Total Neutron Cross Section of Xe^{135} as a Function of Energy*

E. C. SMITH,‡ G. S. PAWLICKI,§ P. E. F. THURLOW,‖ G. W. PARKER, W. J. MARTIN, G. E. CREEK, P. M. LANTZ, AND S. BERNSTEIN,† *Oak Ridge National Laboratory, Oak Ridge, Tennessee*

(Received April 10, 1959)

The total neutron cross section of Xe^{135} as a function of energy has been remeasured at Oak Ridge National Laboratory under more favorable conditions than obtained in earlier measurements. A sample thickness of 2.5×10^{18} atoms of Xe^{135} gas per cm^2 was procured from the gases generated in a homogeneous reactor. A mechanical time-of-flight chopper was used to select neutrons in the energy range from 0.01 ev to several thousand ev. The number of Xe^{135} atoms in the sample was determined by means of mass spectrometer measurements on the long-lived daughter, Cs^{135}. The data of the low-energy resonance were fitted to the single-level Breit-Wigner formula, taking into account Doppler corrections, equally well with the following two sets of parameters: statistical weight factor $g=\frac{3}{8}$; resonance energy $\epsilon_0=0.08472\pm0.00027$ ev; neutron width at energy ϵ_0, $\Gamma_n^0=0.03477\pm0.00021$ ev; capture width, $\Gamma_a=0.083303\pm0.00062$ ev; for $g=\frac{5}{8}$, $\epsilon_0=0.08415\pm0.00028$ ev; $\Gamma_n^0=0.02057\pm0.00012$ ev; $\Gamma_a=0.09493\pm0.00071$ ev. The errors quoted are the standard deviations derived from the statistics of the measurements. Systematic errors are discussed in the body of the paper. No evidence for resonances at energies greater than 0.085 ev was observed. The results described are interpreted in terms of recent considerations on the statistics of the properties of nuclear energy levels.

INTRODUCTION

THE fission product xenon-135 has the largest known neutron cross section. When the spin weight factor, g, is considered, the capture cross section is close to the theoretical maximum, $g\pi\lambda^2=3.8\times10^6$ barns for a statistical weight factor of $\frac{1}{2}$. This is due to a resonance level at 0.085 ev with an abnormally large reduced neutron width which is approximately 95×10^{-3} $(ev)^{\frac{1}{2}}$ at 1 ev. The neutron cross section of Xe^{135} is of special interest because the compound nucleus Xe^{136} contains a magic number of neutrons (82), and because of its pertinence to reactor technology.

The first measurements of the energy dependence of the cross section were completed in 1948 by Bernstein *et al.*[1] A focusing-type single-crystal spectrometer was used as a neutron monochromator. Transmission measurements could be made on samples of 1 mm^2 in area. The xenon was obtained from iodine separated from irradiated uranium, and the samples were assayed by the technique of activity measurements. The maximum sample strength used in these measurements was 5.4×10^{17} atoms/cm^2 of Xe^{135}. This amount of Xe^{135} corresponds to an activity for the whole sample of approximately 3 curies of Xe activity.

About 1954, circumstances at Oak Ridge National Laboratory seemed to offer the opportunity of repeating these measurements under more favorable conditions. A mechanical time-of-flight neutron spectrometer had been built.[2] A copious supply of fission product gases was available from an existing homogeneous reactor. The cross section of Xe^{135} was remeasured using completely different techniques of sample preparation and assay. The spectrometer provided a wider range of energies, with a practical lower limit of 0.01 ev and an upper limit determined principally by the sample strength. In this instance a sample strength of 2.5×10^{18} atoms/cm^2 was achieved with a sample of approximately 500 curies.

The first level was fitted with the Breit-Wigner parameters given below. A search for resonances in the higher energy region with a resolution of 0.5 microsec/meter yielded negative results for the Xe^{135} isotope.

THE TIME-OF-FLIGHT SPECTROMETER

The spectrometer used for this experiment was similar in principle to other fast chopper time-of-flight spectrometers[3–5] and has been described elsewhere.[2] Bursts of neutrons were produced in a collimated beam from a nuclear reactor by a slotted rotor which spun about a horizontal axis parallel to the beam. Neutrons of different energies were sorted by the differences in their time of arrival at a detector 11.39 meters away. The energy of the neutrons is related to the time of flight by the relation

$$E=(72.3/\tau)^2, \quad (1)$$

where τ is the time of flight in microseconds per meter, and E is the energy in electron volts.

An 84-channel time-sorter was used, the timing being

* A preliminary report of the work described here has been given by S. Bernstein and E. C. Smith, *Proceedings of the International Conference on the Peaceful Uses of Atomic Energy, Geneva, August, 1955* (United Nations, New York, 1956). Paper 591, Vol. 4, p. 153.

