Decay of Tb^{156} (5-day)*

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(Received February 16, 1959)

Radioactive Tb^{156} (~5-day) was produced by the bombardment of Eu^{153} with α particles. It was found to decay by K capture to Gd^{156}. A decay scheme has been proposed which includes the following excited states of Gd^{156}: 89 kev (2+), 289 kev (4+), 581 kev (6+), 1134 kev (2+), 1229 kev (3+), 1489 kev (4+), 1604 kev, 1904 kev (3), and 2024 kev (4−). The assignments of spins of levels and multipolarities of gamma-ray transitions are based on the results of measurements of conversion coefficients and gamma-gamma angular correlations. A value of $(1.5\pm0.3)\times10^{-10}$ sec was found for the mean life of the 289-kev level of Gd^{156}. The level scheme is discussed in the light of the unified model. The results support the suggestion that the 1134-kev and 1229-kev levels should be interpreted as collective vibrational levels.

INTRODUCTION

RECENT studies of the properties of a number of even-even nuclei with spheroidal equilibrium shape have shown that the properties of the energy levels of these nuclei are very similar.[1–4] It seems that this similarity originates from the fact that many of the excited levels of these nuclei are of the collective type. The present work was undertaken to find out whether this systematic picture can be extended to cover additional nuclei.

As was already found previously, Tb^{156} decays by K capture to Gd^{156} and its half-life is about 5.5 days.[5,6] It has an isomeric state, 88.2 kev above the ground state, which decays with a half-life of 5.5 hours. Mihelich *et al.*[5] observed conversion electron lines corresponding to transition energies of 89.1, 111.9, 155.2, 199.4, 262.7, 296.7, 356.6, and 422.2 kev associated with the 5-day half-life. The 89.1- and 199.4-kev events were interpreted by them as 2+ to 0+ and 4+ to 2+ transitions between rotational levels, respectively. Additional γ transitions with energies of 1050, 1140, 1210, 1410, 1630, and 1830 kev were observed by Henry *et al.*[6] who also studied coincidences of γ rays associated with the decay of Tb^{156} and suggested a partial decay scheme.

In the present work, the decay scheme of Tb^{156} (~5 days) was re-examined. In addition to the previously reported γ transitions, additional transitions, with energies of 770 kev, 908 kev, 940 kev, 1140 kev, 1315 kev, and 1935 kev have been observed and a decay scheme in agreement with all the results of coincidence measurements is suggested. On the basis of the results of measurements of internal conversion coefficients and gamma-gamma angular correlations, assignments for the spins of excited levels and multipolarities of γ radiations are made. A measurement of the lifetime of the 289-kev level makes possible a comparison of the ratio of this lifetime to the previously measured lifetime of the 89-kev level[7] with the ratio predicted by theory.

The radioactive sources were produced by the bombardment of Eu_2O_3 with α particles in the 60-in. cyclotron of Brookhaven National Laboratory. The α particles were degraded to an energy of about 14 Mev before striking the target. Using the tables of semi-empirical masses of atoms,[8] it was found that the thresholds of the reactions $Eu^{151}(\alpha,n)Tb^{154}$ and $Eu^{153}(\alpha,n)Tb^{156}$ are about 8 Mev and the thresholds of the reactions $Eu^{151}(\alpha,2n)Tb^{153}$, $Eu^{153}(\alpha,2n)Tb^{155}$, and $Eu^{151}(\alpha,np)Gd^{153}$ are about 15 Mev. As expected, no activities characteristic of Tb^{153}, Tb^{155}, and Gd^{153} were detected. The half-lives associated with Tb^{154} are 7 hours and 21 hours[5] and the half-life of Tb^{156m} is ~5 hours. As the

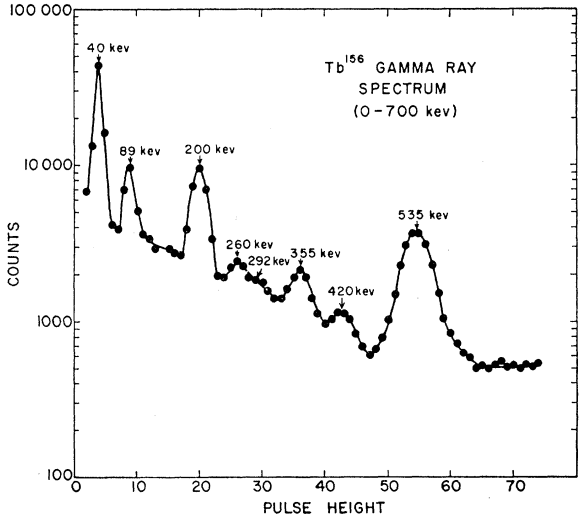

FIG. 1. Spectrum of γ rays in the (0–700) kev region emitted in the decay of Tb^{156}.

* Work performed under the auspices of the U. S. Atomic Energy Commission.
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[1] O. Nathan and M. A. Waggoner, Nuclear Phys. **2**, 548 (1957).
[2] S. Ofer, Nuclear Phys. **4**, 477 (1957).
[3] O. Nathan, Nuclear Phys. **4**, 125 (1957).
[4] Jacob, Mihelich, Harmatz, and Hardley, Bull. Am. Phys. Soc. Ser. II, **3**, 358 (1958).
[5] Mihelich, Harmatz, and Handley, Phys. Rev. **108**, 989 (1957).
[6] Henry, Dillman, Gove, and Becker, Phys. Rev. **113**, 1090 (1959).
[7] O. Nathan, Nuclear Phys. **5**, 401 (1958).
[8] A. G. W. Cameron, Chalk River Project Report No. 690 (unpublished).

measurements were started 10 days after the bombardment, the activity of the source was expected to be entirely due to the decay of Tb^{156} (~5-day). This expectation was shown to be justified: the relative intensities of the γ transitions did not change with time. 5-μC activities were produced by the bombardment of about 50 mg of Eu_2O_3 for 20 hours with 20 μa of α rays.

