Statistics of Slow-Neutron "Negative" Energy Resonances

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Neutron cross sections measured with very slow neutrons often reveal the influence of levels below the neutron binding energy. The published data on 20 isotopes are analyzed in such a way that the properties of the levels below and above the binding energy may be compared. It is found, in particular, that the neutron-width and level-spacing distributions for these (− and +) levels are the same within the limits of the present analysis.

I. INTRODUCTION

WHEN nuclei are bombarded with slow neutrons, effects due to levels below the neutron binding energy in the compound nuclei are sometimes observed. Such levels are frequently denoted by the descriptive phrase "negative-energy levels," and it is usual to assume that they are in every way similar to the levels which are seen with "positive-energy" neutrons. In the following this will be called the equivalence hypothesis. So far this hypothesis has not been tested, and in this paper a simple over-all check of it will be described.

If a comparatively large negative-energy level occurs close to the neutron binding energy, then the well-known effects of such levels will be sufficiently clear cut for approximate values of the level parameters to be determined, (e.g., if there is a level occurring at -2 ev the total neutron cross section will vary roughly as $E^{-\frac{1}{2}}$ from 0 to nearly $+2$ ev and at higher energies will go over to a $E^{-\frac{5}{2}}$ behavior, so producing the characteristic "knee" in the cross-section curve).

At first sight the data on such levels contradict the equivalence hypothesis because in the eight cases of negative-energy levels for which parameters are available in the literature,[1] the neutron width has turned out to be, on the average, six times that of the mean value for corresponding positive-energy levels. However, it is possible that this effect can be explained quantitatively merely in terms of the experimental difficulties in the determination of any but the largest negative-energy levels. This question and the question of the validity of the equivalence hypothesis can be answered only by extracting some information from the neutron cross sections of every nucleus which has been studied. At the present time the only quantity which may be used for this purpose is the thermal-neutron capture cross section; and, in what follows, the details of this quantity will be discussed.

TABLE I. Summary of data.

Isotope	σ_c	σ_+	σ_-	R_+	R_-
Pu^{240}	285 ± 15	300 ± 30	$\leqslant 40$	25 ± 5	$\leqslant 3.5$
U^{238}	2.70 ± 0.04	2.90 ± 0.02	$\leqslant 0.3$	1.4 ± 0.1	$\leqslant 0.13$
U^{236}	8 ± 2	3 ± 1	5 ± 3	1.1 ± 0.5	1.8 ± 0.5
U^{234}	103 ± 8	3 ± 1	100 ± 8	1.1 ± 0.5	37 ± 3
Th^{232}	7.45 ± 0.15	0.78 ± 0.27	6.67 ± 0.43	0.45 ± 0.15	3.8 ± 0.3
Au^{197}	98 ± 1	97.3 ± 3	$\leqslant 5$	9 ± 0.5	$\leqslant 0.5$
W^{186}	34 ± 3	35 ± 7	$\leqslant 10$	27 ± 7	$\leqslant 8$
W^{183}	11 ± 1	5.7 ± 0.5	5.3 ± 1.2	0.8 ± 0.1	0.8 ± 0.2
W^{182}	19 ± 2	12 ± 2	7 ± 3	6.5 ± 1.5	3.8 ± 1.5
Ta^{181}	21 ± 3	13.7 ± 2.5	7.6 ± 3	0.8 ± 0.2	0.5 ± 0.2
Hf^{179}	65 ± 15	9 ± 3	56 ± 15	0.3 ± 0.1	21 ± 5
Hf^{177}	380 ± 30	400 ± 40	$\leqslant 40$	4 ± 0.5	$\leqslant 0.4$
Tm^{169}	118 ± 6	60 ± 20	60 ± 20	2.2 ± 0.8	2.2 ± 0.8
Ho^{165}	64 ± 3	23 ± 6	41 ± 7	3 ± 0.8	5 ± 1
Tb^{159}	44 ± 4	12 ± 4	32 ± 6	0.25 ± 0.08	0.7 ± 0.15
Pr^{141}	11.2 ± 0.6	3 ± 1.5	8.2 ± 1.5	0.8 ± 0.4	2.0 ± 0.8
Cs^{133}	29 ± 1.5	16 ± 3	13 ± 4	1.5 ± 0.2	1.2 ± 0.5
I^{127}	6.7 ± 0.6	2.9 ± 1.0	3.8 ± 1.2	0.2 ± 0.08	0.25 ± 0.08
Ag^{109}	84 ± 7	93 ± 5	$\leqslant 4$	3 ± 0.1	$\leqslant 0.25$
Ag^{107}	30 ± 2	2.6 ± 0.05	27 ± 3	0.3 ± 0.1	3 ± 0.4

II. ANALYSIS OF EXPERIMENTAL CROSS-SECTION DATA

The contribution to the thermal-neutron capture cross section from many noninterfering levels is[2]

$$\sigma_c = \frac{\pi}{kk_0}\sum_r \frac{g\Gamma_n{}^0\Gamma_\gamma}{E_r{}^2}, \tag{1}$$

provided the resonance energies, E_r, are much greater than either the level width or the energy of thermal neutrons, and this is true in most cases. In (1) g is a spin factor, k and k_0 are the neutron wave numbers at thermal and 1 ev, respectively, and $\Gamma_n{}^0$ and Γ_γ are the neutron and radiative capture widths, respectively.

