K^+-Deuteron Scattering in the Impulse Approximation*

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It is suggested that a phenomenological analysis using the impulse approximation of the processes occurring in the scattering of K^+ mesons by deuterons may be used to get the phase-shifts for the $T=0$ isotopic spin state. Typical curves are given for the elastic, elastic plus inelastic and charge exchange scattering differential cross sections of 100-Mev K^+ mesons by deuterons on the assumption that only S-waves contribute and for various ratios of the $T=1$ and $T=0$ isotopic spin states phase shifts.

INTRODUCTION

ALL existing experimental results[1] which give information on K^+ meson-nucleon scattering are consistent with an isotropic and energy-independent differential cross section at energies between 40 and 200 Mev. These results come both from pure hydrogen bubble chamber and from nuclear emulsions experiments.

The pure K^+ meson-proton processes, as observed in hydrogen bubble chambers, can only give information on the amplitude for the scattering in the $T=1$ isotopic spin state. At the energy of 100 Mev a negative S-wave phase shift $\sin\delta_1=-0.37$ (with all the other phase shifts equal to zero) fits well the observed repulsive potential at low energies and the isotropy in the scattering, the total cross section being of $4\pi\sin^2\delta_1/k^2=16$ millibarns.

The direct and charge exchange scattering of K^+ mesons by neutrons observed on nuclear emulsion material depend on the amplitude a_0 for scattering in the $T=0$ state, as well as on the amplitude a_1 for scattering in the $T=1$ isotopic spin state. Assuming charge independence, in a material with a number of protons equal to f times the number of neutrons, the rate of charge exchange to total (non-Coulombian) scattering is $R=(a_1-a_0)^2/[2(a_1^2+a_0^2)+4fa_1^2]$. If there is no interaction in the state $T=0$, that is, if $a_0=0$, we get $R=1/6$ for $f=1$ and $R=1/7$ for $f=5/4$. As the experimentally observed ratio is consistent with these values of R, it has been suggested[2] that at low energies only the $T=1$ isotopic spin state is responsible for the scattering.

These processes observed in nuclear emulsion give the only available information on K^+ meson-neutron processes and on the contribution of the $T=0$ state to the K^+ meson-nucleon interactions. However the complexity of the emulsion material makes the analysis of the experimental results rather difficult. Deuterium bubble chambers and diffusion cloud chambers now in use provide a more convenient sample material containing neutrons.

The impulse approximation[3] has been used with success by Fernbach, Green, and Watson[4] and by Rockmore[5] to express the differential cross sections for scattering of pions by deuterons in terms of the amplitudes of elementary processes of pions with protons and neutrons. In Rockmore's formulas the cross sections for elastic, elastic plus inelastic, and charge-exchange scatterings of the π-mesons by deuterons are expressed in terms of the S- and P-wave phase shifts for scattering in the charge independent $T=3/2$ and $T=1/2$ states of a pion-nucleon system. Comparison of Rockmore's results with experiment[6] has shown the reliability of the application of the impulse approximation to his problem.

We now then suggest that a calculation in impulse approximation similar to that of Rockmore, and experiments on the scattering of a K^+-meson beam by the deuterium of either a bubble or a diffusion chamber, be used together to get information on the phase shifts for the $T=0$ isotopic spin state of the K^+ meson-nucleon system.

In this paper we present the formulas, which are analogous to those of Rockmore, giving the differential cross sections for elastic, elastic plus inelastic, and charge exchange scattering of K^+ mesons by deuterons. In particular we present the curves for these differential cross sections for K^+ mesons of 100-Mev energy in the cases of $T=1$, S wave only contributing to the scattering and of $T=0$, S-wave adding a small contribution to the above (two different signs of the phase shift are considered).

We have suggested explicitly the work with the positive K meson for the obvious reason of the greater simplicity (as compared with the negative K) of its

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[1] See for example the reports of M. Goldhaber *et al.*, *Proceedings of the Sixth Annual Rochester Conference on High-Energy Physics* (Interscience Publishers, Inc., New York, 1956); and Proceedings of the Padua-Venice Conference on Fundamental Particles, 1957 (unpublished); or the more recent reports by M. F. Kaplon and R. H. Dalitz, *Proceedings of the 1958 Annual International Conference on High Energy Physics* (CERN, Geneva, 1958).

