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Single-Particle States of the Neutron from Gross Structure in the Proton Spectra of (d,p) Reactions*

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Systematic gross-structure peaks have been observed in the proton spectra from (d,p) reactions on nuclei in the region of atomic weight 60. Measurements at deuteron energies of 4, 10, and 21.6 Mev were made with the energy resolution in the proton detectors considerably worse than the known spacing of levels in these nuclei. In all these measurements, the peaks in the proton spectra stayed at a fixed energy of the captured neutron. Angular distributions obtained with 10-Mev deuterons were analyzed in terms of the Butler theory of stripping reactions. It was found that each peak in the proton spectrum corresponded to a specific value of the orbital angular momentum of the captured neutron. The peaks are interpreted as being caused by the shell-model single-particle states of the captured neutron, when these states are smeared out among the many actual levels of the nucleus. Thus the observed gross-structure peaks are believed to correspond to the giant resonances of the complex-potential model of Feshbach, Porter, and Weisskopf. Peaks have been assigned to the $1f_{7/2}$, $2p_{3/2}$, $1f_{5/2}$, $2p_{1/2}$, $1g_{9/2}$, $2d_{5/2}$, $3s_{1/2}$, and $2d_{3/2}$ shell-model states, although these states could not all be identified in every nucleus that was studied. The relative intensities of the peaks are also consistent with this interpretation, except for the $l=0$ peaks at high excitation energies. Information on the ground-state configurations of the target nuclei has also been obtained from the intensities.

INTRODUCTION

IN the Bohr model of nuclear reactions[1] it is assumed that the individual particles inside the nucleus interact strongly with an incident nucleon. This means that in a time short compared to the time it would take the incident nucleon to traverse the nucleus it will have been well amalgamated with the nucleus and the resultant compound system will have no recollection of its mode of formation. The probability for forming a particular level of the compound system through a given channel would depend on what fraction of the wave function of this level resembles that of the incident nucleon in a potential corresponding to the ground state of the target nucleus. This probability was expected to be distributed essentially at random among all the complicated levels of the nucleus. This means that the cross section averaged over an energy interval large compared to the spacing between resonance levels will depend on external factors (i.e., barrier penetrabilities) only.[2]

The shell model of the nucleus[3] has been successful in describing low-lying energy levels of nuclei in terms of configurations consisting of individual nucleons moving in a potential representing the rest of the nucleus. This implies that the amalgamated "liquid-drop" picture is not correct for low-lying energy levels.

Barschall's[4] experimental results on average neutron cross sections did not follow the shape predicted from penetrabilities alone. He observed gross structure which was obviously not caused by single resonance levels and which shifted more or less systematically downward in energy as target nuclei with larger and larger radii were studied. Feshbach, Porter, and Weisskopf[5] were the first to offer an explanation for this experimental effect by representing the internucleon interactions by an absorptive, or imaginary, part in an average potential. This implies that the cross section is concentrated in the region of energy at which the incident nucleon would have had an eigenstate in the potential of the target nucleus. Since this eigenstate is no longer pure because of interactions with the individual nucleons, it is spread out to many configurations, all of which contain a certain amount of the original single-particle eigenfunctions. The amount of this spreading is governed by the average strength of internucleon interactions and is represented by the size of the imaginary part of the potential. This latter explanation, in which the single-particle eigenfunction is distributed among the levels of the compound nucleus, has been discussed by Lane, Thomas, and Wigner.[6] Here the transition from the completely amalgamated Bohr compound nucleus to the strict single-particle levels of the shell model was discussed in the framework of the R-matrix theory of nuclear reactions. More recently, further experimental verification of Barschall's type of giant-resonance structure has been found in low-energy neutron data[7] and also in proton cross sections.[8]

In the present experiment a more extensive investigation of the giant-resonance effect in a neutron's

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[1] F. L. Friedman and V. F. Weisskopf, in *Niels Bohr and the Development of Physics* (McGraw-Hill Book Company, Inc., New York, 1955).

[2] Feshbach, Peaslee, and Weisskopf, Phys. Rev. **71**, 145 (1947).

[3] M. G. Mayer and J. H. D. Jensen, *Elementary Theory of Nuclear Shell Structure* (John Wiley & Sons, Inc., New York, 1955).

[4] H. H. Barschall, Phys. Rev. **86**, 431 (1952).

[5] Feshbach, Porter, and Weisskopf, Phys. Rev. **96**, 448 (1954).

[6] Lane, Thomas, and Wigner, Phys. Rev. **98**, 693 (1955).

[7] Hughes, Zimmerman, and Chrien, Bull. Am. Phys. Soc. Ser. II, 4, 35 (1959); Coté, Bollinger, and Le Blanc, Phys. Rev. **111**, 288 (1958); H. Marshak and H. W. Newson, Phys. Rev. **106**, 110 (1957).