‡ Now at Lockheed Aircraft Corporation, Marietta, Georgia.

§ Now at Argonne National Laboratory, Lemont, Illinois.

‖ Now at American Machine and Foundry Company, Greenwich, Connecticut.

† Dr. S. Bernstein of Oak Ridge National Laboratory acted as "editor" and contributor in the preparation of the manuscript of this paper for publication. He did not participate in the experimental work. This procedure seemed desirable because those who took the neutron measurements were no longer at Oak Ridge National Laboratory at the time of writing.

[1] Bernstein, Shapiro, Stanford, Stephenson, Dial, Freed, Parker, Brosi, Hebert, and DeWitt, Phys. Rev. **102**, 823 (1956).

[2] G. S. Pawlicki, Oak Ridge National Laboratory Report ORNL-1526 (unpublished).

[3] W. Selove, Rev. Sci. Instr. **23**, 350 (1952).

[4] Seidl, Hughes, Palevsky, Levin, Kato, and Sjöstrand, Phys. Rev. **95**, 476 (1954).

[5] F. G. P. Seidl, Brookhaven National Laboratory Report BNL-278, July 1954 (unpublished).

established by a calibrated oscillator triggered by photomultiplier pulses derived from an optical system attached to the rotor assembly. The long-term drift of the oscillator was of the order of one percent, but variations in frequency over an interval of several hours were considerably smaller. After pulses derived from the neutron detector were sorted into a channel representing the appropriate time interval, they were recorded by a scale of 8 and mechanical register. Channel widths could be varied from 3.5 μsec to 160 μsec. The full width at half maximum of the neutron burst could be varied from 3.8 to 70 μsec by changing the rotor speed.

The resolution width of the spectrometer depends[6] on the flight path, burst width, channel width, and uncertainty in flight path due to finite detector length, and detector "jitter" (variations in detection time within the counter). In this instance the detector was a parallel plate ionization chamber 30 cm long, and detector "jitter" is a second-order effect. The resolution in energy varied from 4.8% at 0.01 ev to 7.0% at 30 ev.

The distribution of intensities in the spectrum measured with this spectrometer depends not only on the reactor spectrum and detector efficiency, but also on a chopper cutoff effect. The geometry of the rotor slit-detector system was arranged to prevent an overlap of slow neutrons of one burst with the fast neutrons of the succeeding burst. Thus, the dynamic transmission of the shutter varied so that counting rates decreased monotonically with time of flight and vanished at the time of the next burst. Rotor speeds were selected as a compromise between the resolution desired and the effect on counting rate of the chopper cutoff function.

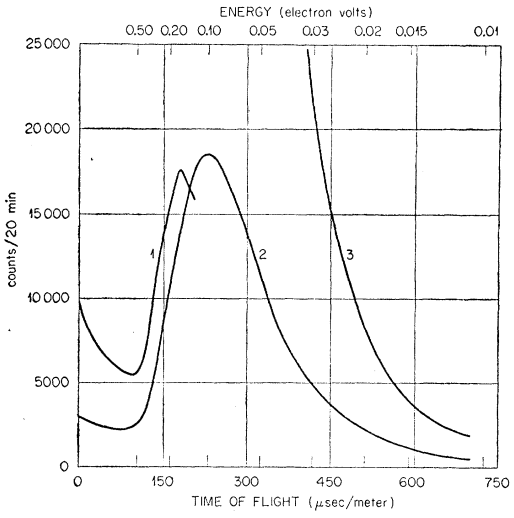

FIG. 1. Typical counting rate curves of the time-of-flight spectrometer for several channel widths and rotor speeds. The data for Curve 1 were taken with 2.45 μsec/meter channel width and 2.02 μsec/meter burst width, resolution width 3.3 μsec/meter. Curves 2 and 3 were obtained with 8.42 μsec/meter channel width and 5.97 μsec/meter burst, resolution width 10.2 μsec/meter.

[6] D. J. Hughes, *Pile Neutron Research* (Addison-Wesley Publishing Company, Inc., Reading, 1953), p. 167.

The total time covered by the 84 channel widths was divided into 84 contiguous parts. This total time could be varied so that the energy region of interest could be covered. Channel widths were selected so that the time of flight to the last channel corresponded to the lowest energy at which measurements were desired. Typical count rate curves for several channel widths and rotor speeds are shown in Fig. 1. Other things being equal, the count rate per channel is proportional to the channel width. As a consequence, when wide channel widths were used to reach low energies, the counting rate in the peak of the thermal neutron spectrum was so high that an appreciable loss of counts occurred due to register dead time. In the instance of the data for curve 2 of Fig. 1, a portion of the beam was blocked with shielding material to reduce the counting rate. When the shield blocks were removed the counting rates in curve 3 were obtained. The shift in the position of the peak between curves 1 and 2 is due to chopper cutoff effects. The very rapid decrease of intensity with time of flight beyond the peak of the Maxwellian distribution is to be noted.