[2] R. H. Dalitz, *Reports on Progress in Physics* (The Physical Society, London, 1957), Vol. 20, pp. 212.

[3] G. F. Chew and M. L. Goldberger, Phys. Rev. **87**, 778 (1952).

[4] Fernbach, Green, and Watson, Phys. Rev. **84**, 1084 (1951).

[5] R. M. Rockmore, Phys. Rev. **105**, 256 (1957). Typographical errors in Rockmore's formulas were later corrected: see R. Rockmore, Phys. Rev. **113**, 1696(E) (1959).

[6] K. C. Rogers and L. M. Lederman, Phys. Rev. **105**, 247 (1957).

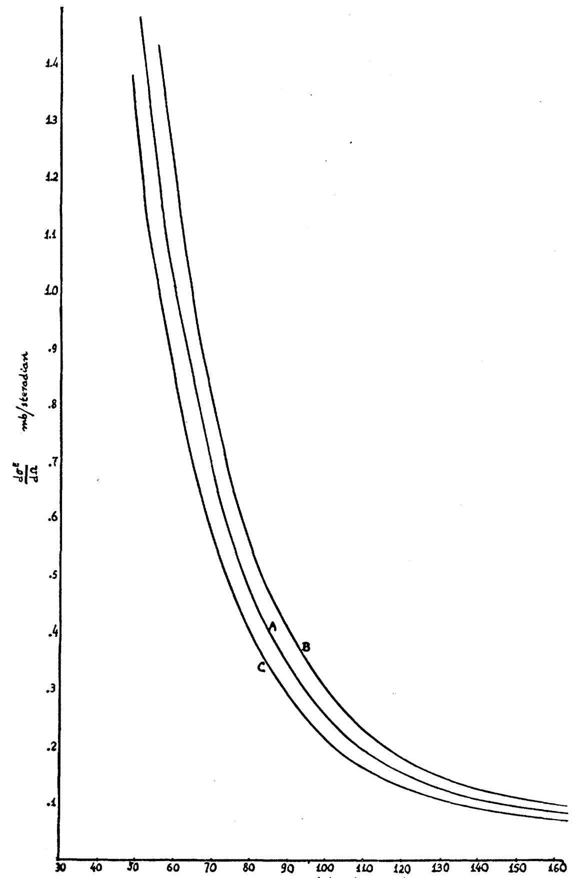

FIG. 1. Differential cross section for elastic scattering of 100-Mev K^+ mesons by deuterons. Only S waves are assumed to contribute. Curve A corresponds to scattering purely in the $T=1$ isotopic spin states. Curves B and C include smaller contributions from the $T=0$ state.

interaction with nucleons. The absence of absorption processes greatly simplifies the phenomenological analysis, and in this respect, the K^+ interaction is even simpler than that of the π mesons.

The multiple-scattering and potential corrections to the impulse approximation calculation have been shown by Rockmore[5] in the pion case to give contributions of opposite signs to the cross sections, so that even if they cannot separately be neglected, their added effect seems not to alter appreciably the values calculated in the pure impulse approximation. In our case of scattering of K mesons the estimation of these corrections by the methods used in the pion case would necessarily be unconvincing. In fact the knowledge of the interactions of K mesons and nucleons is not good enough to provide us with a field-theoretic method for the calculation of the effects of binding and multiple scattering. On the other hand, the Brueckner model, which assumes fixed (infinitely heavy) nucleons, cannot be used in the K-meson case due to the large mass of these particles.

However, we can make some estimates and present some qualitative arguments which show that we can rely on the applicability of the impulse approximation to this problem. We expect the deuteron binding effects (in other words, the nucleon-nucleon interaction in the final state) to be smaller in the scattering of K mesons than they were in the scattering of pions of about the same energy (few hundred Mev). This is because the potential corrections are larger for smaller nucleon recoil energies, and these tend to be larger in a collision by a heavier particle.

As to the multiple-scattering effects, a simple estimation can be made by considering the value of the amplitude of the wave scattered by one nucleon, evaluated at the average position of the other nucleon.[7] In the case of scattering of 100-Mev K mesons we thus estimate (assuming a K-nucleon cross section of 20 millibarns and the usual value for the radius of deuteron) that the error due to neglect of multiple scattering will be probably less than 10%.