EXPERIMENTAL PROCEDURE

1. Spectrum of Gamma Rays

The γ-ray spectrum was examined with a 3-in.×3-in. NaI(Tl) scintillation counter in combination with a hundred-channel analyzer. The spectrum of γ rays whose energy is smaller than 700 kev is shown in Fig. 1. In Fig. 2 the shape of the spectrum of γ rays whose energy is higher than 700 kev is shown. By an analysis of the shape of the spectrum obtained, some of the relative intensities given in Table I were obtained.

2. Coincidence Studies

Gamma-gamma coincidence measurements were carried out with an apparatus employing two 3-in.×3-in. NaI(Tl) crystals. Pulse-amplitude discrimination was made in the "gate" channel pulse-height analyzer and the coincidence spectrum was displayed on a 100-channel pulse-height analyzer. The coincidence circuit was operated at a resolving time of $2\tau=2\times10^{-7}$ sec.

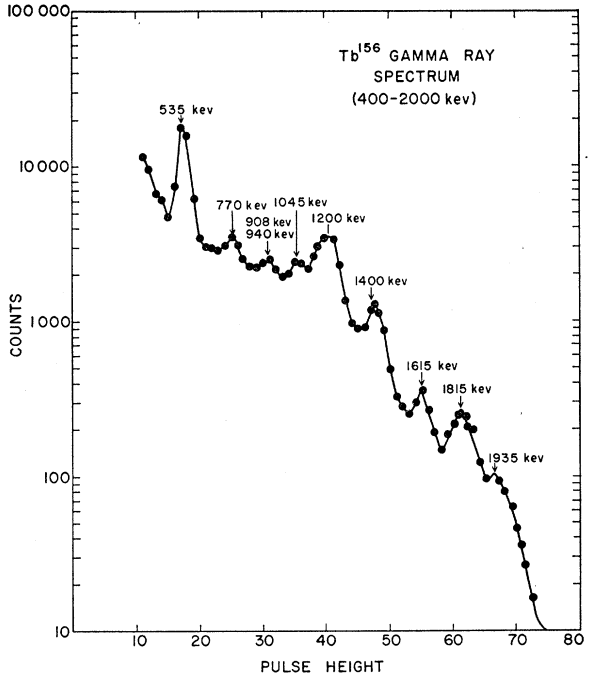

FIG. 2. Spectrum of γ rays with energies higher than 700 kev emitted in the decay of Tb^{156}.

TABLE I. Results of measurements of spectrum of γ rays and of spectra of coincidences.

Energy of transition (kev)	Relative intensity of γ radiation		Energies of γ transitions found to be in cascade with the transition
89	18	(A)	All radiation except the 89-kev and 1134-kev transitions
115	(2)	(G)	89, 200, (260), (355), 420
200	37	(A)	89, 115, 260, 292, 420, 535, 908, 940, 1200, 1315, 1615
260	10	(A)	89, (115), 200, (420), 535, 940, 1140
292	5	(A)	89, 200, 908, (1023)
355	16	(A)	89, (115), (420), 535, 1045, 1134
420	8	(A)	89, 115, 200, (260), (355), (1023), 1315
535	78	(A)	All radiation except 115, 420, 770, 1315, 1615, 1815, 1935
(675)	(1)	(A)	
770	(5)	(A)	89, 1045, 1134
908	4	(A)(C)(E)	89, 200, 292, 535
940	2	(A)(C)(D)	89, 200, 260, 535
(1023)	(1)		(292), (420)
1045	12	(A)(F)	89, 355, 535, 770
1134	11	(F)	355, 535, 770
1140	10	(D)	89, 260, 535
1200	31	(A)(H)	89, 200, (420), 535
1315	(5)	(G)	89, 200, 420
1400	17	(A)	89, 535
1615	3	(A)	89, 200
1815	4	(A)	89
1935	1	(A)	89

The spectra of coincidence of most of the γ rays were measured. In Fig. 3 the spectra of γ rays whose energy is higher than 500 kev in coincidence with the 89-kev, 200-kev, 260-kev, 292-kev, 355-kev, 420-kev, 535-kev, and 770-kev γ rays are shown. All the measurements of coincidence spectra were carried out with an angle of 180° between the detectors. In the measurements of the spectra of coincidences with the 200-kev and 260-kev γ rays, an 8-mm lead absorber was placed between the source and the scintillator which detected the γ rays whose energy was higher than 600 kev, in order to prevent coincidences arising from Compton scattering from one counter into the other.

An analysis of the spectrum of coincidences with the 40-kev x-rays shows that Tb^{156} decays by K capture and that the lifetimes of the excited states are shorter than 10^{-7} sec. The experimental results obtained concerning the energies, the relative intensities, and the cascade relations of the γ rays emitted in the decay of Tb^{156}, are given in Table I. The values given for the energies of the γ radiations may be in error by about 3% and those given for the relative intensities may be in error by about 30%. The values given in brackets are uncertain. Relative intensities derived from the analysis of the singles spectrum are marked by (A). These derived from the analysis of the coincidence spectrum triggered by the 200-kev, 260-kev, 292-kev, 355-kev, 420-kev, or 535-kev γ rays are marked by (C), (D), (E), (F), (G), or (H), respectively. A decay scheme of Tb^{156} consistent with the experimental results is given in Fig. 4.

