Test of Global Symmetry in $K^- - p$ Reactions*

MARC H. ROSS AND GORDON L. SHAW
Indiana University, Bloomington, Indiana
(Received April 17, 1959; revised manuscript received July 21, 1959)

A method of testing the hypothesis that there is global symmetry of the pion-baryon interaction is proposed: Upon analyzing low-energy $K^- - p$ scattering data, one finds a variety of scattering length solutions which are compatible with the elastic scattering, charge exchange scattering, and total charged hyperon production. Our suggestion involves the use of the experimental Σ^+/Σ^- ratios (*a*) to test the global symmetry hypothesis (or any other quantitative description of the pion-hyperon interaction), (*b*) to reduce the ambiguity in the $\bar{K}-N$ scattering length solutions, (*c*) hence, to predict the Σ^0 cross section. We need to know the $\pi - Y$ phase shifts, in addition to the $\bar{K}-N$ scattering lengths in order to predict the Σ^+, Σ^- cross sections. If there is global symmetry of pion-baryon interactions, then we know the $\pi - Y$ phases in the absence of a $\bar{K}-N$ reaction channel. We demonstrate how the actual $\pi - Y$ phase shifts can be obtained, in a *nonperturbative* manner, from idealized $\pi - Y$ phases (i.e., in the absence of $\bar{K}-N$ reactions) and the $\bar{K}-N$ scattering lengths. Earlier proposals making use of the dependence of the hyperon production ratios on the $\pi - Y$ phase shifts are also examined in terms of this result. Certain fits to the present rough data with scattering lengths of negative real part are shown to be incompatible with global symmetry. The proposed analysis involves the assumption that the $\bar{K}$ is an isotopic doublet.

I. INTRODUCTION

It is the purpose of this paper to suggest a quantitative test for various theories of the pion-hyperon interaction. Any theory that gives a definite prediction for pion-hyperon scattering in the absence of a $\bar{K}-N$ reaction channel may be tested by analyzing $K^- - p$ reactions. Our proposal is based on the results of a paper[1] in which we considered the general problem of several open coupled *two-particle* channels near the threshold for one of them. In A, we derived relations (e.g., in the case of two open channels) which enable us to determine the actual phase shift in the second channel, from knowledge of the actual phase shift in the first channel and the phase shift in the second channel if it were decoupled from the first, in a *nonperturbative* manner. These relations should be quantitatively useful for interactions of short, well-defined range.

We want to consider, in this paper, low-energy ($\lesssim 100$ Mev) K^- incident on protons; thus we need only deal with incident *s* waves. We will assume that $K^- - \bar{K}^0$ form an isotopic spin $I=\frac{1}{2}$ (equal parity) doublet. There are five open two-body channels which are coupled to the incident channel. We denote the six cross sections by

$$\sigma(K^-):\quad K^- + p \rightarrow K^- + p, \tag{1}$$

$$\sigma(\bar{K}^0):\quad \rightarrow \bar{K}^0 + n, \tag{2}$$

$$\sigma(\Sigma^+):\quad \rightarrow \Sigma^+ + \pi^-, \tag{3}$$

$$\sigma(\Sigma^-):\quad \rightarrow \Sigma^- + \pi^+, \tag{4}$$

$$\sigma(\Sigma^0):\quad \rightarrow \Sigma^0 + \pi^0, \tag{5}$$

$$\sigma(\Lambda):\quad \rightarrow \Lambda + \pi^0. \tag{6}$$

It has been pointed out by Jackson, Ravenhall, and Wyld[2] that it may be useful to describe (1) and (2) along with the sum of (3) and (4) in the scattering length (and effective range) approximation. They have given convenient expressions for this purpose. On the other hand, Amati and Vitale[3] noted the possibility of relating the scattering in the final pion-hyperon states with the cross sections (3)-(6). We suggested[4] that it seems feasible to combine these ideas and study the $\pi - Y$ interaction from an empirical analysis of the $K^- - p$ reactions.