[8] J. P. Schiffer and L. L. Lee, Jr., Phys. Rev. **109**, 2098 (1958).

interaction with the nucleus has been attempted.[9] The (d,p) reaction for deuterons incident with moderately high energies can be well characterized as a direct capture of the neutron from the incident deuteron into a given state of the final nucleus, the proton acting as a receiver of energy and angular momentum but not participating appreciably in the nuclear part of the interaction.[10] Thus an incident deuteron of a definite energy presents neutrons with a continuous spectrum of energies for capture into final states. Neutrons with negative kinetic energies can be captured into bound states of the final nucleus and neutrons with positive kinetic energies can be captured up to a limiting energy which is determined by the total energy available from the reaction. Each neutron captured into a given level of the final nucleus is associated with a proton of a corresponding energy. Thus the energy distribution of protons directly reflects the distribution of neutron capture probabilities among the levels of the final nucleus.[11,12]

Since virtually all of the (d,p) reaction proceeds by such a stripping mechanism at even moderate deuteron energies, any gross resonance effects in the neutron capture probabilities should be reflected in the proton spectrum. Excitation energies up to 8 Mev can easily be explored by the (d,p) reaction with 10-Mev deuterons on target elements around $A=60$. Thus several giant resonances in the proton spectrum from a single nuclide should be observed representing the various single-particle states of the neutron in this potential. Energies involved in a (d,p) reaction can allow neutrons with high angular momenta to be captured with about as large a probability as the ones with low angular momenta. An estimate based on the formula given by Butler indicates that angular momenta up to 4 should be observable for the captured neutrons. These formulas predict that the outcoming protons should have angular distributions which show characteristic maxima for capture of a neutron with a given angular momentum, so that unique assignments to the single-particle states should be possible. Such assignments are difficult to make from neutron scattering data, except at low energies for which only states with zero angular momentum neutrons can be formed with any intensity.

EXPERIMENTAL PROCEDURE

In order to investigate this field it would have been best to use extremely high resolution and measure each proton group corresponding to a level of the final nucleus. To measure angular distributions for levels in large regions of excitation in each of several nuclei would have required a great deal of time for both the measurement and the analysis of the data. However, the essential features of the distribution of various single-particle amplitudes should be observable in an experiment in which each level is not resolved but the energy spread is still small compared to the spacing of the single-particle levels. NaI(Tl) or CsI(Tl) scintillation crystals provide proton detectors which should satisfy this requirement on resolution. At the same time they permit the use of considerably larger solid angles than is possible with most magnetic spectrometers, and multichannel pulse-height analyzers permit the recording of the proton spectra over the entire region of excitation energy in one measurement. In addition, the yields in an experiment with poor resolution can be increased by letting the deuterons bombard thicker targets, so long as the thickness does not appreciably affect the energy spread of the proton peaks observed by the detectors.

The nuclei chosen in the present investigation were not so light that the number of possible configurations would limit the mixing of a given single-particle state into only a very few levels. On the other hand they were not so heavy that the spacing of single-particle levels would become smaller than the energy interval into which they are spread out, which would make it difficult to observe the giant-resonance effects. Strongly deformed nuclei would also cause complications in the interpretation of the data.[13] The region of nuclei between Ti and Cu was chosen because these nuclei have been found to have a relatively large number of energy levels, the single-particle level spacings would be expected to be of the order of 1–2 Mev, these nuclei are not believed to be strongly deformed, and it is relatively easy to obtain targets of the required thickness in the form of metallic foils.

The 21.6-Mev deuteron beam of the Argonne cyclotron was used in conjunction with a scattering chamber, which has been described elsewhere,[14] in a first investigation of the gross structure effects. Proton spectra observed in a NaI detector at 17° with respect to the incident deuteron beam and recorded in a 100-channel pulse-height analyzer are shown in Fig. 1. It is clear that some gross structure is present and some similarities are discernible in spectra of adjacent nuclei. Foils of metallic Ti, Cr, V, Mn, Fe, and Co were rolled to $\sim$0.0002 in. thickness while enriched foils of the two principal Ni and Cu isotopes were obtained from the

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[9] A preliminary report at an earlier stage of this work has already been published. See Schiffer, Lee, Yntema, and Zeidman, Phys. Rev. **110**, 1216 (1958); also J. P. Schiffer and L. L. Lee, Jr., Bull. Am. Phys. Soc. Ser. II, **3**, 211 (1958).

[10] S. T. Butler, Proc. Roy. Soc. (London) **A208**, 559 (1951).

[11] The first systematic study of proton spectra over large regions of excitation was made by H. E. Gove, Phys. Rev. **81**, 364 (1951). Since this was done before references 5 and 10 were published, the data do not include the forward angles at which the characteristic Butler maxima would occur, so the present interpretation was not possible.

[12] This idea was also suggested independently by A. M. Lane (private communication), and experiments similar to the ones reported here are being done by G. Parry (private communication). Similar interpretations for some experiments were also offered independently by R. A. Peck, Jr., and J. Lowe, Bull. Am. Phys. Soc. Ser. II, **3**, 211 (1958).

[13] B. Margolis and E. S. Troubetzkoy, Phys. Rev. **106**, 105 (1957); Chase, Wilets, and Edmonds, Phys. Rev. **110**, 1080 (1958).

[14] J. L. Yntema, Phys. Rev. **113**, 261 (1959).