Positron emission is energetically possible in the decay of Tb^{156}. A search for β^+ radiations was made using a three-crystal spectrometer, consisting of a 1-cm-thick anthracene scintillator placed between two 3-in. ×3-in. NaI scintillators (lead absorbers shielded the NaI crystals from direct radiations coming directly

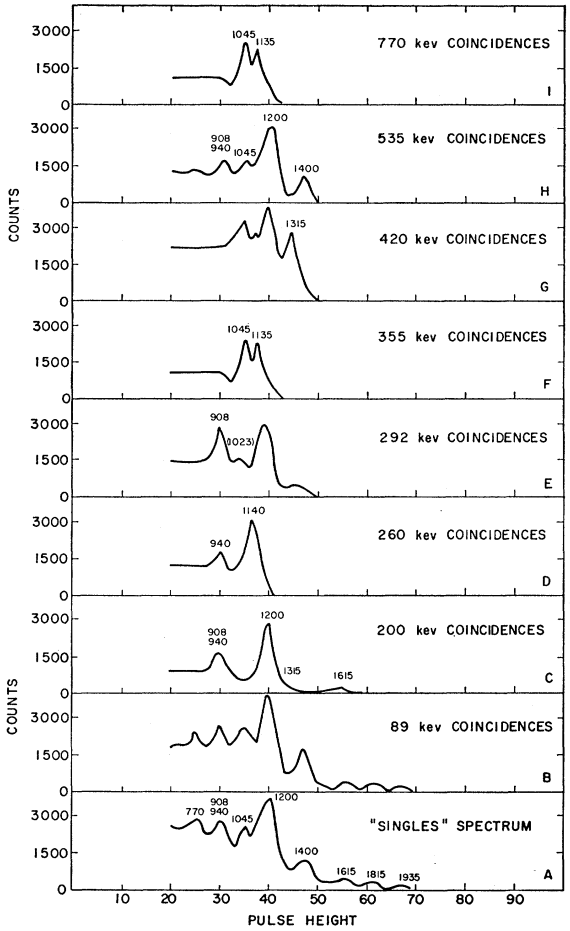

FIG. 3. Spectra of coincidences with 89-kev, 200-kev, 260-kev, 292-kev, 350-kev, 420-kev, 535-kev, and 770-kev γ rays of Tb^{156}.

from the source). The scintillators were operated in triple coincidence, and the analyzers of the two NaI scintillators were channelled on the photopeaks of 510-kev annihilation γ rays. The output pulses from the anthracene scintillator were analyzed by a 100-channel pulse analyzer gated by the output of the fast-slow coincidence circuit. The number of coincidences obtained was so small (and many of them could result from γ rays absorbed by the pair-production effect in the anthracene crystal) that only an upper limit of 5×10^{-4} could be given for the ratio of β^+ emissions to K capture in the decay of Tb^{156}.

3. Angular Correlations of Gamma-Ray Cascades

The angular correlations of gamma-ray cascades following the decay of Tb^{156} were measured using a standard fast-slow coincidence circuit and adjusting the channels of the pulse-height analyzers to select pulses belonging to the full peaks of the two γ rays. The resolving time of the fast coincidence circuit was $2\tau=4\times10^{-8}$ sec. The radiations were detected by two 3-in.$\times$3-in. NaI(Tl) scintillation counters. For both counters the distance between the source and the crystal face was 14 cm. The angle between the counters was changed automatically every 5 minutes by 30° in the region between 90° and 270°. The number of single counts and coincidences were recorded at each angle on registers. A 10-μC source in the form of a dilute aqueous solution of $Tb(NO_3)_3$ was used for all the angular correlation measurements.

The angular correlation function obtained for the 535 kev–1400 kev after correcting for the finite solid angle of the detectors was

$$P(\theta)=A[1+(0.108\pm0.010)P_2(\cos\theta)+(0.00\pm0.01)P_4(\cos\theta)]. \quad (1)$$

In the measurement of the angular correlation of the 535 kev–1200 kev cascade, the channel of one pulse-height analyzer was adjusted to select only pulses belonging to the higher half of the full peak corresponding to the 1200-kev radiation to avoid the counting of coincidences belonging to the 1134-kev and 1140-kev radiations. The angular correlation function obtained for the 535 kev–1200 kev cascade was

$$P(\theta)=A[1-(0.11\pm0.02)P_2(\cos\theta)+(0.00\pm0.01)P_4(\cos\theta)]. \quad (2)$$

The large error given on the coefficient of $P_2(\cos\theta)$ results from the uncertainty in the corrections required due to the fact that about 15% of the coincidences

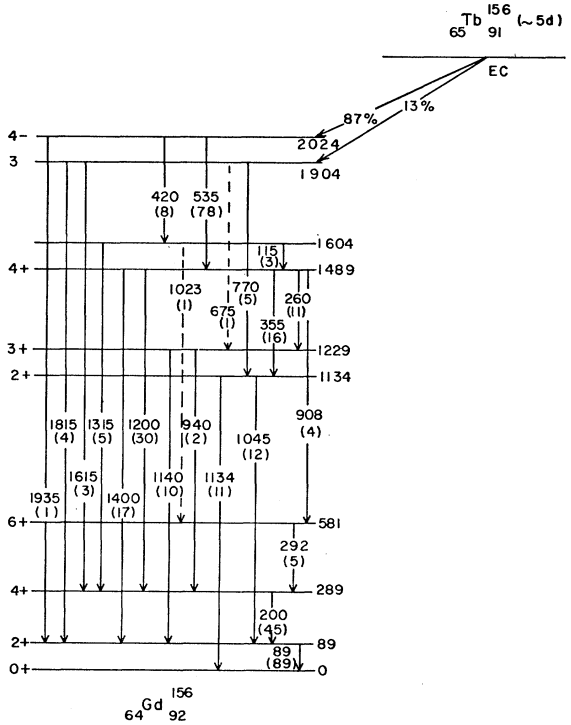

FIG. 4. Decay scheme of Tb^{156}. Gamma-ray energies are given in kev and intensities in percent per disintegration.

counted belonged to the 535 kev–1400 kev, 535 kev–1140 kev, and 535 kev–1134 kev cascades.

The angular correlation function obtained for the 535 kev–355 kev cascade was

$$P(\theta)=A[1+(0.13\pm0.02)P_2(\cos\theta)+(0.00\pm0.02)P_4(\cos\theta)]. \quad (3)$$

In this case, too, the errors given on the coefficients result mainly from the fact that about 20% of the coincidences counted belong to gamma-ray cascades whose second components are γ rays with energies higher than 535 kev absorbed in the crystal by the Compton effect.