The background counts were primarily caused by the leakage of fast neutrons through the shutter when it was "closed." The ratio of the background counts with and without cadmium in the beam was unity. Therefore, background could be measured in the region below 0.4 ev without stopping the rotor by observing the count rate with cadmium in the beam. Backgrounds for the cases represented in Fig. 1 were 800, 126, and 500 counts per channel per 20 minutes for curves 1, 2, and 3, respectively.

An advantage of the fast chopper time-of-flight spectrometer is the small beam area near the slots of the rotor. For transmission measurements, samples could be accurately positioned in the collimator 4.5 inches from the rotor where the beam area was less than 0.25 cm^2. Transmission measurements on gases were made with a sample holder having an area in the beam of 0.315 cm^2 and a volume of approximately 0.9 cm^3. The sample holder could be positioned remotely within a shield enclosure external to the reactor shield.

SAMPLE PREPARATION

The sample of xenon was prepared[7] by bleeding off some of the gases generated in the homogeneous reactor. The reactor was operated at a power of 200 kilowatts for 24 hours to permit buildup of the Xe^{135} from the parent, iodine. The gases generated in the reactor were then collected on a 5-liter silica gel trap. After approximately 5 hours collection time, the trap was warmed, the krypton was eluted to a holding trap, and the xenon was collected in a trap in a mobile shield. The specimen was then transferred to a hot-cell facility for further purification and for loading into the spectrometer sample holder.

[7] Parker, Martin, Creek, and Lantz, *Proceedings of the International Conference on the Peaceful Uses of Atomic Energy* (United Nations, New York, 1956), Vol. 14, p. 96.

At this facility the specimen was collected in a one-liter flask after several elutions from successively smaller silica gel traps and after passage over a hot barium getter to remove impurity gases. The sample was then compressed by filling the flask with mercury and condensed into the sample holder, which was at liquid nitrogen temperature. The capillary tubes connecting the sample holder with the process apparatus were crimped, cut, and soldered by remote operation. The sample was then warmed and tested for leaks by passing air over the holder and onto a monitored charcoal trap. The soundness of the seals having been established, the sample holder was loaded into a lead shield and transported to the chopper. Fifteen minutes were required to transfer the sample to the holder and to seal it. The mid-time of this period was used as an origin for computing sample decay.

SAMPLE ASSAY

In order to convert measurements of transmission to cross section it was necessary to establish the number of atoms per cm^2 (atomic density-thickness) in the sample at any time. This was done by means of mass spectrometer measurements performed by R. Baldock and J. R. Sites of the ORNL Mass Spectrometer group. The number of Xe^{135} atoms in the sample holder at the time it was loaded was determined by the method of isotopic dilution. After the completion of the cross-section measurements, essentially all the xenon was permitted to decay to its daughter, cesium, which consisted principally of the isotopes of masses 133 and 135. The capillaries were cut near the body of the holder and the cesium was removed by rinsing with water solution until no further beta activity could be detected. The solution was then diluted to 10 milliliters. The relative abundance of the isotopes 133 and 135 was measured in samples obtained by spiking 100 microliter aliquots of the solution containing radiogenic cesium with aliquots from a standard solution of normal CsCl containing 66.5 micrograms of Cs^{133} per milliliter.

TABLE I. Mass spectrometer assay.

Spiking ratio p	Abundance ratio R	Number of determinations
0.000	3.59±0.0095	30
0.250	4.46±0.0052	30
0.500	5.35±0.013	29
0.667	5.84±0.021	27
1.000	6.92±0.016	26
1.500	8.57±0.015	30
2.000	10.31±0.020	20

The concentration of Cs^{135} was obtained from these measurements in the following manner: Let x=number of Cs^{135} atoms per milliliter in the solution of radiogenic Cs; y=number of Cs^{133} atoms per milliliter in the solution of radiogenic Cs; and z=number of Cs^{133} atoms per milliliter in the standard solution. Let R=ratio of 133 to 135 atoms in the specimen measured on a mass spectrometer. If we mix u microliters of the unknown with v microliters of the standard solution we have $n_1=ux$ atoms of Cs^{135} and $n_2=uy+vz$ atoms of Cs^{133}, whence

$$R=n_2/n_1=(uy+vz)/ux=a+bp, \qquad (2)$$

in which $p=v/u$, the ratio of parts of standard solution to parts of solution containing the radiogenic Cs.