The point is that the problem becomes even more ambiguous if one just introduces the $\pi - Y$ phase shifts as free parameters to be fitted to the data (i.e., the Σ^+/Σ^- ratios). A whole variety of scattering length solutions, *a*, for the $\bar{K}-N$ system already exist which fit (1), (2), and (3)+(4). What we do is to assume a model for the interaction. This model (e.g., global symmetry) would give us a definite prediction for the "separated" $\pi - Y$ phase shifts, $\delta_0(\pi Y)$, i.e., the phase shifts in the absence of a $\bar{K}-N$ reaction channel.[5] It is the purpose of this paper to discuss the 2 methods[3,4] which have been proposed for determining the actual $\pi - Y$ phase shifts, $\delta(\pi Y)$, in the presence of a $\bar{K}-N$ reaction channel from $\delta_0(\pi Y)$ (and the scattering lengths, *a*), and suggest a much better approximation[1]: The first suggestion[3] was to approximate the phase of the reaction channel by $\delta_0(\pi Y)$. In B, we approximated the pion-hyperon K matrix element, c, by $\tan\delta_0(\pi Y)$. In A, we derived an expression for c which involves

* Work supported in part by the National Science Foundation.
[1] M. Ross and G. Shaw, Ann. Phys. (N. Y.) (to be published). We shall refer to this paper as A.

[2] Jackson, Ravenhall, and Wyld, Nuovo cimento **9**, 834 (1958).
[3] D. Amati and E. Vitale, Nuovo cimento **9**, 895 (1958).
[4] M. Ross and G. Shaw, Midwest Theoretical Physics Conference at Northwestern University, March, 1959 (unpublished). We shall refer to this paper as B.
[5] The "separated" phase is obtained by calculating the scattering omitting all processes where the $\bar{K}N$ system appears as an intermediate state. Thus in the Chew-Low one-meson approximation (for both K and π), we would calculate the "separated" phase omitting all reference to K interactions except that the constants include certain renormalization effects due to K's.

$\delta_0(\pi Y)$ and a. We will refer to these approximations as I, II, and III, respectively.

An analysis of the data would amount to fitting (1), (2), (3), and (4) as a function of energy (over the s-wave region) simply by finding the appropriate a's and using the $\delta_0(\pi Y)$ from the model.

In Sec. II we present the necessary relations, between the scattering lengths for the $\bar{K}-N$ system and c, which one would use for an analysis of the data. In order to discuss the approximations I, II, III in Sec. III, it is useful to present, here, a brief description of the experimental situation.

At present cross sections for processes (1)–(4) have been experimentally determined[6] at low energy ($\lesssim 100$ Mev). The cross sections (5) and (6) for production of the neutral hyperons have not, as yet, been measured. However, it is important to note that the at rest capture of K^- by protons[7] produces very few Λ's. The at rest data, which probably include some p state capture in addition to s state capture, give the branching ratios

$$\Sigma^-/\Sigma^+/\Sigma^0/\Lambda = 4/2/2/\lesssim \tfrac{1}{2}. \tag{7}$$

There are several interesting features of the in-flight data: There is a $\bar{K}^0-K^-$ mass difference[8] which leads to a threshold laboratory energy of 8 Mev for charge exchange scattering. Since the cusp effect on the Σ^+/Σ^- ratio,[4] above the threshold for (2), will not change any general arguments which we shall make, we shall ignore the $K^- - \bar{K}^0$ mass difference in this paper. We refer the reader to Jackson and Wyld[9] for a (straightforward) method for including this mass difference, as well as Coulomb effects.

In addition, the charge exchange cross section is small over the entire energy range. A qualitative aspect is the rapid variation of the Σ^+/Σ^- ratio. However, the data are too preliminary to consider this large energy dependence as established.