In the angular correlation measurements of cascades including the 200-kev γ radiation, the crystal which detected the higher-energy γ rays was shielded with a 6-mm lead absorber in order to prevent coincidences arising from Compton scattering from one counter into the other. The angular correlation function obtained for the 1615 kev–200 kev cascade was

$$P(\theta)=A[1-(0.115\pm0.015)P_2(\cos\theta)+(0.015\pm0.015)P_4(\cos\theta)]. \quad (4)$$

The angular correlation of the 1200 kev–200 kev cascade was measured by adjusting the channel of one pulse-height analyzer to select pulses belonging to the photopeak of the 200-kev radiation and the channel of the second pulse-height analyzer to select pulses belonging to the higher half of the full peak corresponding to the 1200-kev radiation, in order to avoid the counting of coincidences belonging to the 1134 and 1140 radiations. The angular correlation function found was

$$P(\theta)=A[1+(0.04\pm0.01)P_2(\cos\theta)+(0.11\pm0.02)P_4(\cos\theta)]. \quad (5)$$

When one of the pulse-height analyzers was channelled on the 200-kev photopeak and the second pulse-height analyzer was channelled on the composite photopeak of the 908-kev and 940-kev radiations, the angular correlation function found was

$$P(\theta)=A[1+(0.09\pm0.01)P_2(\cos\theta)+(0.03\pm0.01)P_4(\cos\theta)]. \quad (6)$$

The spectrum of coincidences with the 200-kev γ rays showed that when the angle between the detectors was 180°, about 50% of the coincidences counted belonged to the 1200 kev–200 kev cascade. Taking into account the function obtained for the 1200 kev–200 kev cascade one gets for the angular correlation of the mixture of 908 kev–200 kev and 940 kev–200 kev cascades the function

$$P(\theta)\sim A[1+0.14P_2(\cos\theta)-0.05P_4(\cos\theta)]. \quad (7)$$

The angular correlation function obtained when the pulse-height analyzers were channelled on the 89-kev and 200-kev peaks was

$$P(\theta)=A[1+(0.024\pm0.004)P_2(\cos\theta)]. \quad (8)$$

An analysis of the spectra of coincidences with the 89-kev γ ray and with the 200-kev γ ray shows that about a third of the pulses counted belonged to coincidences with γ-ray energies higher than 200 kev which were absorbed in one of the crystals by the Compton effect. Because of the complexity of the spectrum of γ rays emitted in the decay of Tb^{156}, it is reasonable to assume that the angular correlation of these coincidences is isotropic. Hence the angular correlation function corresponding to the 200 kev–89 kev cascade is:

$$P(\theta)\sim A[1+0.04P_2(\cos\theta)]. \quad (9)$$

4. Internal Conversion Coefficients

The source used for conversion electron studies was prepared by bombarding 10 mg of a Eu_2O_3 powder in the cyclotron with 14-Mev α particles for about 30 hours. The intensity of the beam was about 30 μa. A sodium amalgam reduction method[9] for separating terbium from europium was used for improving the specific activity of the source by a factor of about 10. At the end of the chemical procedure the source was in a form of a dilute aqueous solution of $Eu(NO_3)_3$. The solution was evaporated onto a 1-mil-thick Al foil over an area of about 1 cm in diameter. The thickness of the source was about 1 mg/cm^2 and its intensity about 2 μC.

The measurements were performed with an iron-free intermediate-image spectrometer designed by Alburger.[10] The resolution setting of the spectrometer was 3% and its transmission about 5.5%. No continuous β^- spectrum was found and an upper limit of 1% was calculated for the ratio of β^- emissions to K captures in the decay of Tb^{156} (5 days). This result disagrees with some previous results.[11]

The conversion lines of many of the γ transitions listed in Table I were identified. But because of the complexity of the spectrum and the low specific activity of the source used, the conversion coefficients of only the 260-kev, 292-kev, 355-kev, 420-kev, 535-kev, 1200-kev, and 1400-kev transitions could be determined. The K-conversion coefficients were determined by comparing the relative intensities of the conversion lines with the relative intensities of the γ rays. If one assigns an $E2$ character to the 200-kev γ transition (the assignment is justified in the following paragraphs), the theoretical value of its K-conversion coefficient is 0.15. Relying on this value and on the experimental relative intensities of the γ transitions and their conversion lines, the conversion coefficients given in Table II were

[9] Shirley, Smith, and Rasmussen, Nuclear Phys. **4**, 395 (1957).
[10] D. E. Alburger, Rev. Sci. Instr. **27**, 991 (1956).
[11] Henry, Dillman, Gove, and Becker, Bull. Am. Phys. Soc. Ser. II, **2**, 341 (1957).

TABLE II. Conversion coefficients of Tb^{156} γ rays.

Energy of γ transition	Type of conversion	Relative intensity of conversion line	Conversion coefficient
200 kev	K	100	0.15
200 kev	$L+M$	49	0.07
260 kev	K	12	0.07
292 kev	K	(6)	(0.07)
355 kev	K	9	0.031
355 kev	$L+M$	2.7	0.009
420 kev	K	(5)	(0.008)
535 kev	K	6	0.0042
1200 kev	K	1.1	0.0019
1400 kev	K	0.3	0.0011

deduced. The possible errors on the conversion coefficients are estimated to be 35%. An upper limit of 3.5×10^{-3} was found for the K-conversion coefficients of the 908-kev, 940-kev, 1045-kev, 1135-kev, and 1140-kev transitions.