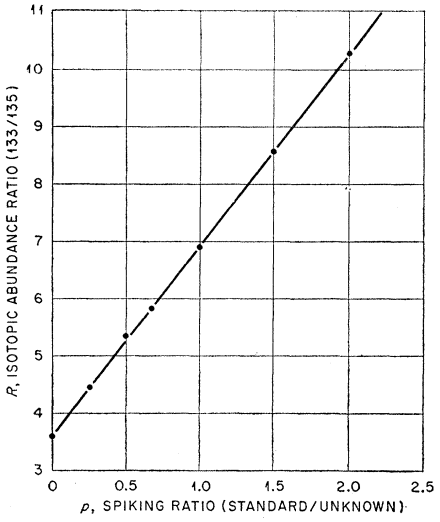

FIG. 2. The ratio of the numbers of Xe^{133} atoms to Xe^{135} atoms as a function of "spiking ratio," the parts of standard solution to parts of radiogenic solution.

Thus R is a linear function of p, and the parameters of the line may be determined by the method of least squares. The spectrometer measurements are shown in Table I and plotted in Fig. 2.

The measured isotopic abundance ratio, R, is given in the second column of Table I for each value of p, the ratio of parts of the solution added to the aliquot of the unknown solution. The errors quoted are the standard deviations of the means of the number of observations of the abundance ratio which were made on a single sample for each value of the spiking ratio. The slope and intercept of the least squares line are $a=3.623\pm0.017$ and $b=3.327\pm0.026$.

There were 66.5 μg/ml in the standard, corresponding to 3.013×10^{17} atoms per ml. Thus the concentration of Cs^{135} in the specimen was 9.06×10^{16} atoms/ml, or a total of 9.06×10^{17} atoms of Cs^{135} in the 10 milliliters containing all the sample. This corresponds to an activity of 514 curies of Xe^{135} activity at the time the holder was loaded.

The volume of the sample holder was calculated from dimensions carefully measured during fabrication. It was 0.900 cm^3. Therefore, the atomic density of Xe^{135} in the holder was 1.007×10^{18} atoms/cm^3. The length of neutron path through the sample was 2.86 cm, so the density-thickness of the sample when it was loaded was 2.88×10^{18} atoms/cm^2.

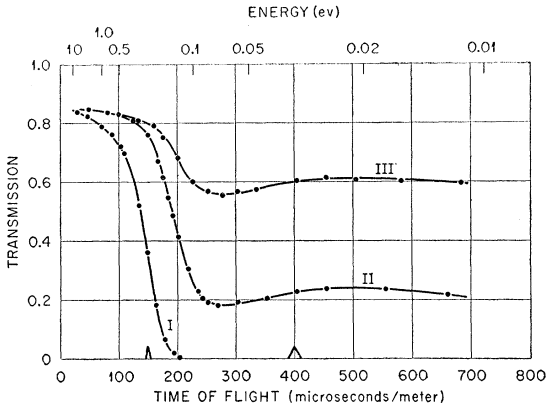

FIG. 3. Typical transmission measurements of Xe^{135} sample as a function of neutron time-of-flight. Curves I, II, and III were obtained for values of $(t-t_0)=0$, 1300 minutes, 2400 minutes. The resolution for I was 3.3 μsec/meter, and for II and III, 10.2 μsec/meter.

CROSS-SECTION MEASUREMENTS

Transmissions were measured by comparison of the counting rate when the sample was positioned in the beam with the counting rate observed when the sample and its holder were removed from the collimator. A typical cycle for transmission measurements was as follows: sample in 20 minutes, open beam 20 minutes, sample in 20 minutes, background 20 minutes. The effect of the decay of the sample during the 20 minutes required for a count with the sample in the beam was negligible. Typical transmission measurements are shown in Fig. 3. Curves II and III were taken with a resolution of 10.2 μsec per meter. The energy region above 0.12 ev was measured, also, with a resolution of 3.3 μsec per meter. The data using this resolution are shown in curve I of Fig. 3. The lowest energy at which measurements were taken was 0.0105 ev.

The transmission of the sample is given by

$$T=T_0e^{-n\sigma}, \tag{3}$$

where

$$n=n_0e^{-\lambda(t-t_0)} \tag{4}$$

and T_0 = transmission of sample holder; t = mid-time of sample-in count; t_0 = origin of time measurements, chosen here to be the mid-time of the first sample-in count; n = number of Xe^{135} atoms per cm^2 at time t; n_0 = number of Xe^{135} atoms per cm^2 at time t_0; λ = decay constant of Xe^{135}.

A weighted mean of 7 independent determinations of the half-life of Xe^{135} is 9.177±0.023 hours.[8] Therefore, $\lambda=(1.2588\pm0.0032)\times10^{-3}$ min^{-1}.