II. RELATIONS BETWEEN THE CROSS SECTIONS AND $\pi-Y$ SCATTERING

The scattering operator $\mathbf{T}$ and the reaction operator $\mathbf{K}$ are given by the usual relations:

$$\mathbf{S} \equiv 1+\mathbf{T} \equiv (1-\tfrac{1}{2}i\mathbf{K})(1+\tfrac{1}{2}i\mathbf{K})^{-1}. \tag{8}$$

The Hermiticity of $\mathbf{K}$ insures the unitarity of $\mathbf{S}$. Also, the matrix elements of $\mathbf{T}$ and $\mathbf{K}$ are defined as

$$\begin{aligned}\mathbf{T}_{ij} &= -2\pi i\delta(E_i-E_j)T_{ij}', \\ \mathbf{K}_{ij} &= 2\pi\delta(E_i-E_j)K_{ij}',\end{aligned} \tag{9}$$

where E is the total energy in the center-of-mass system. In a given isotopic spin state I, we expand T' and K' in terms of eigenvalues of orbital angular momentum l and total angular momentum J to obtain

$$T_{\alpha\beta}'(lJ) - K_{\alpha\beta}'(lJ) = 2i\sum_\gamma K_{\alpha\gamma}'(lJ)\rho_\gamma T_{\gamma\beta}'(lJ), \tag{10}$$

where the Greek subscripts refer to channels. When γ is a two-particle channel (which is the case we will be interested in), then

$$\rho_\gamma = k_\gamma^2 dk_\gamma/dE_\gamma, \tag{11}$$

where k_γ is the momentum in the center-of-mass system. (We use units $\hbar = c = 1$.) If we make the transformation

$$T_{\alpha\beta}' = -\tfrac{1}{2}\rho_\alpha^{-\frac{1}{2}} T_{\alpha\beta}\rho_\beta^{-\frac{1}{2}} \tag{12}$$

along with a similar one for $K_{\alpha\beta}'$, we get the matrix equation

$$T = (1-iK)^{-1}K, \tag{13}$$

where there is a submatrix along the diagonal for each set of quantum numbers l, J, I, etc. The diagonal element of this T matrix has the usual form

$$T_{\alpha\alpha}(lJ) = \{\exp[2i\delta_{\alpha\alpha}(lJ)]-1\}/2i, \tag{14}$$

where δ is the phase shift. The total cross section from α to β, for the case where α consists of one particle with spin $\frac{1}{2}$ and one with spin zero, is, for a particular isotopic spin state,

$$\sigma_{\alpha\beta} = 4\pi k_\alpha^{-2}\sum_{l,J}(J+\tfrac{1}{2})|T_{\alpha\beta}(lJ)|^2. \tag{15}$$

We shall be dealing, in this paper, only with an initial $K^- - p$ state $l=0$. The following modification must be made if the intrinsic parity of β, $P(\beta)$, is not equal to that of α: If $P(\bar{K}-N)$ is the same as $P(\pi-Y)$, the relevant $\pi-Y$ phase shifts are the $s_{\frac{1}{2}}$ ones. If $P(\bar{K}-N)$ is minus $P(\pi-Y)$, then the $\delta(\pi-Y)$ that enter, enter are the $p_{\frac{1}{2}}$ ones.

In order to describe the $\bar{K}-N$ processes, we need the T matrix in both isotopic spin zero and one states. For example, we write,[10] for $I=0$,

$$T^0 = \begin{pmatrix} T^0(\bar{K}) & |T^0(\Sigma)|\exp(i\phi^0) \\ |T^0(\Sigma)|\exp(i\phi^0) & T^0(\pi) \end{pmatrix}, \tag{16}$$

where the first row refers to $\bar{K}-N$ and the second to $\pi-\Sigma$. Thus,[11] $T^0(\pi)$ is the amplitude for $\pi+\Sigma \rightarrow \pi+\Sigma$ in the $I=0$ state. Note that ϕ is a real number. We then

[6] Nordin, Rosenfeld, Solmitz, Tripp, and Watson, Bull. Am. Phys. Soc. 4, 24 (1959); A. H. Rosenfeld, Bull. Am. Phys. Soc. 3, Ser. II, 363 (1958); Ascoli, Hill, and Yoon, Nuovo cimento 9, 813 (1958); R. S. White (private communication).