5. Measurement of Mean Life of 289-Kev Level of Gd^{156}

The half-life of the 289-kev level of Gd^{156} was measured using the electronic apparatus for lifetime measurements designed by Sunyar and described elsewhere.[12,13] Two plastic scintillators, one with a thickness of about 2 mm and the other with a thickness of about 2 cm, were used. The channel of the analyzer of the pulses of the thin scintillator was adjusted to select pulses corresponding to the K-conversion line of the 200-kev transition. The channel of the second pulse-height analyzer was adjusted to select pulses corresponding to electrons with energies in the region of 450 kev–500 kev obtained from γ rays with energies higher than $\sim$600 kev absorbed in the crystal by the Compton effect. With these adjustments no coincidences with the 535-kev γ rays were detected and the analysis of the results was independent of the lifetime of the 1489-kev level.[14] The output pulses from the fast coincidence circuit were analyzed by a 100-channel pulse-height analyzer gated by the output of the slow triple coincidence circuit. The points marked in Fig. 5 give the number of coincidences obtained with pulse heights higher than the corresponding values on the abscissa. The numbers were normalized so that the total number of coincidences was taken as 1. The spectrum of coincidence pulses obtained was compared to the spectrum obtained with a Cs^{134} source using the same channels as used with the Tb^{156} source. Curve A in Fig. 5 shows the integral spectrum obtained with the Cs^{134} prompt source. In order to measure the sensitivity of the shape of the coincidence spectrum to the lifetime of the intermediate state, 1×10^{-10}-sec, 2×10^{-10}-sec, and 3×10^{-10}-sec delay cables were introduced between the detector of the 200-kev conversion line and the fast coincidence circuit and the coincidence spectra were measured using a Cs^{134} source as shown in curves B, C, and D of Fig. 5. An analysis of the integral spectra shows that the mean lifetime of the 289-kev level is about 1.5×10^{-10} sec. A more accurate determination of the lifetime was made by calculating the positions of the centroids of the curves of differential spectra of coincidence pulse heights obtained with the Tb^{156} and Cs^{134} sources. In Fig. 6 a plot of the dependence of the position of the centroids obtained with a Cs^{134} source on the length of the delay inserted between the thin plastic scintillator and the fast coincidence circuit is shown. A value of $(1.5\pm0.3)\times10^{-10}$ sec was obtained for the mean lifetime of the 289-kev level from the position of the centroid of the curve obtained with the Tb^{156} source. The error given is not purely statistical but includes an estimate of possible experimental systematic errors.

ANALYSIS OF EXPERIMENTAL RESULTS

1. Internal Conversion Coefficients and Multipolarities

The values obtained by Henry *et al.*[6] for the K-conversion coefficients of the 89-kev and 200-kev transitions and the reported conversion electron relative intensities for these transitions[5,15] prove that both are $E2$ transitions. Table III presents the experimental K-conversion

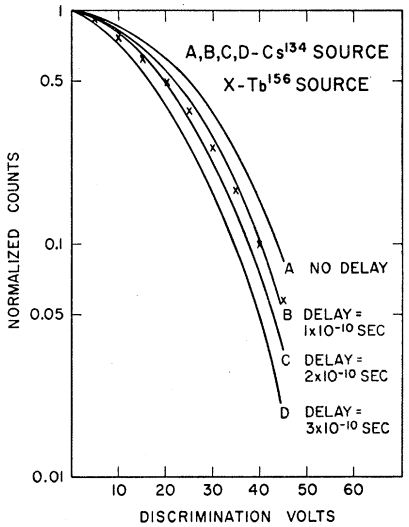

FIG. 5. Normalized integral spectra of coincidence pulse heights in the measurement of the 289-kev level lifetime.

[12] A. W. Sunyar, Bull. Am. Phys. Soc. Ser. II, 2, 37 (1957).

[13] A. W. Sunyar, *Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1958* (United Nations, Geneva, 1958).

[14] In the first stage of this measurement, the analyzer of the pulses from the thick plastic scintillator was adjusted to select pulses corresponding to the 535-kev and higher energy γ rays. When we became aware of the measurement of R. E. Bell and M. H. Jorgensen [Nuclear Phys. (to be published)] who found that the 1489-kev level has a half-life of about 2×10^{-10} sec, this adjustment was changed.

[15] Ewan, Knowles, and MacKenzie, Bull. Am. Phys. Soc. Ser. II, 2, 259 (1957).

coefficients together with Sliv and Band's theoretical values[16] for $E1$, $E2$, $M1$, and $M2$ radiations and the multipolarities assigned by comparing these sets of values. Only the dominant multipole order is indicated in the table, since the experimental uncertainties of the conversion coefficients preclude a more precise determination of the multipole mixtures.

2. Spins of the Excited States of Gd^{156}

A 2+ spin and parity assignment to the first excited level of Gd^{156} is strongly supported by the conversion data[5,6,15] and by the fact that this level is reached by Coulomb excitation of Gd^{156}. Energy considerations make it very likely that the 289-kev level can be classified as the third member of a rotational band associated with the ground state, in which case its spin is 4 and its parity even. This assignment is supported by the $E2$ character of the 200-kev transition[5,6,15] and the absence of a 289-kev cross-over to the ground state.

Energy considerations make it also likely that the 581-kev level can be classified as the fourth member of the rotational band associated with the ground state, in which case its spin and parity are 6+. The energy of this level does not show an appreciable deviation from proportionality with $I(I+1)$. If the energies of the rotational levels are represented by the expression $aI(I+1)+bI^2(I+1)^2$, as suggested by Bohr and Mottelson,[17] an excellent agreement is found between the experimental energies of the three first excited states of Gd^{156} and the calculated ones. A value of $a/b=-560$ is needed for the best agreement. The 6+ assignment is also in agreement with the absence of crossover transitions from the 581-kev level to the 89-kev level and to the ground level.

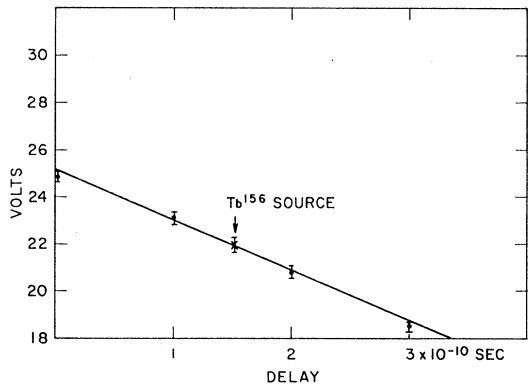

FIG. 6. Plot of the dependence of the position of the centroid of the curve of the differential spectrum of coincidence pulse heights obtained with a Cs^{134} source on the length of the delay inserted between the thin plastic scintillator and the fast coincidence circuit.