[8] F. Brown and L. Yaffee, Can. J. Chem. **31**, 242 (1953); Newton, Sullivan, Johnson, and Nattorf, *Radiochemical Studies: The Fission Products* (McGraw-Hill Publishing Company, Inc., New York, 1951), National Nuclear Energy Series, Plutonium Project Record, Vol. 9, Book 2, Part V, p. 1028; E. J. Hoagland and N. Sugarman, *ibid.*, p. 1031; Turkevitch, Adams, and Freedman, *ibid.*, p. 1034; Thulen, Bergstrom, and Hedgron, Phys. Rev. **76**, 871 (1949); J. E. Faulkner, Hanford Works (unpublished); P. E. F. Thurlow, Oak Ridge National Laboratory (unpublished).

If the natural logarithm of Eq. (3) is taken, then

$$-\ln T=-\ln T_0+n_0\sigma e^{-\lambda(t-t_0)}. \tag{5}$$

Thus $(-\ln T)$ is a linear function of the variable $\exp[-\lambda(t-t_0)]$ with slope $n_0\sigma$ and intercept $(-\ln T_0)$. Typical data are plotted in this manner in Fig. 4 for several neutron energies. The data at each energy were fitted to Eq. (5) by the method of least squares. The slope, intercept, and their standard deviations were obtained on the Oracle, the ORNL fast digital computer.

Cross sections were calculated by dividing the slopes by n_0 which is given by

$$n_0=Ne^{-\lambda(t_f-t_0)}=2.50\times10^{18}\ \text{atoms/cm}^2. \tag{6}$$

In Eq. (6), (t_f-t_0) = time interval between the mid-time of sample loading and the first sample-in count (113 minutes). $N=2.88\times10^{18}$ atoms per cm^2 is equal to the number of Cs^{135} atoms per cm^2 of the neutron beam, as determined from the assay. The number of Xe^{135} atoms at mid-time of loading is taken to be equal to N. The measured cross sections are shown in Fig. 5 as a function of neutron energy. The resonance character of the cross section is more apparent in the plot of $\sigma\sqrt{E}$ against E which appears in Fig. 6.

ANALYSIS OF THE RESONANCE

The measured cross sections were fitted to a single-level Breit-Wigner formula:

$$\sigma_t=\frac{4\pi\lambda^2g\Gamma_n\Gamma}{4(\epsilon-\epsilon_0)^2+\Gamma^2}, \tag{7}$$

where σ_t = total cross section; λ = neutron wavelength/2π relative to the target nucleus; ϵ = energy of the neutron relative to the nucleus; ϵ_0 = relative energy of the level; $\Gamma_n=\Gamma_n{}^0(\epsilon/\epsilon_0)^{\frac{1}{2}}$ = neutron width; $\Gamma=\Gamma_n+\Gamma_a$ = total level width, full width at half-maximum; Γ_a = capture

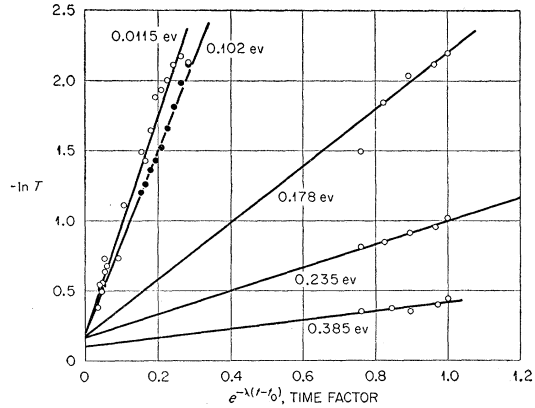

FIG. 4. Logarithm of the sample transmission as a function of the relative number of Xe^{135} atoms for several values of neutron energy. The total cross section of Xe^{135} is proportional to the slopes of the fitted straight lines.

width; $g=(2J+1)/2(2I+1)$; I, J=spins of target and compound nucleus, respectively.

In (7), terms involving potential scattering have been neglected, since they affect the cross section by at most 0.3%. Also an error of 0.75% is introduced into (7) by our use of relative energies rather than center of mass energies. Since the spin of Xe^{135} is probably $\frac{3}{2}$, there are two possible values for the statistical weight factor g, $\frac{3}{8}$ or $\frac{5}{8}$. Doppler effects due to the thermal motion of the Xe nuclei were taken into account in the following manner. Let m=neutron mass, M=mass of target atom, T=target temperature, E=neutron bombarding energy in the laboratory system. Then the distribution function for ϵ may be approximated by[9]

$$w(\epsilon,E)d\epsilon=\tfrac{1}{2}\left(\frac{M}{\pi mkT}\right)^{\frac{1}{2}} \times\frac{\sqrt{\epsilon}}{E}\exp\left[-\left(\frac{M}{mkT}\right)(\epsilon^{\frac{1}{2}}-E^{\frac{1}{2}})^2\right]d\epsilon. \quad (8)$$