On the other hand, the interaction time of a K^+-nucleon system is certainly shorter than that of nucleon-nucleonic or pion-nucleonic systems (which are of the

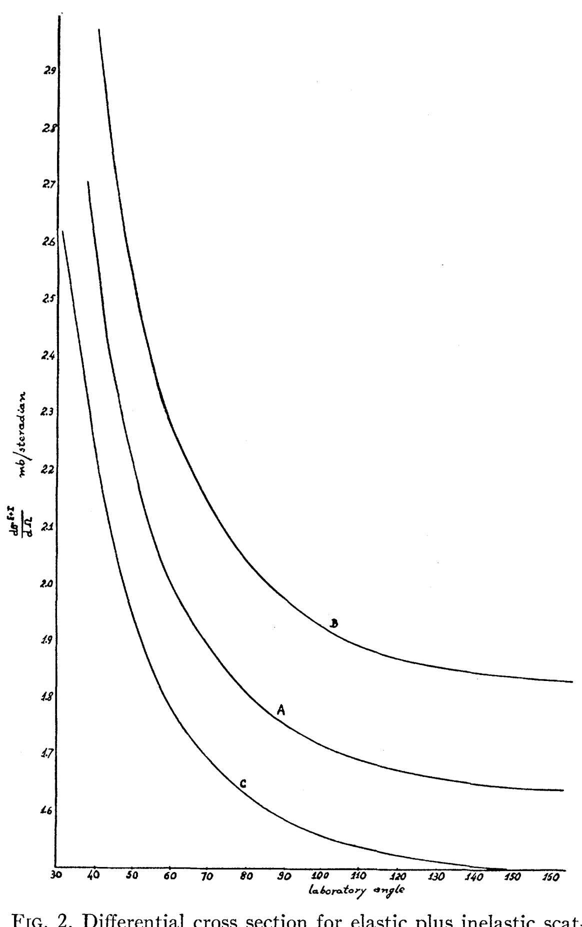

FIG. 2. Differential cross section for elastic plus inelastic scattering of 100-Mev K^+ mesons by deuterons. Curve A corresponds to scattering of S waves only in the $T=1$ isotopic spin state. Curves B and C admit contributions from the S waves in the $T=0$ state.

[7] G. F. Chew and G. C. Wiek, Phys. Rev. **85**, 636 (1952).

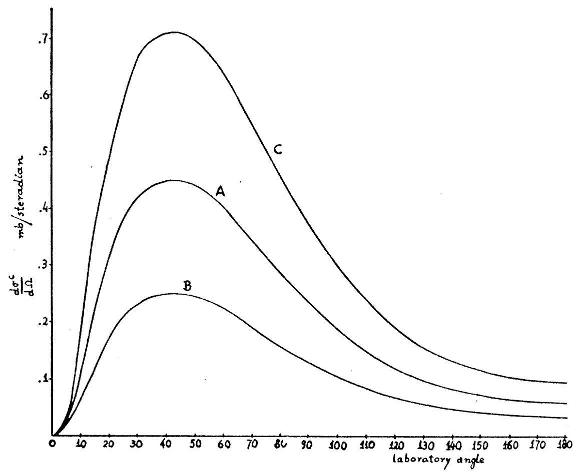

FIG. 3. Differential cross section for charge exchange scattering of 100-Mev K^+ mesons by deuterons. Curve A corresponds to scattering only in $T=1$ state. Curves B and C are calculated with small contributions from the S waves of the $T=0$ state.

order of $\hbar/m_\pi c^2$) and can be neglected compared to the much longer time involved in the nucleon motion inside the deuteron. So the neglect of the effects of the forces between the two nucleons during the "collision time" (this is the properly called impulse approximation) is well justified here, as it was in the nucleon-deuteron and pion-deuteron cases (see reference 7).

Summing up, we estimate the main error, due to multiple scattering, to be less than 10%. Other effects should not change this error by more than a few percent.

FORMULAS AND CURVES

Our notation is the same as that of Fernbach[4] and Rockmore.[5] Phase shifts are indicated in a manner similar to that introduced by Fermi, the first index being either 1 or 0 according to whether the isotopic spin state is $T=1$ or $T=0$. The second index is omitted for S waves, and for P waves is either 3 or 1 according to whether the total angular momentum is 3/2 or 1/2.