TABLE III. K-conversion coefficients and multipolarity assignments.

Energy (kev)	Experimental	Theoretical $E1$	$E2$	$M1$	$M2$	Assigned multipolarity
260	0.07	0.019	0.074	0.12	0.54	$E2$
292	(0.07)	0.014	0.051	0.086	0.37	$(E2)+(M1)$
355	0.031	0.0088	0.028	0.051	0.19	$E2$
420	0.008	0.006	0.018	0.033	0.115	$E1$
535	0.0042	0.0035	0.010	0.019	0.054	$E1$
1200	0.0019	0.0007	0.0016	0.0026	0.0062	$(E2)+(M1)$
1400	0.0011	0.0005	0.0012	0.0012	0.0042	$E2$

The existence of $E2$ transitions from the 1489-kev level to the 289-kev and 89-kev levels whose spins are 4 and 2, respectively, proves that the spin of the 1489-kev level is 2, 3, or 4. The value 3 must be rejected because of the sign obtained for the coefficient of $P_4(\cos\theta)$ in the correlation function of the 1200 kev–200 kev cascade. A 2 assignment has to be ruled out because of the value 0.07 ± 0.01 obtained for the coefficient of $P_2(\cos\theta)$ in the 1200 kev–200 kev cascade which does not fit a 2(2)4(2)2 cascade. Hence, the conclusion is that the 1489-kev level has a 4+ character.

The $E2$ character of the 355-kev transition from the 1489-kev level to the 1134-kev level, together with the existence of transitions from the 1134-kev level to the ground state and first excited state, show that the spin and parity of the 1134-kev level are 2+. This assignment is also in agreement with the experimental angular correlation function obtained for the 535 kev–355 kev cascade.

The existence of transitions from the 1904-kev level to the 289-kev and 89-kev levels whose spins are 4 and 2, respectively, indicates that the spin of the 1904-kev level is 2, 3, or 4. The experimental angular correlation function obtained for the 1615 kev–200 kev cascade can be fitted to the theoretical one only if a spin 3 is assigned to this level.

The fact that the 535-kev transition going from the 2024-kev level to the 1489-kev (4+) level is predominantly of the $E1$ type indicates that the spin and parity of the 2024-kev level is 3−, 4−, or 5−. A 5− assignment is unlikely because of the existence of a transition from the 2024-kev level to the 89-kev (2+) level. The angular correlation functions found for the 535 kev–1400 kev, 535 kev–1200 kev, and 535 kev–355 kev cascades cannot be fitted to the theoretical ones if a value of 3 is assigned to the spin of the 2024-kev level. Hence, the most likely spin and parity assignment to the 2024-kev level is 4−.

The $E2$ character of the 260-kev transition going from the 1489-kev level to the 1229-kev (4+) level, the existence of transitions from the 1229-kev level to the 289-kev (4+) and 89-kev (2+) levels and the absence of a transition from this level to the ground level indicate that the spin and parity of the 1229-kev level are 4+ or 3+. The experimental angular corre-

[16] L. A. Sliv and J. M. Band, Leningrad Physico-Technical Institute Report, 1956 [translation: Report 57 ICCK1, issued by Physics Department, University of Illinois, Urbana, Illinois (unpublished)].

[17] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **27**, No. 16 (1953).

lation function (4) obtained for the mixture of 908 kev–200 kev cascade and 940 kev–200 kev cascades cannot be fitted to theoretical functions if a spin 4 is assigned to the 1229-kev level. The theoretical function calculated for the one-three gamma-gamma angular correlation of the 908 kev–200 kev cascade is

$$P(\theta)=A[1+0.163P_2(\cos\theta)+0.044P_4(\cos\theta)]. \quad (10)$$

assuming that it is a 4(2)6(2)4(2) cascade. Taking into account the relative intensities of the 908-kev and 940-kev transitions and the functions (4) and (9), one gets the following angular correlation function for the 940 kev–200 kev cascade:

$$P(\theta)\sim A[1+(0.09\pm0.05)P_2(\cos\theta) - (0.24\pm0.12)P_4(\cos\theta)]. \quad (11)$$

The negative sign obtained for the coefficient of $P_4(\cos\theta)$ rules out the possibility that the spin of the 1229-kev level is 4. The suggested spin and parity assignment for the 1229-kev level is therefore 3+.

The most likely spin and parity assignments for the 1604-kev level are 4+ or 5+.

3. Analysis of the Results of Angular Correlation Measurements

From the previous discussions it follows that the 535 kev–1400 kev cascade is of the 4(1,2)4(2)2 type. The experimental angular correlation function (1) obtained for this cascade fits the theoretical function for such a cascade if $\delta^2=0.045\pm0.015$, where δ^2 denotes the $M2/E1$ ratio for the 535-kev transition.

The 535 kev–1200 kev cascade is of the 4(1,2)4(1,2)4 type. The experimental angular correlation function (2) obtained for this cascade fits the theoretical function for such a cascade if $\delta=-2.3_{+0.8}{}^{-2.7}$, where δ denotes the ratio between the $E2$ and $M1$ matrix elements corresponding to the 1200-kev transition. This value of δ was derived by using the value found for the mixing ratio of the 535-kev transition.

The experimental angular correlation function (3) obtained for the 535 kev–355 kev cascade is in agreement with the theoretical function for a 4(1,2)4(2)2 cascade, if the value found for the mixing ratio of the 535-kev transition is used.