The Doppler-corrected Breit-Wigner formula becomes

$$\sigma_D(E)=\int_0^\infty w(\epsilon,E)\sigma_t(\epsilon)d\epsilon, \quad (9)$$

with w given by Eq. (8) and σ_t by Eq. (7). Those values for the parameters Γ_n^0, Γ_a, and ϵ_0 were determined which minimized the weighted sum of squares of deviations of the measured cross section from those computed by Eq. (9). Weights for the data were based on standard deviation of the slopes of the lines used to determine relative cross sections in Eq. (5). A standard procedure was used.[10]

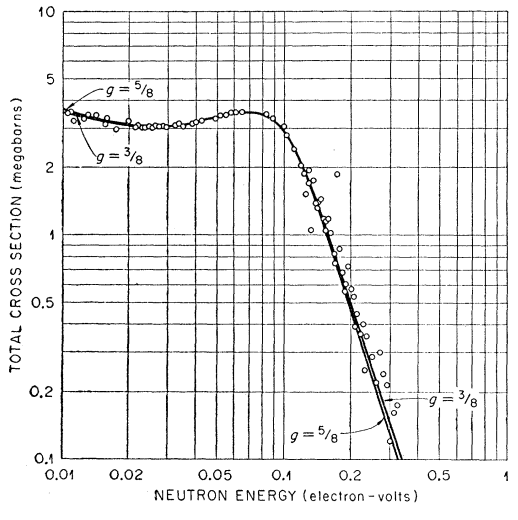

FIG. 5. The total cross section of Xe^{135} as a function of energy. The circles represent the data. The curves are the Doppler-corrected Breit-Wigner formula fitted to the data by least squares for two possible values of the statistical factor, g.

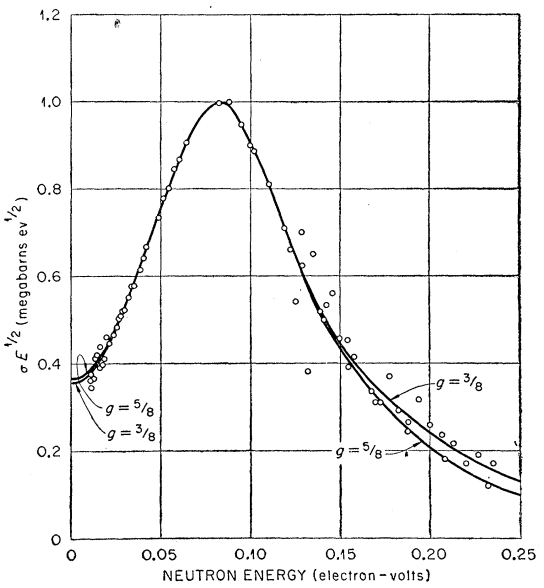

FIG. 6. $\sigma\sqrt{E}$ as a function of neutron energy, E. The data are shown as circles. The curves are derived from the Doppler-corrected Breit-Wigner formula fitted to the data for two possible values of the statistical factor, g.

The data were equally well fitted by the following two sets of parameters:

$$\begin{aligned} g&=\tfrac{3}{8},\\ \epsilon_0&=0.08472\pm0.00027\text{ ev},\\ \Gamma_n^0&=0.03477\pm0.00021\text{ ev},\\ \Gamma_a&=0.08303\pm0.00062\text{ ev},\\ \Gamma^0&=0.11780\pm0.00065\text{ ev}; \end{aligned} \quad (10)$$

or

$$\begin{aligned} g&=\tfrac{5}{8},\\ \epsilon_0&=0.08415\pm0.00028\text{ ev},\\ \Gamma_n^0&=0.02057\pm0.00012\text{ ev},\\ \Gamma_a&=0.09493\pm0.00071\text{ ev},\\ \Gamma^0&=0.11550\pm0.00072\text{ ev}. \end{aligned} \quad (11)$$

The standard deviations given were calculated from the deviations of the data points from the calculated values. $\Gamma^0=\Gamma$ at $\epsilon=\epsilon_0$. If center-of-mass coordinates instead of relative coordinates had been used in (7), then ϵ_0, Γ_0, and Γ_n^0 should be reduced by approximately 1%. However, values of cross sections computed at corresponding relative and center-of-mass energies would remain unchanged.

Curves calculated from these values are shown in Fig. 5 and Fig. 6, together with the experimental data. It is to be noted that the curves differ in shape only at

[9] We are indebted to Dr. R. K. Osborn of Oak Ridge National Laboratory for the derivation of Eq. (8). It differs from some others in that all three components of the thermal motion were taken into consideration.