[7] L. Alvarez *et al.*, University of California Radiation Laboratory Report UCRL-3775, 1957 (unpublished).

[8] Rosenfeld, Solmitz, and Tripp, Phys. Rev. Letters 2, 110 (1959); F. Crawford *et al.*, Phys. Rev. Letters 2, 112 (1959).

[9] J. D. Jackson and H. W. Wyld, Phys. Rev. Letters 2, 355 (1959); Nuovo cimento (to be published).

[10] The T matrix for a given J and J_z is symmetric if time reversal holds.

[11] All numerical superscripts (with the exception of those on the hyperons or mesons which refer to charge) denote the isotopic spin state I. The subscript 0 [with the exception of that on the effective range (31)] refers to the "separated" quantity, i.e., the quantity which would exist if the coupling between channels vanished.

have

$$\sigma\begin{pmatrix} K^- \\ \bar{K}^0 \end{pmatrix} = 4\pi k^{-2} \times \tfrac{1}{4} | T^1(\bar{K}) \pm T^0(\bar{K}) |^2, \tag{17}$$

$$\sigma(\Sigma^\pm) = 4\pi k^{-2} \times \tfrac{1}{2} | (1/\sqrt{2}) | T^1(\Sigma) | \times \exp(i\phi^1) \mp (1/\sqrt{3}) | T^0(\Sigma) | \exp(i\phi^0) |^2, \tag{18}$$

$$\sigma(\Sigma^0) = 4\pi k^{-2} \times \tfrac{1}{6} | T^0(\Sigma) |^2, \tag{19}$$

$$\sigma(\Lambda) = 4\pi k^{-2} \times \tfrac{1}{2} | T^1(\Lambda) |^2, \tag{20}$$

where k is the momentum of the incident K^- in the center-of-mass system.

Let us consider how many independent quantities we need to describe the $\bar{K}-N$ processes. For $I=0$, we have

$$K^0 = \begin{pmatrix} d^0 & e^0 \\ e^0 & c^0 \end{pmatrix}, \tag{21}$$

where d^0, e^0, and c^0 are real, since K is symmetric as well as Hermitian. Thus three real parameters determine the T matrix (16). Let us introduce the complex scattering length a^I for the incident channel in the isotopic spin state I:

$$k \cot\delta^I(\bar{K}N) \equiv -1/a^I(k) = -[A^I(k) - iB^I(k)]^{-1}, \tag{22}$$

where $\delta(\bar{K}N)$ is the complex phase shift in the $\bar{K}-N$ channel and A and B are real (note that B is a positive quantity). We choose to express all the scattering amplitudes in terms of A, B, and c. Then, from (13) using (14), (16), (21), and (22), we obtain (after some algebra) for the two-channel problem:

$$T^0(\bar{K}) = -k\left(\frac{1}{a^0(k)} + ik\right)^{-1}, \tag{23}$$

$$T^0(\Sigma) = \exp(i\phi^0)(kB^0)^{\frac{1}{2}} | 1 + ika^0 |^{-1}, \tag{24}$$

where

$$\phi^0 = \tan^{-1}\left[c^0 - \frac{kA^0[1+(c^0)^2]}{1+k(B^0+c^0A^0)}\right], \tag{25}$$

$$T^0(\pi) = [\cot\delta^0(\pi\Sigma) - i]^{-1}, \tag{26}$$

where

$$\tan\delta^0(\pi\Sigma) = c^0 + \frac{ikB^0[1+(c^0)^2]}{1+ik(A^0 - c^0B^0)}, \tag{27}$$

$$d^0 = k(B^0c^0 - A^0), \tag{28}$$

$$e^0 = \{kB^0[1+(c^0)^2]\}^{\frac{1}{2}}. \tag{29}$$

For energies below the $\bar{K}-N$ threshold, we continue (23)–(27) according to the relation

$$k \to i|k|. \tag{30}$$

We may exhibit the energy dependence of $a(k)$ in the effective-range approximation; i.e., to order k^2 we have