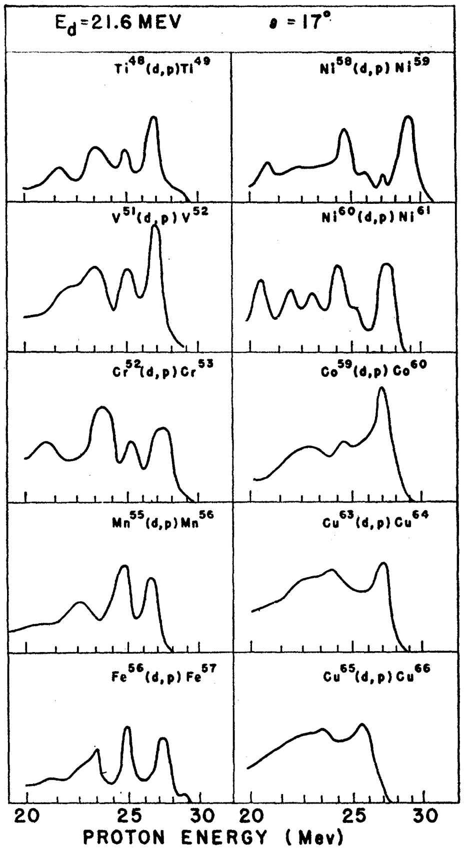

FIG. 1. Proton spectra from the (*d*,*p*) reaction as measured in a NaI(Tl) scintillation detector and recorded in a 100-channel pulse-height analyzer. The bombarding energy, angle of observation, target nuclides, and approximate proton energy scale are shown in the figure.

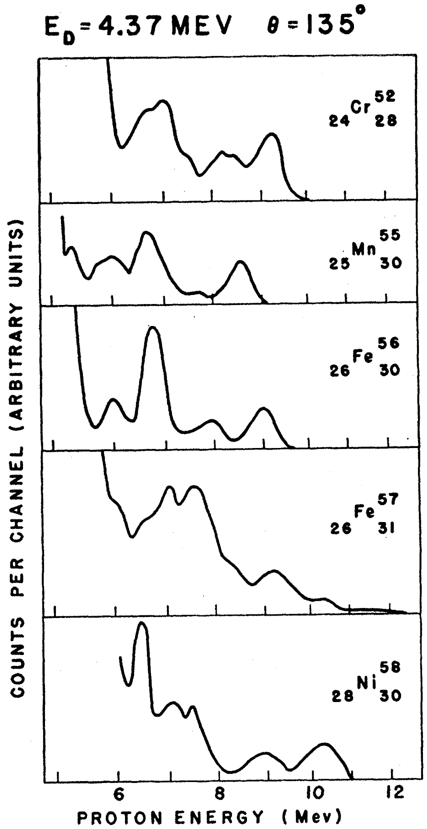

FIG. 2. Proton spectra from the (*d*,*p*) reaction as measured in a CsI(Tl) scintillation detector and recorded in a 256-channel pulse-height analyzer. The bombarding energy, angle of observation, target nuclides, and approximate proton energy scale are indicated in the figure.

Atomic Energy Research Establishment at Harwell. The energy scale was obtained by calibrating against reactions with known Q values.

The experiment was repeated on the Argonne Van de Graaff and qualitatively similar effects were obtained, as is shown in Fig. 2. Results were obtained with deuteron energies between 3 and 4.5 Mev. At these lower deuteron energies the absolute energy resolution of the proton detector was considerably better than at 21.6 Mev and much of the detailed structure that was observed may be due to fluctuations in the positions of individual levels rather than evidence for more gross structure. However, it seems from Fig. 2 that the envelopes one would draw for the various proton spectra still are similar to the 21.6-Mev data. Unfortunately, at the low bombarding energies the effects of the contaminants became serious, and in fact did not permit the exploration of the region of excitation which fell below the Q values of the $O^{16}(d,p)$ and $C^{12}(d,p)$ reactions. The effects of contaminants were negligible at the higher bombarding energies on the cyclotron.

At the low energies the (*d*,*p*) reaction on these nuclei does not proceed by the simple stripping mechanism discussed by Butler, presumably largely because of the strong distorting effects of the Coulomb field of the nucleus.[15] Therefore, it was not possible to make definite angular momentum assignments to the observed gross structures from the angular distribution data for low energies. The theory of the (*d*,*p*) reactions does not yet take account of modifications by the Coulomb field and other effects in sufficient detail to permit assignment of l values. Indeed some detailed experimental work on the $Ca^{40}(d,p)Ca^{41}$ reaction, where the l value is known, indicates that assignments on the basis of angular distribution measurements may not be at all meaningful at these energies.[16]

At 21.6 Mev the Butler theory is expected to be a good description of the reaction mechanism. Here, however, it predicts angular distributions which, for at least the first three l values, are peaked in the forward direction and are therefore difficult to distinguish from each other.

The best deuteron energy, at which the Butler

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[15] W. Tobocman and M. H. Kalos, Phys. Rev. **97**, 132 (1955).

[16] L. L. Lee, Jr., and J. P. Schiffer, Phys. Rev. **107**, 1340 (1957).