The 1615 kev–200 kev cascade is of the 3(1,2)4(2)2 type. The experimental angular correlation function (4) obtained for this cascade fits the theoretical function if $\delta<1/20$ where δ is the mixing ratio between the $M2$ and $E1$ matrix elements corresponding to the 1615-kev transitions, i.e., the $M2/E1$ ratio in the 1615-kev transition is smaller than 1/400.

The experimental angular correlation function (5) obtained for the 1200 kev–200 kev cascade, is in agreement with the theoretical function for a 4(1,2)4(2)2 if $\delta=2.3\pm0.3$ where δ denotes the ratio between the $E2$ and $M1$ matrix elements corresponding to the 1200-kev transition. This value of δ is in agreement with the value obtained from the analysis of the angular correlation function of the 535 kev–1200 kev cascade. The analysis of the angular correlation function was made using the formulas and tables of Biedenharn and Rose[18] and the fact that different signs of δ were found for the 1200-kev transition in a case where it was the first component of a cascade and in a case where it was the second component is in agreement with theory.[19] The value obtained for δ indicates that for the 1200-kev radiation the $E2$ transition probability is (5.3 ± 1.3) times larger than the $M1$ transition probability, although $\Delta I=0$.

It is difficult to determine the mixing ratio of the 940-kev γ ray from the analysis of the angular correlation function found for the 940 kev–200 kev cascade, because of the large possible errors in the values of the coefficients of this angular correlation. In spite of these large errors, it can be concluded that the $E2/M1$ ratio for the 940-kev transition is larger than 2, although $\Delta I=1$.

The 200 kev–89 kev cascade is of the 4(2)2(2)0 type. The theoretical angular correlation function for such a cascade is

$$P(\theta)=A[1+0.102P_2(\cos\theta)+0.009P_4(\cos\theta)]. \quad (12)$$

A comparison between this function and the experimental function (9) shows that the angular correlations of cascades whose intermediate state is the 89-kev level are strongly attenuated even when the source is in the form of a dilute aqueous solution and that the attenuation factor is ~0.4. This strong attenuation is not very surprising in the light of the fact that it was shown that the angular correlations of Tb^{160} γ-ray cascades whose intermediate state is the first excited level of Dy^{160} and the angular distribution of Coulomb-excited γ rays of Gd^{156} are strongly attenuated.[20,21]

DISCUSSION

The properties of the energy levels of Gd^{156} are very similar to those of neighboring even-even nuclei such as Sm^{152},[1,2] Gd^{154},[22] Tb^{160},[3,20] W^{182},[23] and W^{184}.[24] It seems that this similarity originates from the fact that many of the excited states of these nuclei are of the collective type.

It is possible to interpret the 89-kev, 289-kev, and 581-kev levels of Gd^{156} as rotational levels belonging to the ground state with $K=0$ (K is the quantum number which represents the projection of the total angular momentum on the axis of symmetry of the nucleus[17]).

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[24] Gallagher, Jr., Strominger, and Unik, Phys. Rev. **110**, 725 (1958).

According to the collective model, the transition probability for an $E2$ transition $(I+2\rightarrow I)$ between 2 states of a rotational band belonging to the ground state of an even-even nucleus is given by[17]

$$B_e(2)=\frac{15}{32\pi}e^2Q_0^2\frac{(I+1)(I+2)}{(2I+3)(2I+5)}. \quad (13)$$

From this follows

$$\frac{B_e(2)(4+\rightarrow 2+)}{B_e(2)(2+\rightarrow 0+)}=\frac{10}{7}. \quad (14)$$

Nathan measured the half-life of the 89-kev level of Gd^{156} and found it to be 1.9×10^{-9} sec.[7] In the present work the mean life of the 289-kev level was found to be $(1.5\pm0.3)\times10^{-10}$ sec. In order to compare the "reduced" γ-transition probabilities from the 4+ and 2+ levels, it is necessary to know the values of the conversion coefficients of the 200-kev and 89-kev transitions. Values of 4.05 and 0.21 were found for the conversion coefficients of the 89-kev and 200-kev γ rays by interpolation from Rose's K, L, and M internal conversion coefficients.[25] The experimental γ-transition probabilities ratio obtained for Gd^{156} was

$$\frac{B_e(2)(4+\rightarrow 2+)}{B_e(2)(2+\rightarrow 0+)}=1.4\pm0.4. \quad (15)$$

The large error given on the ratio results from the possible errors on the two half-lives and the conversion coefficient values. Such ratios were already measured for Hf^{180}, Os^{186}, and Os^{190}. The ratio obtained for Hf^{180} was 1.21 ± 0.5,[26] that obtained for Os^{186} was 1.0 ± 0.4, and that obtained for Os^{190} was 0.6 ± 0.3.[26] The conclusion is that the agreement with the theoretical value (1.43) is better for Gd^{156} and Hf^{180} which belong to the strong-coupling region of nuclei than for Os^{190} which lies in the transition region between the nuclei with rotational and those with near-harmonic level schemes.[26]

The following properties of the 1134-kev and 1229-kev level support the suggestion to interpret them as collective vibrational levels with $K=2$. The spins and parities of these levels are 2+ and 3+, respectively, their excitation energies are about 1 Mev, the energy separation between them is almost equal to the energy separation between the 89-kev level and the ground level, the $E2/M1$ ratio in the 940-kev γ transition is large, and the branching ratios of the γ transitions from these levels are in agreement with the theoretical ratios if it is assumed that $K=2$ for them. These are the properties predicted by the unified model for gamma vibrational levels.[27] The weakness of the $M1$ transitions from these levels to the levels belonging to the rotational band of the ground state is explained by the K-forbiddenness of such transitions. The theoretical branching ratios according to the unified model for the transitions from the 1134-kev level to the ground level, the 89-kev level, and the 289-kev level, assuming that $I=2$ and $K=2$ for the 1134-kev level, are $I(1134\text{ kev}):I(1045\text{ kev}):I(845\text{ kev})=1.04:1:0.017$, and the experimentally obtained ratios are $0.92:1:<0.05$. The theoretical branching ratio for the transitions from the 1229-kev level, assuming that $I=3$ and $K=2$ for this level, is $I(1140\text{ kev}):I(940\text{ kev})=6.5$ and the experimental ratio is 5. Taking into account the errors possible in the experimental values, the agreement is good.