We define

$$J_0=\int_{E_f}\delta(E_f-E_0)q^2dq$$

the kinematical factors being those of a collision of a K meson with a nucleon, and

$$J_0'=\int_{E_f'}\delta(E_f'-E_0)q^2dq,$$

where the kinematics is that of a collision of a K meson with a deuteron.

The curves are traced for 100-Mev energy K mesons and for the following set of phase shifts. Curves labeled by A correspond to $\sin\delta_1=-0.37$, all the other phase shifts being set equal to zero (only $T=1$, S-waves contributing to the scattering). They correspond to a ratio R of the charge exchange to the total number of processes equal to 1/6 (for $f=1$).

Curves labeled by B correspond to $\sin\delta_1=-0.37$, $\sin\delta_0=-0.1$, and in this case we have $R=1/5$. For the curves C we have $\sin\delta_1=-0.37$, $\sin\delta_0=+0.1$, and $R=1/9$.

The differential cross section for elastic scattering is

$$\frac{d\sigma^E}{d\Omega}=\frac{Mq_0J_0H^d(\theta)}{4k^4wW_{N(\mathrm{lab})}}\left\{\left|3\eta_1+\eta_0-\frac{\alpha}{v\sin^2(\theta_0/2)}+(6\eta_{13}+2\eta_{03}+3\eta_{11}+\eta_{01})\cos\theta_0\right|^2+\tfrac{2}{3}\sin^2\theta_0|3\eta_{13}+\eta_{03}-3\eta_{11}-\eta_{01}|^2\right\}.$$

For elastic plus inelastic scattering we have

$$\frac{d\sigma^{E+I}}{d\Omega}=\frac{J_0Mq_0}{k^4wW_{N(\mathrm{lab})}}\left\{\left|\eta_1+(2\eta_{13}+\eta_{11})\cos\theta_0-\frac{\alpha}{2v\sin^2(\theta_0/2)}\right|^2+\sin^2\theta_0|\eta_{13}-\eta_{11}|^2+\tfrac{1}{4}|\eta_1+\eta_0+(2\eta_{13}+2\eta_{03}+\eta_{11}+\eta_{01})\cos\theta_0|^2+\tfrac{1}{4}\sin^2\theta_0|\eta_{13}+\eta_{03}-\eta_{11}-\eta_{01}|^2\right\}$$
$$+\frac{H_2J_0'Mq_0}{8k^4wW_{N(\mathrm{lab})}}\left\{\left|3\eta_1+\eta_0-\frac{\alpha}{v\sin^2\theta_0/2}+\cos\theta_0(6\eta_{13}+2\eta_{03}+3\eta_{11}+\eta_{01})\right|^2-\left|\eta_1-\eta_0-\frac{\alpha}{v\sin^2(\theta_0/2)}+\cos\theta_0(2\eta_{13}-2\eta_{03}+\eta_{11}-\eta_{01})\right|^2+\tfrac{1}{3}\sin^2\theta_0|3\eta_{13}+\eta_{03}-3\eta_{11}-\eta_{01}|^2-\tfrac{1}{3}\sin^2\theta_0|\eta_{13}-\eta_{03}-\eta_{11}+\eta_{01}|^2\right\},$$

and for the charge-exchange differential cross section

$$\frac{d\sigma^C}{d\Omega}=\frac{Mq_0J_0}{4k^4wW_{N(\mathrm{lab})}}\{(1-H_2)[|\eta_1-\eta_0+(\eta_{11}-\eta_{C1}+2\eta_{13}-2\eta_{03})\cos\theta_0|^2+\tfrac{2}{3}\sin^2\theta_0|\eta_{11}-\eta_{01}-\eta_{13}+\eta_{03}|^2]+\tfrac{1}{3}(1+H_2)\sin^2\theta_0|\eta_{11}-\eta_{01}-\eta_{13}+\eta_{03}|^2\}.$$

Note added in proof.—While the present work was in the process of publication, an equivalent paper by M. Gourdin and A. Martin appeared in Nuovo cimento **XI**, 670 (1959).

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