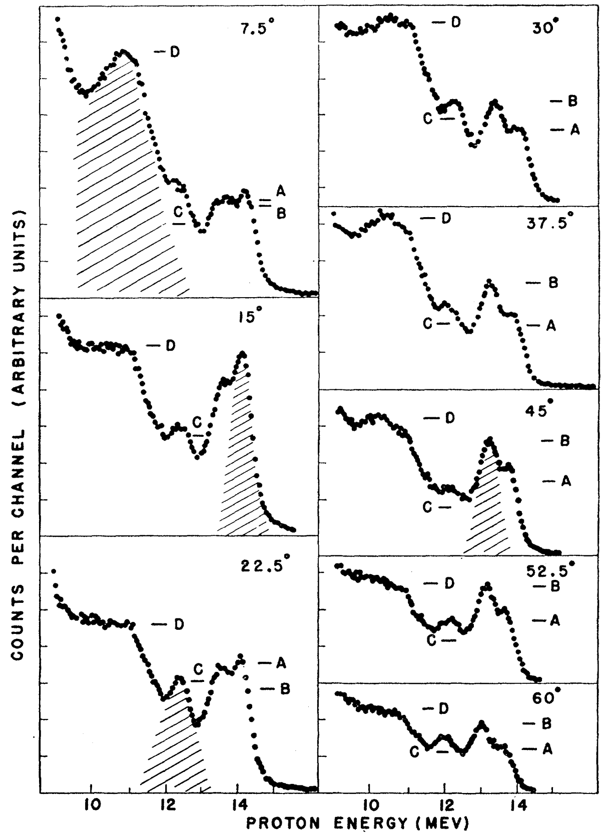

FIG. 3. Proton spectra from the $Cu^{65}(d,p)Cu^{66}$ reaction at $E_d=10$ Mev as detected in a CsI(Tl) scintillation crystal and recorded in a 256-channel pulse-height analyzer. The angular resolution was 5°. The group shaded in the 15° data was called group *A*, the one in the 45° data was group *B*, the 22.5° one was group *C*, and the 7.5° one, group *D*. The peak shape assigned to each group is shown by the shape of the shaded area. The lines marked by these letters show the heights assigned to these peaks in the analysis of the data.

theory is expected to be valid and at which it predicts strongly different angular distributions for different l values, is around 10 Mev. The experiment was therefore repeated at this energy and the pulse-height data subjected to fairly detailed analysis in order to obtain angular distributions and thus to assign l values.

The cyclotron of the Physics Department at Washington University in St. Louis was used in conjunction with a scattering chamber of 6-inch diameter. The protons were observed at 7.5° intervals between 7.5° and 60° with respect to the incident beam. An absorber, sufficiently thick to stop the deuterons, was used between the target and detector. A typical set of data obtained with a Cu^{65} target is shown in Fig. 3. The shapes of the various peaks (*A*, *B*, *C*, and *D*) that were observed in the spectra were determined by studying the spectra at the angles at which they were the most intense. A certain amount of arbitrariness in these assignments was inevitable; it is estimated that the total areas are not in error by more than 20% for the more intense peaks and low excitation energies, nor by more than 50% for the weaker peaks and the high excitation energies. The relative heights of the peaks that were determined at the various angles are designated by lines drawn next to the appropriate letters in each spectrum. The angular distributions that were obtained from these data are shown in Fig. 4 along with the Butler curves calculated for the appropriate energies and a radius of 6.4 fermis [1 fermi (f) $=10^{-13}$ cm]. Figure 5 shows angular distributions, observed by analyzing the data in the same way, from the $Ni^{60}(d,p)$-Ni^{61} reaction. The Butler formula was coded for the Argonne fast digital computer GEORGE[17] and a range of parameters was explored (radius between 5 and 7 fermis and Q value within 1 Mev of the peak) in order to insure the uniqueness of the assignments. The intensities were normalized by assuming that the group for $l=1$ corresponded to the $p_{1/2}$ single-particle state. These normalizations gave reasonably good fits to the peaks with $l=2$ and $l=4$ when it was assumed that they were the $d_{5/2}$ and $g_{9/2}$ single-particle states. The agreement in intensity for the peak with $l=0$, on the other hand, is poor. This discrepancy for the peak with $l=0$ was observed for most of the elements studied. It can be seen then that while the Butler theory by no means

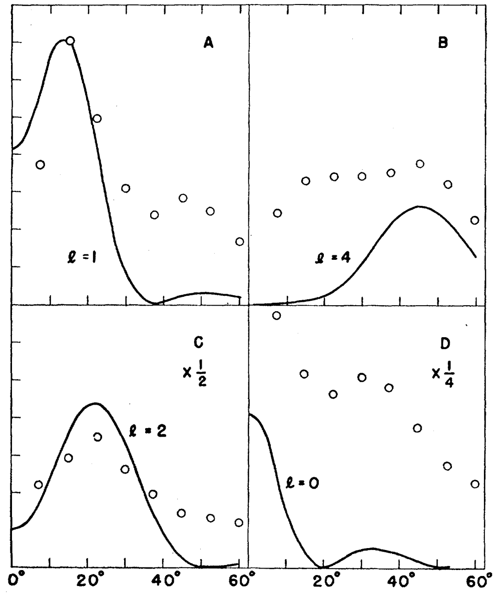

FIG. 4. Angular distributions of protons from the $Cu^{65}(d,p)Cu^{66}$ reaction. The points represent the areas of the various groups shown in Fig. 3. The statistical error in the points is less than their size; the error introduced by the method of analysis cannot be estimated accurately, but should be less than 10% for relative errors between adjacent points in one distribution. The theoretical curves are calculated for single-particle states from the Butler theory with a radius of 6.35 fermis and are normalized to give the proper intensity for the $p_{1/2}$ state.