[10] F. Garwood, Biometrika **33**, 46–48 (1941, 1942).

extreme energies. The cross section at the peak is computed from Eq. (7) to be, for $g=\frac{3}{8}$,

$$\sigma_t=3.40\times10^{-18}\text{ cm}^2,\quad \sigma_a=2.40\times10^{-18}\text{ cm}^2; \tag{12}$$

and for $g=\frac{5}{8}$,

$$\sigma_t=3.42\times10^{-18}\text{ cm}^2,\quad \sigma_a=2.81\times10^{-18}\text{ cm}^2. \tag{13}$$

The 2200-m/sec values are, for $g=\frac{3}{8}$,

$$\sigma_t=3.07\times10^{-18}\text{ cm}^2,\quad \sigma_a=2.50\times10^{-18}\text{ cm}^2; \tag{14}$$

and for $g=\frac{5}{8}$,

$$\sigma_t=3.08\times10^{-18}\text{ cm}^2,\quad \sigma_a=2.76\times10^{-18}\text{ cm}^2. \tag{15}$$

ANALYSIS OF ERRORS

The errors quoted for the Breit-Wigner parameters are the standard deviations obtained from the least-squares fit alone. There are in addition a number of other sources of error which were not included in the statistical analysis. Uncertainties in the energy scale would be reflected in the value of ϵ_0 and the total width Γ. Although the oscillator controlling the timing was calibrated a short time before the measurements were made, it was subject to drifts which make the energy scale, or the values of ϵ_0 and Γ, uncertain to perhaps 1.5%. Uncertainty in the absolute magnitude of the cross section is reflected most sensitively in the value of Γ_n^0. The principal random sources of error of these are the following: mass spectrometer assay, 0.8%; volume to which sample was diluted, 0.5%; volume of sample holder, 1%; neutron path length in sample, 0.1%; Xe^{135} half-life, 0.25%; effective time of sample loading, 0.5%. Considering all the random sources of error, the estimate of the errors in the Breit-Wigner parameters are about 1.5% for ϵ_0 and Γ, and about 1.5% for Γ_n^0. Cross sections computed from Eq. (10) and Eq. (11) for values of ϵ very near or very far from ϵ_0 will then be about $\pm2.0\%$ in error.

In addition to random errors, there are possible systematic errors. One error is due to the possibility that not all of the Cs^{135} was removed from the sample holder. It is estimated that about 1% of the sample used for the neutron measurements may not have appeared in the assay samples. To take this possibility into account, the cross sections given by the use of the constants (10) and (11) should be reduced about 1%. A second error is the possibility that CsCl used in the standard solution retained some water of hydration in spite of the fact that it was repeatedly baked and weighed. It may have contained small amounts of impurity solids, also. If water or other impurities were present, then the values of the cross section given by the use of the constants (10) and (11) should be increased.

MEASUREMENTS AT HIGHER ENERGIES

Measurements were made at energies greater than 1.5 ev with a resolution of 0.5 microsecond/meter. No additional resonances in Xe^{135} were detected. The significance of these measurements can be evaluated by the following considerations. The area under a resonance for a thin sample is

$$A_E=(2\epsilon_0^{\frac{3}{2}}/72.3)A_\tau=2\pi^2 g\lambda\!\!\!^{-}_0{}^2\Gamma_n^0 n, \tag{16}$$

in which A_E is the area of the transmission dip in ev, A_τ is the area in microseconds per meter, and the remaining symbols are defined above. Using the appropriate numerical factors, Eq. (16) may be rearranged to give an expression for the reduced width, γ_n^0, in terms of the area of the transmission dip:

$$\gamma_n^0\equiv\Gamma_n^0/\epsilon_0^{\frac{1}{2}}=8\times10^{-3}A_\tau\epsilon_0^2, \tag{17}$$

in which an average value of $\frac{1}{2}$ has been used for g. Under the conditions of the measurements, an A_τ of about 0.025 microsecond per meter could certainly be detected. Thus, all resonances having a reduced width

$$\gamma_n^0\geqslant 2\times10^{-4}\epsilon_0^2 \tag{18}$$

would have been detected. For example, at 10 ev, 30 ev, and 100 ev, all resonances with reduced widths greater than 0.02, 0.2, and 2.0 $(\text{ev})^{\frac{1}{2}}$, respectively, would have been detected. In the energy interval zero to 1 ev, all levels of reduced neutron width greater than 2×10^{-4} $(\text{ev})^{\frac{1}{2}}$ should certainly have been observed, except perhaps those which might occur within about one resolution width or $\frac{1}{2}\Gamma$ of the 0.08-ev resonance.

DISCUSSION

On the basis of current statistical theories of the properties of nuclear energy levels, some qualitative statements can be made about the experimental observations over the range of the measurements, zero to about 6000 ev. For the purpose of these qualitative considerations, the assumption of a random distribution of energy levels should be adequate.