Sufficient levels have been studied now with high-resolution neutron spectrometers that the sum at (1) may be evaluated fairly accurately for positive-energy levels in many heavy nuclei. The spin factor g does not present a problem since in the neutron spectrometer work the product $g\Gamma_n{}^0$ is determined. Also due to the $E_r{}^{-2}$ factor the sum rapidly converges, and the part of the energy scale above the upper limit of the measurements may be summed over with sufficient accuracy by replacing the sum with an integral involving only average values of the level parameters.

In this way, a quantity σ_+, being the thermal-neutron capture cross section due to positive-energy levels can be obtained, and by subtracting σ_+ from the capture cross section measured at thermal energies, the corresponding capture cross section due to negative-energy levels (σ_-) may be derived. In most cases it is found

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[1] *Neutron Cross Sections*, compiled by D. J. Hughes and J. Harvey, Brookhaven National Laboratory Report BNL-325 (Superintendent of Documents, U. S. Government Printing Office, Washington, D. C., 1955; second edition, 1958).

[2] H. A. Bethe, Revs. Modern Phys. 9, 69 (1937).

that σ_+ can be determined to an accuracy of about $\pm 20\%$. In twenty heavy isotopes[1] extending from silver to plutonium, only six cases were found in which σ_- was indeterminate, and so can be quoted only as being less than a certain limit. These data are given in Table I.

Unfortunately σ_+ or σ_- cannot be compared directly for different nuclei because they are proportional to the radiation width, strength function, and level spacing of the nuclei concerned and contain also some statistical factors. To reduce these cross sections to standard quantities, we make three assumptions regarding the properties of the two possible spin sequences of levels: (a) the two states have the same strength function (S), (b) the level spacings are proportional[2] to $(2J+1)^{-1}$, and (c) an average radiation width (Γ_γ) may be used for all levels in both states.

With these assumptions Eq. (1) reduces to

$$\sigma_c = \frac{4\pi\bar{\Gamma}_\gamma S}{kk_0\bar{D}} R, \tag{2}$$

where $\bar{D}$ is the average level spacing for all levels observed with s-wave neutrons and R is a nondimensional statistical factor given by

$$R = \sum_r \frac{g\Gamma_n{}^0/\langle g\Gamma_n{}^0\rangle_{\text{Av}}}{(2E_r/\bar{D})^2}. \tag{3}$$

It will be assumed further than the R so defined may be compared for nuclei of zero or nonzero spin.

Equation (2) shows that σ_c is proportional to a fixed factor $\bar{\Gamma}_\gamma S/\bar{D}$ and a statistical factor R. A quantity like σ_+/σ_c describes the distribution of cross section between σ_+ and σ_-, and if it has a mean of 0.5 and is symmetrically distributed about the mean then it is very probable that both factors behave in the same way for positive- or negative-energy levels. The mean of σ_+/σ_c for the 20 cases considered is 0.56 and it is symmetrically distributed about 0.5 within the error introduced by the small number of isotopes taken. Thus to conclude this analysis the measured value of $\bar{\Gamma}_\gamma S/\bar{D}$ for positive-energy levels is assumed to hold for both positive and negative levels and the distribution of the statistical factor R can then be considered. Corresponding to the values of σ_+ and σ_- pairs of values for R (R_+ and R_-) are derived from Eq. (2), and the probability integrals for R_+ and R_- obtained from the data on the twenty heavy isotopes mentioned above are plotted in Figs. 1(a) and 1(b). As expected following the discussion of σ_+/σ_c, the distribution of R is, broadly speaking, the same for positive- or negative-energy levels.

III. THE DISTRIBUTION OF THE STATISTICAL FACTOR R

First the largest values of R are due to large levels near the origin and thus the value of R in these cases is fixed by the first term in the sum at (3). The inte-