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[17] The formulas used in the computation were given in a simplified form in W. J. Childs, thesis, University of Michigan, 1956 (unpublished). We are indebted to L. Kassel for coding these for GEORGE.

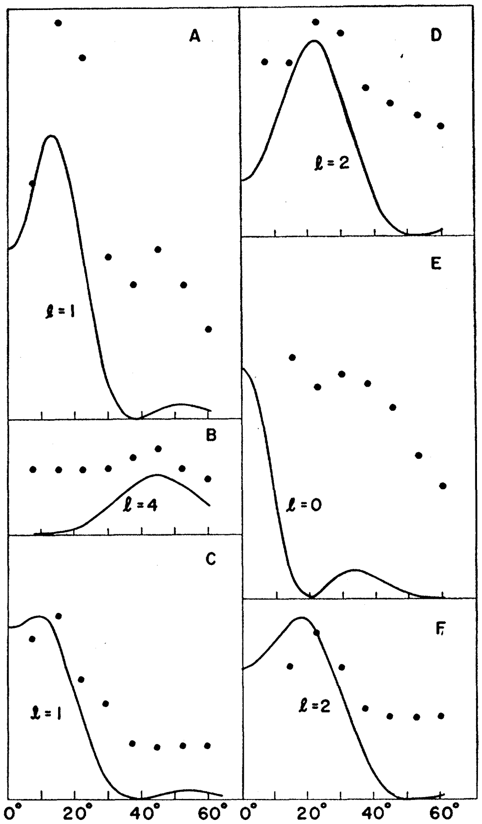

FIG. 5. Angular distribution of protons from the $Ni^{60}(d,p)Ni^{61}$ reaction. The points were obtained in the manner indicated for Cu^{65} in Fig. 3. The uncertainties in the points and the angular resolution were the same as in the data shown in Fig. 4. The theoretical curves were also calculated in the same manner.

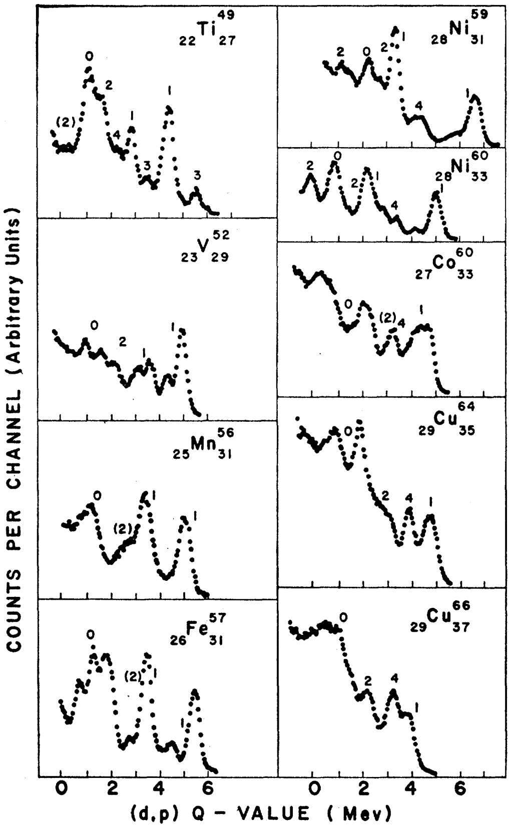

FIG. 6. Pulse-height spectra from the (*d*,*p*) reaction obtained at a deuteron energy of 10 Mev at 30° with respect to the incident beam. The angular momentum assignments obtained for the various peaks by the method shown in Figs. 3 and 4 are indicated by numbers next to the peaks.

gives perfect fits to the angular distributions, it does seem to approximately reproduce the peaks at the proper angles and furthermore it seems to give the right relative intensities for the different l values. Thus it is possible to arrive at assignments which have a high probability of being correct.

The assignments were facilitated to some extent by the knowledge of the sequence of levels as they appear at the ground states of nuclei. Thus in ${}_{29}Cu_{37}{}^{66}$ the last neutron is expected to be in the $f_{5/2}$ shell in the ground state. Very close above this would be the $p_{1/2}$ state followed by the 1*g*, 2*d*, and 3*s* states. In this experiment the expected $f_{5/2}$ state was not observed near the ground state. This peak was probably obscured by the much stronger $p_{1/2}$ group which occurs at very nearly the same energy. It is indeed probable that some of the levels near the ground state of this nucleus are formed by capture of neutrons with a mixture of $l=1$ and $l=3$, and other levels are formed either only by capture of neutrons with $l=1$ or only by neutrons with $l=3$. This matter could, however, be settled only if the angular distributions for individual levels were studied in this reaction with high energy resolution.[18] The next three single-particle states expected do indeed correspond to the identifications from the angular distributions.