$$1/a(k) = (1/a) - \tfrac{1}{2}r_0k^2, \tag{31}$$

where r_0 is the effective range. For the $K^- - p$ reactions it is very likely that we may ignore r_0 for $k < 0.6\ \mathrm{f}^{-1}$ ($1\ \mathrm{f} \equiv 1$ fermi $\equiv 10^{-13}$ cm); we shall discuss this in Sec. III. In general, we also have

$$c(k) = c + c_1k^2 + \cdots.$$

However, for the energy region in which we are interested, it is a good approximation to neglect the energy dependence of c. Thus for small k, we consider A^0, B^0 and c^0 in (23)–(29) to be constants. (Therefore $d^0 \propto k$ and $e^0 \propto k^{\frac{1}{2}}$.)

Now, as noted previously, the first approximation, I, for ϕ^0 is given by

$$\tan\phi^0 = \tan\delta_0{}^0(\pi\Sigma) \equiv c_0{}^0, \tag{32}$$

where $\delta_0{}^0(\pi\Sigma)$ is the "separated" pion-sigma $I=0$ phase shift, i.e., that calculated when there is no coupling to the $\bar{K}-N$ channel.[5] The second approximation, II, is

$$\tan\phi^0 = c_0{}^0 - \frac{kA^0[1+(c_0{}^0)^2]}{1+k(B^0+c_0{}^0A^0)}. \tag{33}$$

In A, we derived the following expression for c^0:

$$c_A{}^0 = \frac{B^0 + c_0{}^0(R - A^0)}{(R - A^0) - c_0{}^0B^0}, \tag{34}$$

where R is a constant approximately equal to the range of the diagonal interaction in the $\bar{K}-N$ channel. (See A for an exact definition of the constant R—it is referred to as L in A.) The relation (34) for c^0 should be accurate for interactions of short, well-defined range. This is the basis for approximation III; knowing $c_0{}^0$, A^0, B^0 (and R), we determine $c_A{}^0$ from (34) and use this c^0 in (25) to calculate ϕ^0. Thus to completely determine the T matrix in the $I=0$ state, we need to know the two observable quantities A^0 and B^0 and the theoretical phase shift $\delta_0{}^0(\pi\Sigma)$.

The $I=1$ case is more complicated. The real K matrix can be represented as

$$K^1 = \begin{bmatrix} d^1 & e^1 & g^1 \\ e^1 & c^1 & h^1 \\ g^1 & h^1 & j^1 \end{bmatrix}.$$

The first row corresponds to the $\bar{K}-N$ channel, the second to $\pi-\Sigma$, and the third to $\pi-\Lambda$. Here, we need to know six parameters to determine T^1, e.g., $c_0{}^1$, $h_0{}^1$ and $j_0{}^1$ (the pion K^1 matrix elements in the absence of any coupling to the $\bar{K}-N$ channel), the complex scattering length a^1 ($\equiv A^1 - iB^1$), and the ratio

$$\alpha \equiv | T^1(\Lambda) |^2 / | T^1(\Sigma) |^2, \tag{35}$$

which is a constant for small momentum k. We then could find relations, similar to (34), from which we could calculate the actual K^1 matrix elements c^1, h^1, and j^1, and hence determine T^1.

If we consider the situation where c_0^1, h_0^1, and j_0^1 are negligible as essentially they would be with global symmetry (both the $s_{\frac{1}{2}}$ and $p_{\frac{1}{2}}$ pion-nucleon phases shifts are small), then we obtain[1] the simple results:

$$T^1(\Sigma)=\exp(i\phi^1)\frac{(kB^1)^{\frac{1}{2}}}{|1+ika^1|(1+\alpha)^{\frac{1}{2}}}, \qquad (36)$$

where

$$\tan\phi^1=c^1(1+\alpha)-\frac{kA^1\{1+[c^1(1+\alpha)]^2\}}{1+k[B^1+c^1(1+\alpha)A^1]}, \qquad (37)$$

and our approximation $c_A{}^1$ to c^1 is given by

$$c_A{}^1=\frac{B^1}{(1+\alpha)(R-A^1)}. \qquad (38)$$

Thus for the $I=1$ system, we need to know the three observables A^1, B^1, and α. It might be implied from the at-rest data, (7), that $\alpha\ll 1$.