The neutron cross section of Xe^{135} is high at low energies because of the concurrence of two factors: the existence of a resonance at low energy; a high value for the ratio (Γ_n^0/Γ). There are a number of resonances at low energies in this mass range, but both the capture and reduced neutron widths of these are not unusually large. The average number of energy levels in the energy interval dE is (dE/D), in which D is the average distance between levels. As shown below, for the compound nucleus Xe^{136}, D is probably of the order of magnitude of 600 ev. The probability of finding a level between 0 and 1 ev (where the neutron wavelength has its greatest values) is roughly (1/600). The occurrence of the level in this interval at 0.08 ev is, therefore, quite fortuitous.

Porter and Thomas[11] give a distribution function for reduced neutron widths in terms of $(\gamma_n^0/\bar{\gamma}_n^0)$, the ratio

[11] C. E. Porter and R. G. Thomas, Phys. Rev. **104**, 483 (1956).

of the reduced neutron width at resonance energy to the average reduced neutron width for all the levels of a given nuclide. According to this distribution, 10% of the γ_n^0's are greater than $2.71\bar{\gamma}_n^0$, and 10% less than $(1/64)\bar{\gamma}_n^0$. The cloudy-crystal-ball model of the nucleus predicts[12] a value of $(\bar{\gamma}_n^0/D)$ of about 2×10^{-4} $(\text{ev})^{\frac{1}{2}}$ for nuclei of mass number about 135. Using this relationship, the probabilities from the Porter-Thomas distribution given above and the measured value 0.1 ev for γ_n^0, the qualitative statement can be made that D has a value between 180 ev and 10^5 ev with a probability of 80%. The semiempirical theory of Newton[13] gives an expression for D in terms of the mass number and excitation energy of the compound nucleus after neutron capture. Using the measured values[14] for the masses of Xe^{135} and Xe^{136}, and the mass of the neutron, the binding energy of a neutron in Xe^{136} is 7.89 Mev. Using this value in Newton's expression along with his corrections and tables, a value of about 600 ev is predicted for the observed level spacing. This value is perhaps good to within a factor of 3.

The significant feature of the measurements at higher energies is that no resonances other than the large one at 0.08 ev were observed between zero and 6000 ev. Since D is roughly 600 ev, and $6000/600=10$, at least "several" levels should exist in this interval. Only one was found. Any others were probably not observed because the thickness of sample used (corresponding to 514 curies), and the resolution of the apparatus, were not adequate.

A rough estimate of the average number of levels, m, in the interval 0 to E, is given by

$$m=\int^{E}\left[\int^{\infty}P(x)dx\right]\frac{dE}{D}. \tag{19}$$

The $\int_b^\infty P(x)dx$ gives the fraction of reduced neutron widths greater than the lower limit of observable widths determined by conditions of our experiment. The symbol $b=2\times10^{-4}(E^2/\bar{\Gamma}_n^0)$ is the lower limit as determined by Eq. (18). $P(x)$ is the Porter-Thomas distribution function. We have calculated m for three assumed values, $\bar{\Gamma}_n^0=0.03$, 0.1, and 0.3 $(\text{ev})^{\frac{1}{2}}$. The value 0.1 is approximately the average of the two possible values measured for the low-energy resonance. For these values of $\bar{\Gamma}_n^0$ we find $m=0.02$, 0.03, and 0.06, respectively. Thus for $\bar{\Gamma}_n^0=0.1$, the probability, according to the Poisson distribution law, of observing only 1 level is 0.029. The probability of observing 2 or more levels is 5×10^{-4}. Thus, on the average, if the cross section of 30 nuclides with a D of about 600 ev, were measured under our experimental conditions, a single resonance should be found between 0 and 6000 ev in only one of these nuclides. Xe^{135} happens to be the one. The chances of observing 2 or more levels under our conditions are so small that it is not at all surprising that they were actually not observed. In order to observe additional levels at higher energies, a much thicker sample and an instrument of much higher resolution are needed. Of course, all of the probabilities mentioned here are subject to variation by a factor of about 3 because of the uncertainty in D.

ACKNOWLEDGMENTS

The computations of the least-squares fit to the Breit-Wigner equation were programmed for the ORACLE and carried through by N. M. Dismuke. Thelma Arnette assisted in obtaining the data and the programming. F. J. Muckenthaler and F. N. Watson assisted in taking the measurements. We are indebted to Dr. J. A. Harvey and Dr. L. Dresner for valuable discussion.

[12] V. F. Weisskopf, Revs. Modern Phys. **29**, 174 (1957).
[13] T. D. Newton, Can. J. Phys. **34**, 804 (1956).
[14] W. H. Johnson and A. O. Nier, Phys. Rev. **105**, 1014 (1957).