A summary of data obtained for all the target nuclei is shown in Fig. 6 in the form of pulse-height distributions at 30°, and the l-value assignments are indicated by the numbers next to the peaks. The identifications were made in the same way as for Cu^{65} and Ni^{60}. The lines corresponding to high angular momenta were generally more difficult to identify. The levels with $l=3$ were found only in Ti, where they are relatively isolated. In all the other nuclei studied the $f_{5/2}$ state with $l=3$ seems to fall too close to the states with $l=1$ and is obscured by them. The shell-model assignments are summarized in the form of an energy-level diagram in Fig. 7, with dashed lines indicating uncertain assignments, and are listed in Table I.

The information on relative intensities is summarized

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[18] Evidence for this sort of mixture has been found in high-resolution angular-distribution studies of a very similar case, the reaction $Co^{59}(d,p)Co^{60}$. Massachusetts Institute of Technology, Laboratory for Nuclear Science Annual Progress Report June 1, 1956–May 31, 1957 (unpublished).

TABLE I. Approximate energies of single-particle states in terms of the neutron binding energy (Mev).[a]

Shell-model assignment \ Final nucleus	$_{22}Ti_{27}{}^{49}$	$_{23}V_{29}{}^{52}$	$_{25}Mn_{31}{}^{56}$	$_{26}Fe_{31}{}^{57}$	$_{28}Ni_{31}{}^{59}$	$_{27}Co_{33}{}^{60}$	$_{28}Ni_{33}{}^{61}$	$_{29}Cu_{35}{}^{64}$	$_{29}Cu_{37}{}^{66}$
$3s_{1/2}$	−3.2	−3.3	−2.6	−2.8	−4.3	−3.6	−3.5	−4.1	−3.6
$2p_{1/2}$	−5.0	−5.5	−5.3	−5.3	−5.3	−7.1	−5.6	−7.5	−6.7
$2p_{3/2}$	−6.5	−6.9	−6.8	−7.2	−7.5		−7.4		
$2d_{3/2}$	(−2.4)				(−3.3)		−2.4		
$2d_{5/2}$	−3.8	(−4.8)	(−4.5)	(−5.2)	−5.3	(−4.8)	−3.8	−5.5	−5.1
$1f_{5/2}$	−5.6								
$1f_{7/2}$	−8.1								
$1g_{9/2}$	−4.3			(−4.3)	−6.4	(−6.0)	−6.0	−6.6	−6.2

[a] Parentheses indicate uncertain assignments.

in Table II. Here the results are normalized to the $p_{1/2}$ group which was observed in all the nuclei studied. The results are given in terms of relative reduced widths, on the assumption that the assignments stated are correct and that the Butler theory is applicable.

DISCUSSION

The first question that arises in connection with these data is whether they could be due to single levels which are in fact the single-particle states. That this is not the case is demonstrated by the rather extensive (d,p) data which have been obtained with high resolution in this region of the periodic table by Buechner *et al.*[19] Typical results are shown in Fig. 8 for the levels obtained in Ni^{59} and Co^{60} by magnetic analysis of the protons from the (d,p) reaction on Ni^{58} and Co^{59}. Obviously many levels are excited in this reaction and it is clear from the high-resolution results that the gross structures observed in the present experiments contain many of the actual levels of the nucleus. Additional information, which is reasonably evident from the high-resolution data, is that the gross structure observed is not due to bunching of the levels of the nucleus but rather to the effect of the average amplitudes becoming larger at certain energies; the levels seem to be spaced more or less at random. This is the expectation from the paper of Lane, Thomas, and Wigner, but is not clear from the complex-potential considerations[20] of Feshbach, Porter, and Weisskopf.

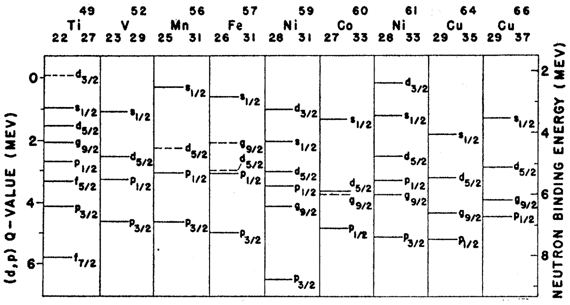

FIG. 7. Summary of shell-model level assignments from the data. All energy values are to be regarded as approximate with an uncertainty which is approximately equal to the width of the corresponding peaks. Levels indicated by dotted lines have a greater degree of uncertainty in their assignment than the solid ones.

The data summarized in Fig. 7 can be compared to the sequence of shell-model levels as observed in the ground states of nuclei. It is interesting to note that the energies of these levels, and in some cases even their sequence, seems to vary somewhat from nucleus to nucleus. This presumably is due to variations in the details of the nuclei which are only approximately represented by average potentials for the incident neutrons. Another quantity which can be observed from this diagram is the size of the spin-orbit term in the potential since the splitting of the $p_{3/2}$ and $p_{1/2}$ levels was measured in six nuclei, the $d_{5/2}$ and $d_{3/2}$ in three nuclei and the $f_{7/2}$ and $f_{5/2}$ in one.