III. ANALYSIS OF DATA AND DISCUSSION

In this section, we will present a *rough* analysis of the $K^- - p$ data, to illustrate our proposal, and discuss some of the approximations made.

Our first step is to analyze the data $\sigma(K^-)$, $\sigma(\bar{K}^0)$, and $\sigma(\Sigma^+)+\sigma(\Sigma^-)$. We shall assume that the Λ production is negligible, or more specifically that $\alpha=0$. Then, the two phase shifts $\delta^I(\bar{K}N)$ are sufficient to portray this data. In fact, within the fairly large errors of the experimental points, we find that a scattering length approximation $[a^I(k)=a^I]$ is sufficient to determine the energy dependence of the $\bar{K}-N$ s-state scattreing phase shifts:

$$k\cot\delta^I(\bar{K}N)=-1/a^I=-(A^I-iB^I)^{-1}, \quad I=0,1.$$

We find a variety of solutions which work: The A's must both be the same sign [to keep $\sigma(\bar{K}^0)$ small], either positive or negative. The absorption can be divided in any way between the $I=0$ and 1 states. We illustrate the families of solutions to in-flight data with a few cases. We can have all $I=1$ absorption[12,4]:

$$a^1=\pm 1.20-0.87i, \quad a^0=\pm 0.58; \qquad (39)$$

roughly equal $I=0$ and $I=1$ absorption, as discussed by Dalitz and Tuan,[13] for example,

$$a^1=\pm 0.40-0.41i, \quad a^0=\pm 1.88-0.82i, \qquad (40)$$

and

$$a^1=\pm 1.62-0.39i, \quad a^0=\pm 0.20-0.78i; \qquad (41)$$

or all $I=0$ absorption[4]:

$$a^1=\pm 0.88, \quad a^0=\pm 0.53-1.19i. \qquad (42)$$

On the basis of the $K^- - p$ capture at rest and $K^- - d$ capture at rest,[14] regardless of the complexity of the analysis of this data, we can exclude the extreme solution (39). We must consider, however, all cases with $B^0 \gtrsim B^1$.

Now we come to the problem of fitting the $\sigma(\Sigma^+)/\sigma(\Sigma^-)$ ratio. This ratio appears to change from $\approx\frac{1}{4}$ at $k=0.3$ to ≈ 2 at $k=0.4$ to ≈ 1 at $k=0.6$ and then stays ≈ 1. Since the data are preliminary, let us content ourselves with exploring the implications of $\sigma(\Sigma^+)/\sigma(\Sigma^-)$ being approximately equal to one at $k=0.6$. Whether a solution leads to a rapidly changing ratio as a function of energy will also be discussed.

To separate the charged hyperons, we need to know something about pion-hyperon scattering. Due to our knowledge of the pion-nucleon system, the most convenient assumption to make is that there is global symmetry in the pion-baryon interactions. Then we have the pion K-matrix elements, in the absence of the $\bar{K}-N$ reaction channels given by[15]

$$\begin{aligned} c_0{}^1&=\tfrac{1}{3}[\mathbf{K}^{\frac{3}{2}}+2\mathbf{K}^{\frac{1}{2}}],\\ c_0{}^0&=\mathbf{K}^{\frac{1}{2}},\\ h_0{}^1&=\tfrac{1}{3}\sqrt{2}[\mathbf{K}^{\frac{3}{2}}-\mathbf{K}^{\frac{1}{2}}],\\ j_0{}^1&=\tfrac{1}{3}[2\mathbf{K}^{\frac{3}{2}}+\mathbf{K}^{\frac{1}{2}}] \end{aligned} \qquad (43)$$

where $\mathbf{K}^I$ are the pion-nucleon K-matrix elements. Our results turn out to be essentially independent of the parity $P(\bar{K}-N)$ since both the $s_{\frac{1}{2}}$ and $p_{\frac{1}{2}}$ pion-nucleon phase shifts are small. Thus for illustrative purposes, let the $\bar{K}$ particles be pseudoscalar so that