The fact that the 1489-kev level lies above the 1134-kev and the 1229-kev levels and that its spin and parity are 4+ supports a suggestion to interpret it too as a vibrational level with $K=2$. But on the other hand, the following experimental facts disagree with such an assignment: (a) The energy separation between this level and the 1229-kev level is about two times larger than that expected from the separation between the 1229-kev and 1134-kev levels, assuming that the three levels belong to the same band with $K=2$. (b) The theoretical branching ratios from the transitions from the 1489-kev level assuming that $I=4$ and $K=2$ for this level are $I(1400\text{ kev}):I(1200\text{ kev}):I(908\text{ kev})=0.735:1:0.021$ whereas the experimental ratios are $I(1400\text{ kev}):I(1200\text{ kev}):I(908\text{ kev})=0.57:1:0.133$. Hence the relative intensity of the 908-kev transitions is about 6 times larger than that expected from theory. (c) The reduced probabilities of the transitions from the 1489-kev level to the 1229-kev and 1134-kev level with $K=2$ are about 1000 times larger than the reduced probabilities of the transitions from this level to the 289-kev and 89-kev levels with $K=0$. (d) The probability of the transition from the 2024-kev (4−) level to the 1489-kev was found to be at least 50 times larger than the probability of the transition to the 1229-kev level. (e) The 260-kev transition from the 1489-kev level to the 1229-kev level is mainly of the $E2$ type although $\Delta I=1$. It is difficult to explain all these experimental facts if $K=2$ for the 1489-kev level. $K=4$ assignments to both the 1489-kev (4+) level and the 2024-kev (4−) level are in agreement with most of the experimental results. The weakness of the $E1$ transition from the 2024-kev level to the 1229-kev level and the weakness of the transitions from the 1489-kev level to the $K=0$ band levels can be explained by the K-forbiddenness of such transitions. The transition from the 2024-kev (4−) level to the 289-kev (4+) level is highly forbidden by K-selection rules ($\Delta K=4$) and this explains why such a transition was not found and its

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reduced probability was calculated to be at least 15 000 times smaller than the reduced probability of the 535-kev transition. The fact that the 260-kev transition from the 1489-kev level to the 1229-kev level is mainly of the $E2$ type can be explained by the K-forbiddenness of $M1$ transitions between these levels ($\Delta K=2$).

The fact that no $K=0$ bands associated with quadrupole and octupole vibrations are observed in the decay of Tb^{156} (5-day) can be accounted for by the action of K-forbiddenness. From the proposed decay scheme it seems reasonable to assign $I=K=3$ or 4 to the ground state of Tb^{156} in which case transitions from it to $K=0$ levels of Gd^{156} are K-forbidden. K selection rules and competition of more favorable transitions can explain why high-lying $K=0$ levels are not populated by γ transitions from excited states of Gd^{156} lying above them.

ACKNOWLEDGMENTS

I would like to thank Dr. D. E. Alburger for the use of his spectrometer, Dr. A. W. Sunyar for the use of his apparatus for short-lifetime measurements, and Dr. C. P. Baker for arranging the cyclotron bombardments.

PHYSICAL REVIEW VOLUME 115, NUMBER 2 JULY 15, 1959

Photonuclear Reaction Energies*

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(Received December 15, 1958)

The general problem of determining photonuclear reaction energies in the region below 25 Mev is discussed, and the need for absolute calibration of betatron energy scales is demonstrated. A system for accomplishing such a calibration is briefly described and the results of some measurements made with it are given. The $Cu^{63}(\gamma,n)Cu^{62}$ threshold was observed to be at 10.78±0.03 Mev, and two well-defined "breaks" in the $O^{16}(\gamma,n)O^{15}$ yield curve are at 16.19±0.04 and 17.25±0.04 Mev. These "breaks" lead to the assignment of energy levels in O^{16} which are in excellent agreement with those observed in the $N^{15}(p,n)O^{15}$ reaction.

INTRODUCTION

IN recent years it has been demonstrated that the operating energy of a betatron can be controlled to 0.15% over a period of many days and 0.025% for a period of hours.[1–5] Unfortunately, while it is possible to control the operating energy with this precision it has not been possible to assign a value to this energy which is commensurate with even the long-term stability. For example, it will be seen in Table IV that disparities as large as 1.4% in energy assignments have occurred in the literature since 1955.

The energy scale of a betatron is usually determined by comparing observed reaction thresholds to the values of those thresholds predicted from mass and beta-decay data. This comparison serves to determine a calibration curve for the energy controlling system of the betatron. The method requires that the thresholds of the "standard" reactions be accurately known, and that the experimental determination of the threshold be unambiguous. These requirements have not always been fulfilled, as is shown below.

Even recently (1957) the accepted values for the "standard" thresholds have shifted due to changes in the accepted beta decay energies. For example, the $O^{16}(\gamma,n)O^{15}$ threshold has jumped from 15.605±0.012 Mev[6] to 15.657±0.007 Mev[5]; a change of over four times the larger quoted error. The threshold values have also changed due to changes in the accepted mass values. For example, Table I shows mass data for copper, whose (γ,n) threshold has frequently been used as a calibration standard. Here again, the difference between the two measurements is many times the larger quoted error.

Aside from the above difficulties, in the case of the light elements there is another problem. For these elements it is difficult to experimentally determine the

TABLE I. The $Cu^{63}-Ni^{62}-1$ mass difference.

Collins *et al.*[a]	2.446±0.1 mMU
Quisenberry *et al.*[b]	1.574±0.01 mMU

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* Research supported by a joint program of the Office of Naval Research and the U. S. Atomic Energy Commission.
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