The intensities can also be used to deduce ground-state configurations of the target nuclei. From statistical considerations alone, one would expect the proton group corresponding to neutron capture in the $p_{3/2}$ state to show up twice as strongly as the one corresponding to capture in the $p_{1/2}$ state. If, however, the $p_{3/2}$ shell were partially filled, then there would be fewer available substates into which the neutron could be captured, and thus the peak would be weaker.[21] After the filling of the $f_{7/2}$ shell with 28 neutrons, the $p_{3/2}$ and $f_{5/2}$ shells compete with each other. In comparing the data for target nuclei with 30 neutrons ($_{25}Mn_{30}{}^{55}$, $_{26}Fe_{30}{}^{56}$, and $_{28}Ni_{30}{}^{58}$) one observes that the $p_{3/2}$ proton group is roughly twice as intense as the $p_{1/2}$ group for Fe^{56} and Ni^{58}, but it is the same intensity for Mn^{55}. This implies that in the first two nuclei the 29th and 30th neutrons are in the $f_{5/2}$ shell in the ground state while in Mn they are the $p_{3/2}$ shell. Similarly, by observing the data for $_{27}Co_{32}{}^{59}$ and $_{28}Ni_{32}{}^{60}$ one observes only one $l=1$ peak for Co, indicating that the $p_{3/2}$ shell is filled with four neutrons; while there are two $l=1$ peaks in Ni^{60} with the one at lower energy about twice as intense as the other, thus indicating that all four

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[19] Massachusetts Institute of Technology, Laboratory for Nuclear Science Annual Progress Report, June 1, 1956–May 13, 1957 and June 1, 1957–May 31, 1958 (unpublished); Green, Smith, Buechner, and Mazari, Phys. Rev. **108**, 841 (1957).

[20] C. Porter (private communication).

[21] The calculations on the reduced widths for partially filled shell-model states were supplied to us by B. J. Raz.

TABLE II. Approximate reduced widths of single-particle states normalized to give $\theta_n^2=1$ for the $2p_{1/2}$ state.[a]

Shell-model assignment \ Final nucleus	$_{22}Ti_{27}{}^{49}$	$_{23}V_{29}{}^{52}$	$_{25}Mn_{31}{}^{56}$	$_{26}Fe_{31}{}^{57}$	$_{28}Ni_{31}{}^{59}$	$_{27}Co_{33}{}^{60}$	$_{28}Ni_{33}{}^{61}$	$_{29}Cu_{35}{}^{64}$	$_{29}Cu_{37}{}^{66}$
$3s_{1/2}$	2.2	1.4	5.2	3.6	2.7	>1	3.0	>1.6	>2.7
$2p_{3/2}$	1.1	1.2	0.6	1.4	1.4		1.3		
$2d_{3/2}$	$\sim$0.8				($\sim$1.1)		1.0		
$2d_{5/2}$	1.0	($\sim$0.8)	(0.7)	(0.8)	1.1	(0.4)	1.0	$\sim$0.7	0.7
$1f_{5/2}$	$\sim$0.6								
$1f_{7/2}$	$\sim$0.3								
$1g_{9/2}$	$\sim$0.7				1.3	$\sim$0.8	1.2	0.8	1.4

[a] Parentheses indicate uncertain assignments.

neutrons are in the $f_{5/2}$ shell. These results are summarized in Table III.

TABLE III. Ground state configurations of neutrons in the target nuclides.

$_{22}Ti_{26}{}^{48}$	$(f_{7/2})^6$		
$_{23}V_{28}{}^{51}$	$(f_{7/2})^8$		
$_{25}Mn_{30}{}^{55}$	$(f_{7/2})^8$	$(p_{3/2})^2$	
$_{26}Fe_{30}{}^{56}$	$(f_{7/2})^8$		$(f_{5/2})^2$
$_{28}Ni_{30}{}^{58}$	$(f_{7/2})^8$		$(f_{5/2})^2$
$_{27}Co_{32}{}^{59}$	$(f_{7/2})^8$	$(p_{3/2})^4$	
$_{28}Ni_{32}{}^{60}$	$(f_{7/2})^8$		$(f_{5/2})^4$
$_{29}Cu_{34}{}^{63}$	$(f_{7/2})^8$	$(p_{3/2})^4$	$(f_{5/2})^2$
$_{29}Cu_{36}{}^{65}$	$(f_{7/2})^8$	$(p_{3/2})^4$	$(f_{5/2})^4$

There seems to be a tendency for the $p_{3/2}$ level to be below the $f_{5/2}$ level for nuclei with odd proton number and for the order to be reversed for even proton number. It also appears that the $g_{9/2}$ level occurs slightly below the $p_{1/2}$ level in the two Ni isotopes and above it in Co and Cu isotopes. In both cases the state with higher angular momentum seems to occur lower in energy for nuclei with even proton numbers.

Our results are then qualitatively consistent with the optical model in that the expected gross structure caused by single-particle states of the neutron was observed, and with the shell model in showing that roughly the appropriate sequence of levels was observed. The facts that the observed details of the gross structure are not identically the same from nucleus to nucleus and that fluctuations in the levels seem to occur indicates that the replacement of the nucleus by an average potential is, after all, an approximation, and small variations in this average potential evidently occur between nuclei.[22] The rate at which the observed levels move down in energy as atomic weight increases indicates that a potential with slowly tapering edges, such as a Saxon potential,[23] is a better approximation than the more conventional square-well or oscillator potential. The observed spin-orbit splitting is 1.52 Mev for the $2p_{3/2}$ and $2p_{1/2}$ states and 1.5–2 Mev for the $d_{5/2}$ and $d_{3/2}$ states. The one observation of the splitting between $f_{7/2}$ and $f_{5/2}$ is $\sim$2.5 Mev.