$$\mathbf{K}^I=\tan\delta^{2I}(\pi N),$$

where $\delta(\pi N)$ are the s-state $\pi-N$ phase shifts. Evaluated at the appropriate pion momentum, which is essentially a constant over the range in K^- energy we are working with, we get

$$\mathbf{K}^{\frac{3}{2}}=-0.14, \quad \mathbf{K}^{\frac{1}{2}}=0.22,$$

with which we obtain

$$\begin{aligned} c_0{}^0=0.22, &\quad c_0{}^1=0.10,\\ h_0{}^1=-0.17, &\quad j_0{}^1=-0.02. \end{aligned} \qquad (44)$$

As we note in Sec. II, for the $I=0$ system, we need the three quantities A^0, B^0, and $c_0{}^0$ (and R) to determine the T^0 matrix, whereas for $I=1$, we need six quantities: A^1, B^1, $c_0{}^1$, $h_0{}^1$, $j_0{}^1$, and α. Since $c_0{}^1$, $h_0{}^1$, and $j_0{}^1$ are so small, we may use Eqs. (36)–(38). Actually, we will assume that $j_0{}^1$ and $h_0{}^1$ are negligible and $c_0{}^1$ is small but not negligible. Then with $\alpha=0$, the 3×3 T^1 matrix has been effectively reduced to a system of two coupled channels. In the following discussion we shall assume that the $I=0$ and 1 systems are described by similar two-channel T matrices, i.e., we ignore the Λ channel.

[12] All lengths will be measured in fermis (f), and all momenta in f^{-1}. 1 fermi$\equiv 10^{-13}$ cm.

[13] R. H. Dalitz and S. F. Tuan, Phys. Rev. Letters **2**, 425 (1959); Ann. Phys. (to be published).

[14] Horwitz, Miller, Murray, Schwartz, and Taft, Bull. Am. Phys. Soc. Ser. II, **3**, 363 (1958).

[15] M. Ross, Phys. Rev. **112**, 986 (1958).

Let us consider the various methods for determining the Σ^+/Σ^- ratio. We can discuss the extreme type of solution (42) before going further. The ratio is equal to one, independent of the phases; it can show no variation with energy. Now, we consider the more complex cases (40) and (41) at the various levels of approximation discussed in Sec. II:

I: $\tan\phi^I = c_0{}^I$, i.e., the phase of the production T matrix element is taken as the "separated" $\pi-\Sigma$ scattering phase. Referring to (25) we see that I embodies two approximations:

$$\text{(i)} \quad kA(1+c^2)/(1+kB+kAc) \ll 1,$$

$$\text{(ii)} \quad c = c_0.$$

The first approximation, (i), is good at zero energy. It becomes very poor for the observed scattering lengths at about $k \gtrsim 0.3$ (i.e., for lab lenergies of 10 Mev and above). Approximation (ii) can be studied using (34). We see that $c=c_0$ if

$$R - A \gg B \quad (\text{and } c_0 B). \tag{45}$$

In certain cases this relation is roughly satisfied. If the range R is about the pion Compton wavelength, 1.4, it will be satisfactory for small or negative A. If $A<0$, it will usually be satisfied. However, we probably should consider $R \approx 0.5$, near the $\bar{K}$ Compton wavelength.[1] It is clear that (45) will not be satisfied for many solutions we want to consider,

II: $c^I = c_0{}^I$, i.e., the diagonal K matrix element in the $\pi-\Sigma$ channel is taken from the "separated" $\pi-\Sigma$ scattering. This approximation, which involves only approximation (ii) of I (it is equivalent to I at zero energy), formed the basis of the analysis B. Both procedures I and II lead to a Σ^-/Σ^+ ratio about 10 or greater unless we adopt the extreme type solution (42).