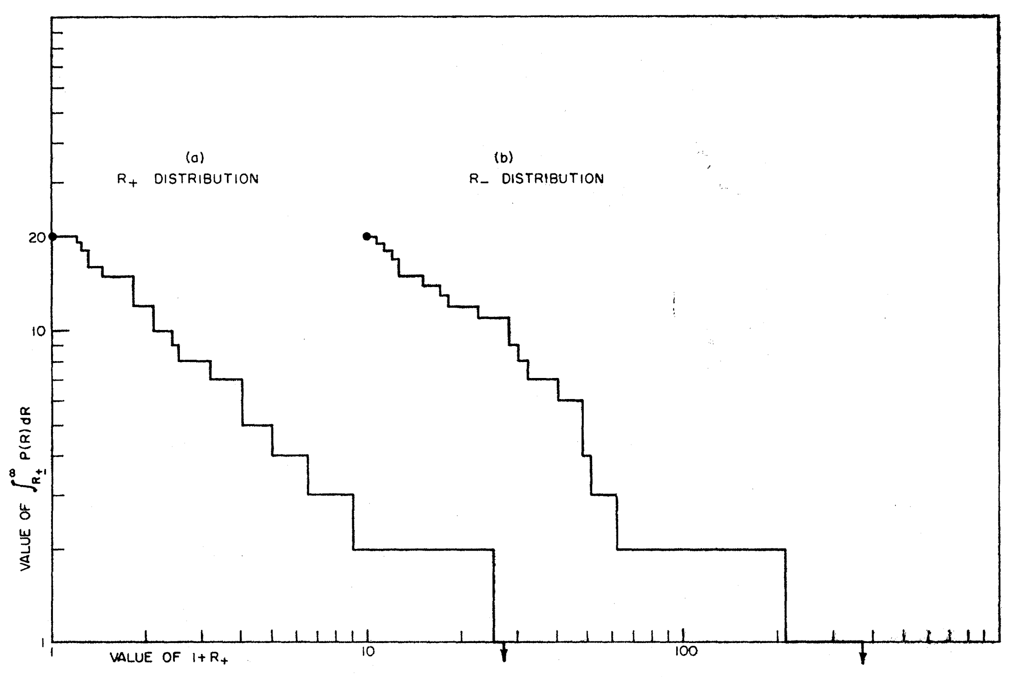

FIG. 1. The integrated probability distribution for the statistical factor R. (The six indeterminate values of R_-, have been plotted at half their upper limits.)

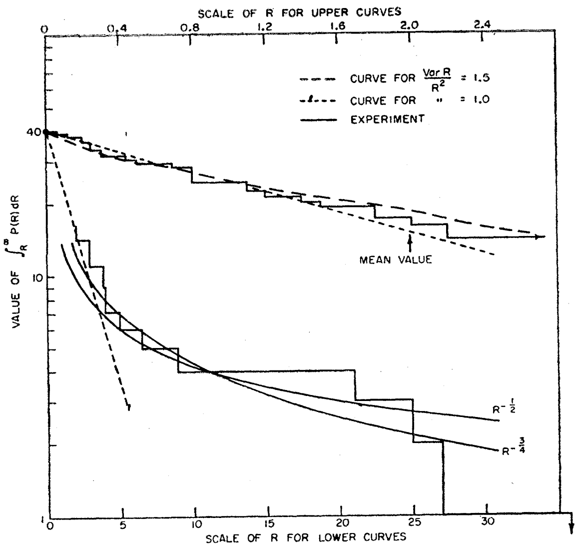

FIG. 2. The combined probability distributions for R_+ and R_-.

grated probability distribution for this case goes roughly as $R^{-\frac{1}{2}}$ if $\Gamma_n{}^0$ follows a Porter-Thomas distribution and E_r is random. [The randomness of E_r is a good approximation if small values of E_r only are considered, which is the condition for large R. Actually we must have $\Gamma_n+\Gamma_\gamma \ll E_r \ll \bar{D}$, and $R \gtrsim 3$. This approximation appears to be valid for the largest $\frac{1}{3}$ of R values—see below.]

Secondly, for small R several levels contribute to the sum at (3), and from an examination of the details of the positive energy levels a rough division of values of R into large R and small R groups may be made. If one fixes arbitrarily a level of significance of $\geq 20\%$ of R, it is found for the positive levels that in $\frac{2}{3}$ of the cases more than one level is significant. Thus as stated above we may expect that the $R^{-\frac{1}{2}}$ distribution is valid approximately for the largest $\frac{1}{3}$ of observed R values. The mean number of significant levels for the smallest $\frac{2}{3}$ of observed R_+ values is 2.2, and for the isotopes considered the number of significant levels is never greater than 3. Thus only 2 or 3 terms in (3) are important for small R, and using a Porter-Thomas distribution for $\Gamma_n{}^0$ and a random distribution for E_r with a cutoff at $\bar{D}$, we find for small R that Var. $R/\bar{R}^2$ is between 1 and 2, and $\bar{R}\sim 2$.

The combined distribution for R_+ and R_- is shown in Fig. 2, compared with the above predictions. It is found that the $\frac{2}{3}$, $\frac{1}{3}$ division is realistic and that for small R, Var. $R/\bar{R}^2$ is between 1 and 1.5, and $\bar{R}=2$, while for large R the distribution is between $R^{-\frac{1}{2}}$ and $R^{-\frac{3}{4}}$. (The mean value for large R is, from the nature of the distribution, very large.)

Thus the distribution of R and absolute values of R are quite near the theoretically expected behavior.

IV. CONCLUSION

The neutron width distribution for negative-energy levels is very broad (consistent with Porter-Thomas distribution), and the distribution of positions of levels and the value of $\bar{\Gamma}_\gamma S/\bar{D}$ on either side of the neutron binding energy is closely the same. The equivalence hypothesis is verified to a first approximation, and it is confirmed that experimental difficulties must account for the fact that observable negative-energy levels have large parameters.

The accuracy of the present analysis is poor. As more data become available, a more detailed analysis will be attempted, including a proper calculation of the complete distribution of R.