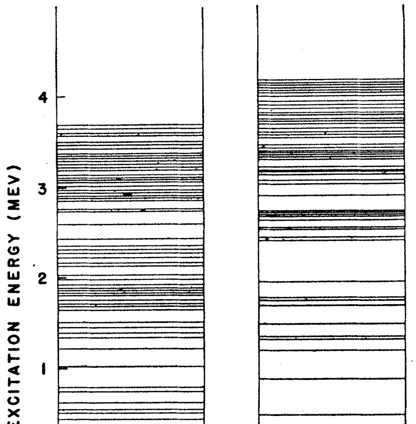

FIG. 8. Known energy levels in the nuclei Ni^{59} and Co^{60} as given in reference 19.

In conclusion it appears that detailed information of this type might be obtained in similar experiments with many more nuclei. It would also be of interest to interpret other stripping or pickup reactions in terms of single-particle amplitudes. Such an interpretation has been offered in a qualitative way for gross structure observed in (p,d) reactions[24] and in (n,p) reactions.[25] There is also a qualitative similarity between the gross structure observed here and the effects which occur in inelastic scattering of protons,[26] deuterons,[27] and alpha particles.[28] Some efforts have been made in an attempt to interpret these also in terms of single-particle

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[22] It has been suggested by A. M. Lane (private communication) that a term proportional to $(N-Z)/A$ should be added to the potential representing variations in the potential with the neutron excess. This is qualitatively consistent with some of the fluctuations obtained here. In particular Ni^{58}, which has the smallest neutron excess of any of the nuclei studied, has its single-particle levels occurring at lower energies indicating that the potential for neutrons tends to be deeper in nuclei where the neutron excess is smaller.

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processes, but it is not clear that these interpretations can account for all the results from inelastic scattering.[29]

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Nuclear States in the RaE β-Decay*

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It is shown that the β-transforming neutron of Bi^{210} most probably has an $i_{11/2}$ character, despite a $g_{9/2}$ character of the ground-state neutron in Pb^{209}. This makes a critical difference to the RaE spectrum parameter, $\xi = i\langle\mathbf{r}\rangle/\langle\boldsymbol{\sigma}\times\mathbf{r}\rangle$, yielding $\xi\approx+1$ rather than $\xi\approx-1/10$. The effect of configuration mixing is also investigated but does not affect ξ appreciably.

To arrive at the above conclusions, it is necessary to show that the neutron-proton attraction in the $(h_{9/2}i_{11/2})_1$ state is the large amount, 840 kev, greater than in the $(h_{9/2}g_{9/2})_0$ state. The resulting shell-model problem has interest independent of the β-theory application which was the original objective of this work. True and Ford had found that two neutrons extra to the doubly-magic core, Pb^{208}, as against nucleons deep in the core matter, interact with about the same strength and range of force as do two free nucleons. The problem here checks the extension of that important finding to neutron-proton and proton-proton pairs. The force strength is consequently *not* used as an adjustable parameter, as it has been in previous approaches to such problems.

The True-Ford problem involved only singlet, central forces between like nucleons in an essential way. The RaE daughter, Po^{210}, investigated here, has only Coulomb repulsion superposed. The resultant comparison with experiments is about as good as that obtained by True and Ford.

The extension to the neutron proton pair of Bi^{210} is far more complex, since triplet forces, an exchange character, and noncentral forces, may now come into play. We find that the finite-range, central forces alone cannot give substantially more attraction in the $(h_{9/2}i_{11/2})_1$ state than in $(h_{9/2}g_{9/2})_0$. Even using the strength as a parameter cannot help significantly. However, the tensor forces produce attraction in the former and repulsion in the latter state. Hence, the two-body forces must be imitated even in this detail in order to yield an explanation of the RaE level scheme. This is unfortunate for quantitative results, because the strength of the two-body tensor force seems never to have been determined unambiguously for potentials without cores.

Our final conclusion is that the two-body neutron-proton force may be well represented by zero-range forces of the same volume energy as found experimentally. Tensor effects vanish identically in this limit and so an unambiguous representation of the strength can be obtained. The results for the relative positions of the $J=0$ and 1 states, used as the test above, now turn out in almost perfect agreement with the observations. Configuration mixing plays a role in this result, and, in consequence, the work includes a generalization of de-Shalit's formulas, for the interaction energies with zero-range forces, to nondiagonal matrix elements.

INTRODUCTION

THE distinctive characteristics of the RaE decay (nonstatistical spectrum, anomalous electron polarization, prolonged lifetime) have made it an important test case.[1] The radiation seems to be generated through at least the β-moments,[2] $\langle\mathbf{r}\rangle$, $\langle\boldsymbol{\sigma}\times\mathbf{r}\rangle$, and $\langle\boldsymbol{\alpha}\rangle$. These have been treated as completely arbitrary parameters, with attendant uncertainties of interpre-

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