III: c^I is given by $c_A{}^I$, Eq. (34). Let us consider the four solutions (40) and (41), and assume that the $\bar{K}-N$ interaction is characterized by a range equal to the $\bar{K}$ Compton wavelength. Using (34) and the "separated" $\pi-\Sigma$ phases, $c_0{}^I$, from (44), we calculated the Σ^-/Σ^+ ratio. The results are given in Table I for an energy corresponding to $k=0.6$. We see that: (1) The negative A solutions are not compatible with weak "separated" $\pi-Y$ scattering, as predicted by global symmetry; (2) The positive A solutions may be compatible with the present data and small $\delta_0(\pi Y)$. The great role of the energy-dependent second term on the right-hand side of (25) is evidenced by the difference between $\tan\phi$ and c. Add to this the sensitivity of the Σ^+/Σ^- ratio to the phases, ϕ, and we see that there will be very rapid variation of Σ^+/Σ^- as a function of energy. It should be hard to fit a particular energy dependence and we may hope that only few satisfactory solutions could be found. Such fitting must await more detailed data.

An important feature of any detailed analysis is the effective range, r_0, in the expansion (31) for $a(k)$. We have shown, in A, that a necessary condition for $|r_0{}^I| \gg R$ is that Im $(1/a^I)$ (measured in f^{-1}), times some kinematical factors which are ≈ 1 for this problem, is much greater than 1. The data indicate that the Im $(1/a^I) < 1$. Thus we feel that $r_0{}^I$ should prove to be small.

TABLE I. The ratio Σ^-/Σ^+ calculated at $k=0.6$ (using approximation III) with $R=0.5$, $c_0{}^0=0.22$ and $c_0{}^1=0.10$ for the scattering length solutions (40) and (41).

I	A	B	c	ϕ	$\cos(\phi^1-\phi^0)$	Σ^-/Σ^+
0	1.88	0.82	−0.33	2.17		
1	0.40	0.41	7.1	1.24	0.60	3.9
0	−1.88	0.82	0.61	1.19		
1	−0.40	0.41	0.59	0.17	0.90	17
0	0.20	0.78	6.5	1.33		
1	1.62	0.39	−0.24	2.24	0.61	4.0
0	−0.20	0.78	1.73	1.13		
1	−1.62	0.39	0.29	0.93	0.98	56

We can summarize our proposal by stating that a given theory of $\pi-Y$ interactions may be tested by evaluating the $\pi-Y$ scattering omitting $\bar{K}-N$ reaction channels, and then combining the results of this "separated" scattering theory with empirical observation of $\bar{K}-N$ processes via (34) in order to predict the hyperon production ratio Σ^+/Σ^-. In the example shown above, the global symmetry model was considered. We concluded only that the scattering length solutions with negative A of Dalitz and Tuan[13] are not compatible[16] with present data. Data on the energy dependence of Σ^+/Σ^- and an understanding of the capture at rest data should allow us to draw some strong conclusions with this type of analysis. It is important to note that we need not be restricted to an analysis which neglects Λ production: (35)–(38) indicated how the generalization is made to a three channel T^1 matrix when the "separated" $\pi-Y$ phase shifts, $\delta_0{}^1(\pi Y)$, are small. To embody a theory in which the $\delta_0{}^1(\pi Y)$ are large, similar (but more involved) relations may be derived (see A).

In conclusion, we want to emphasize the importance of relation (34): Dalitz and Tuan[13] have noted that a perturbation treatment of the effect of the $\bar{K}$-baryon coupling on $\pi-Y$ scattering is a very poor approximation. We agree with this statement, and point out that (34) enables us to calculate $\pi-Y$ scattering from the "separated" $\pi-Y$ phase shifts, $\delta_0(\pi Y)$, in a nonperturbative manner. Thus one can theoretically investigate $\pi-Y$ scattering in the absence of a $\bar{K}-N$ reaction channel without assuming that the coupling to this reaction channel is small.

ACKNOWLEDGMENT

We would like to thank Dr. N. Horwitz for discussion of the experimental situation.

[16] It is interesting to note that Jackson and Wyld[9] rejected solutions with negative A on the basis of Coulomb effects on (1).