, we obtain $\delta=0.33$.

The Nilsson energy level diagram indicates that the proton should be in an $\Omega=\frac{7}{2}$ state (level 25), which agrees with the observed spin of $\frac{7}{2}$ for Lu^{175}. The neutron level available is also an $\Omega=\frac{7}{2}$ state (level 41). Thus, the collective model predicts a spin of 0 or 7, the latter value being consistent with the experimental data. The value calculated from Eq. (4) for the Lu^{176} moment is $\mu_{\text{theor}}=+2.93$, which is not in good agreement with the observed $+4.2\pm 0.8$ nm.

This poor agreement with experiment is not surprising in this case, since the use of level 25 in calculating the Lu^{175} moment yields the correct spin but gives a theoretical moment far different from that observed for Lu^{175}. In fact, the disagreement in this case is larger than any other odd-even nucleus we have studied in the strong-coupling region.[15]

We can, however, use a semiempirical approach in this case. Thus, the procedure we shall follow is to solve for $g_{sp}\langle s_{p3}\rangle+\langle l_{p3}\rangle$ using the odd-even strong-coupling moment relation[5]

$$\mu=[I/(I+1)][g_{sp}\langle s_{p3}\rangle+g_{lp}\langle l_{p3}\rangle], \tag{5}$$

together with the observed magnetic moment of Lu^{175} of $+2.9\pm 0.5$ nm. Using this empirically obtained information concerning the proton state together with the neutron state assumed above (level 41), we obtain $\mu_{\text{theor}}=+4.59\pm 0.56$, which is consistent with the observed value of $\mu=+4.2\pm 0.8$. The uncertainty in the theoretical value results from the uncertainty in the observed value of the Lu^{175} magnetic moment. This indicates that both of the errors in the theoretical predictions for Lu^{175} and Lu^{176} arise from the use of the proton wave function associated with level 25.

[11] Abraham, Kedzie, and Jeffries, Phys. Rev. **108**, 58 (1957).

[12] M. R. Lee and R. Katz, Phys. Rev. **93**, 155 (1954); R. L. Graham and J. Walker, Phys. Rev. **94**, 794(A) (1954); N. Martz, Compt. rend. **238**, 2516 (1954).

[13] H. Schuler and H. Gollnow, Z. Physik **113**, 1 (1939).

[14] P. F. A. Klinkenberg, Physica **17**, 715 (1951).

[15] The energy level diagram of Gottfried also predicts the $\frac{7}{2}$ spin for Lu^{176}, but the eigenstate involved is different from the Nilsson level mentioned above. The theoretical value in this case, however, differs by about the same amount from the experimental result as does the value based on the Nilsson level scheme.

TABLE I. Comparison of experimental and calculated magnetic moments.[a] The theoretical values are obtained by using Eq. (4) in conjunction with the Nilsson wave functions corresponding to the assumed value of the deformation parameter. The values used here for this parameter are based on the results given in reference 8. The level numbers refer to the particular Nilsson level assumed.

Nucleus	Assumed deformation	Proton level and Ω-value	Neutron level and Ω-value	μ (theoretical)	μ (observed)
Eu^{152}	0.30	33, $\frac{3}{2}$	57, $\frac{3}{2}$	+2.14	±2.0
	0.16	33, $\frac{3}{2}$	52, $\frac{3}{2}$	+2.33	±2.0
Eu^{154}	0.30	33, $\frac{3}{2}$	52, $\frac{3}{2}$	+1.95	±2.1
Lu^{176}	0.33	25, $\frac{7}{2}$	41, $\frac{7}{2}$	+2.93	+4.2±0.8
	0.33		41, $\frac{7}{2}$	+4.59±0.6[b]	+4.2±0.8
Au^{192}	0.15	42, $\frac{3}{2}$	71, $-\frac{1}{2}$	+0.50	±0.008
Au^{194}	0.12	42, $\frac{3}{2}$	71, $-\frac{1}{2}$	+0.46	±0.07
Au^{198}	0.10	42, $\frac{3}{2}$	71, $\frac{1}{2}$	−0.46	±0.50

[a] W. M. Hooke, Bull. Am. Phys. Soc. 3, 186 (1958).
[b] Semiempirical value. See text.

Au^{192}, Au^{194}, and Au^{198}

Recently, atomic beam magnetic resonance measurements of the spins and moments of Au^{192}, Au^{194}, and Au^{198} have been obtained.[16–19] Studies of nuclear excited states[20] have shown that the strong particle-core coupling decreases sharply in the region of 114 neutrons. The gold nuclei discussed here, therefore, may be described more accurately by an intermediate-coupling model rather than the strong-coupling model assumed here.

The distortions assumed for these nuclei have been estimated by extrapolating the curve given by Mottelson and Nilsson[8] showing nuclear core distortion as a function of atomic number. We have assumed that the proton is an $\Omega=\frac{3}{2}$ level (level 42), since the odd-even gold nuclei, Au^{191}, Au^{193}, Au^{197}, and Au^{199}, are all observed to have spin $\frac{3}{2}$.

The results of applying Eq. (4) are shown in Table I. The theoretical values of the Au^{192} and Au^{194} moments differ from one another by less than 0.1 nuclear magneton as do the observed moments. There is a disagreement in absolute value, however, of around $\frac{1}{2}$ a nuclear magneton, an error typical of odd-even moment calculations mentioned previously. The theoretical value of the Au^{198} magnetic moment is consistent with the observed moment and indicates a negative sign. It would not be surprising, however, if this moment were found to be positive, since this nucleus is probably outside the strong-coupling region.

CONCLUSION

In obtaining the strong-coupling magnetic moment relation for odd-odd nuclei [Eq. (4)], we have assumed that the odd proton and odd neutron are tightly coupled to the core but do not interact with each other. The effect of the spin-orbit interaction for each odd particle has been taken into account by the use of Nilsson wave functions corresponding to finite values of core distortion.

We have evaluated the strong-coupling moment relation for every odd-odd nucleus in the strong coupling region for which the value of the magnetic moment is known. The theoretical values of the moments differ no more from the experimental values than do the corresponding predictions for odd-even nuclei. Thus, in these few cases, we find no evidence that the odd neutron–odd proton interaction is comparable with the odd nucleon–core interaction.

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[16] Ewbank, Marino, Shugart, and Silsbee, Bull. Am. Phys. Soc. Ser. II, 2, 383 (1957).
[17] Reynolds, Christensen, Hooke, Hamilton, Stroke, Ewbank, Nierenberg, Shugart, and Silsbee, Bull. Am. Phys. Soc. Ser. II, 2, 317 (1957).
[18] Hooke, Christensen, Hamilton, Reynolds, and Stroke, Bull. Am. Phys. Soc. Ser. II, 2, 344 (1957).
[19] Christensen, Hamilton, Lemonick, Pipkin, Reynolds, and Stroke, Phys. Rev. **101**, 1389 (1956).
[20] *Proceedings of the University of Pittsburgh Conference on Nuclear Structure, June 6–8, 1957*, edited by S. Meshkov (University of Pittsburgh and Office of Ordnance Research, U. S. Army, 1957), p. 497.